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A Robust Winner Determination Problem for Combinatorial Transportation Auctions under Uncertain Shipment Volumes

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Abstract. Combinatorial auctions are widely used for the procurement of transportation services. In these auctions, shippers act as auctioneers who need to outsource a number of transportation services to external carriers. Carriers compete by submitting bids on packages of shippers' requests. After receiving all carriers' bids, the shipper solves the well-known winner determination problem (WDP) in order to determine winning bids. This paper considers the WDP in a context where shipment volumes are not known with certainty. Based on the bi-level characteristic of the problem, a 2-stage robust formulation is proposed and solved using a constraint generation algorithm. Experimental results show a good performance of the proposed approach. We also evaluate, through an experimental analysis, the benefits of considering a robust rather than a deterministic WDP.

Keywords. Combinatorial auctions, truckload procurement, uncertain shipment volumes, 2-stage robust optimization, constraint generation.

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1. Introduction

During the last decade, combinatorial auctions have become very attractive market mechanisms for the procurement of truckload transportation services. In these auctions, shippers act as auctioneers who need to outsource a number of transportation services to some external carriers. Carriers, the bidders of the auction, compete by submitting bids on shippers' requests. The output of the auction is a set of winning carriers with which the shipper will engage on a long-time period (one to three years).

Shipper requests derive from a whole process in which the shipper tries to forecast its transportation needs for the upcoming period (Caplice and Sheffi, 2006). A shipper request is a transportation contract in which the shipper specifies the pick-up and delivery location pair, generally called a lane, a volume to be shipped on this lane, and some other information on shipping conditions, specific equipments, etc. Total truckload (TL) transportation, considered in this paper, assumes that shipments are delivered directly from pick-up to delivery locations.

Several carriers are invited to participate in the auction. The shipper may restrict the set of participating carriers through a pre-auction selection phase. A scoring system can be used in this case to rate and rank carriers with respect to some attributes such as on-time delivery, on-time loading, billing accuracy, financial stability, or equipment compatibility (Gibson et al., 1995). The carriers with the highest score are certified as partners and will be invited to participate into the auction. Commonly, the shipper includes most incumbent carriers and a small number of new carriers (Caplice and Sheffi, 2006).

In combinatorial auctions, carriers are permitted to submit bids on a package of lanes. This would encourage carriers to multiply bids and allows them to express their preferences for any combination of lanes they want to acquire. For instance, to reduce empty repositioning costs, a carrier may prefer to move shipments from i to j, jointly with shipments from j to i

rather than serving them separately. A bid submitted by a carrier can be defined in several ways depending on the bidding language used and on the amount of information required. Caplice and Sheffi (2006) identified seven types of bids currently used within transportation auctions. For our problem, we consider the so-called *flexible package bid*, that is a bid in which the carrier specifies for each lane within the package, the price asked for serving a one unit volume on that lane as well as some restrictions on per/lane or total volume shipments. Section 2.1 gives more detail on the bids considered in this paper. The way these bids are constructed by each carrier, known as the *carrier bid construction problem*, is not addressed in this paper. We refer the reader to the papers by Song and Regan (2005) and Lee et al. (2007), for example, for more details on this topic.

After receiving all carriers' bids, the shipper solves the well-known winner determination problem (WDP) in order to determine winning bids. Winning bids must satisfy the shipper's transportation needs while minimizing its total transportation cost. Combinatorial WDP was proven to be NP-hard (Rothkopf et al. (1998) established its equivalence with the weighted set packing problem). Different variants of the problem have been investigated depending on the trading context in which the combinatorial auction is applied. Surveys and discussions on WDP in different contexts are available, for example, in De Vries and Vohra (2003), Sheffi (2004) and Abrache et al. (2007). Caplice and Sheffi (2006) focus on TL transportation markets and propose different formulations for the WDP in this particular context. Real-life application studies for transportation markets are also reported in Ledyard et al. (2002) for Sears Logistics, and Elmaghraby and Keskinocak (2002) for Home Depot.

Caplice and Sheffi (2003) pointed out that uncertainty in problem data may have bad consequences and compromise the efficiency of a solution. Uncertainty may concern carriers' ask prices, shippers' demands, carriers' capacities, etc. This paper deals with the WDP in the context where the shipper's demand on the volume to be transported on each lane is not known with certainty. Taking into account the uncertainty on shipment volumes is of great importance in the shipper decision process. Indeed, when lanes are proposed to external carriers, the shipper has only forecast demands and poor estimates can result in considerable losses. For example, over-estimating lanes volumes can be damaging when the shipper guarantees a minimum amount to the carrier. Under-estimating these volumes may yield large costs as the shipper would ask a carrier on a spot market (at extra costs) to ship the remaining non-served volume. We propose a 2-stage robust formulation to address this issue. The decisions to be made are indeed of two types: a lane-allocation decision which defines the lanes won by each carrier (this decision is taken in an uncertain environment, before the actual volumes are known), and a volume-allocation decision that specifies the volume assigned to each carrier on each lane won, computed once the lanes volumes have been revealed (the recourse variables).

Multi-stage robust optimization has been introduced by Ben-Tal et al. (2004) and Minoux (2009) for linear and convex programming and is inspired by stochastic programming. Basically, there are two major differences between these two approaches. The first one concerns the description of the uncertainty. In fact, unlike stochastic programming, in robust optimization, there is no probability law available on the uncertain parameters and the uncertainty is described by discrete scenarios or polyhedral sets (generally interval numbers). The second major difference comes from the recourse actions that are taken on the worst case event for robust optimization instead of the expectancy of all the possibilities, as is the case for stochastic programming.

In the literature, a few studies attempt to handle uncertainty in combinatorial auction problems. Ma et al. (2010) propose a two-stage stochastic formulation to address the uncertainty on shipment demands. The problem addressed in our paper considers almost the same environment as Ma et al.

(2010) but proposes another solution approach. Boutilier et al. (2004) and Kameshwaran and Benyoucef (2009) consider the maximum regret criteria to handle uncertain costs in the bid evaluation. Recently, Tsai et al. (2011) estimate costs uncertainty in the spot market. To the best of our knowledge, this is the first time multi-stage robust optimization is used to model uncertainty in combinatorial auctions. The objective of this paper is to show the relevance of such an approach to produce robust solutions immunized against uncertainty. As the uncertainty is located on the right hand side of the constraints, we use a 2-stage robust formulation introduced by Thiele et al. (2009) and Gabrel et al. (2011).

The paper is structured as follows. In Section 2, we present the assumptions and notations of the deterministic winner determination problem considered. Section 3 describes the uncertainty model and the 2-stage robust formulation. A constraint generation algorithm for solving the resulting min-max problem is also presented. Experimental results are reported and discussed in Section 4. Section 5 concludes the paper.

2. Definition of the deterministic problem

2.1. Context and assumptions

We consider a combinatorial auction mechanism where a single shipper (the auctioneer) has to outsource a number of its transportation operations to a set of participating carriers (the bidders), denoted T. A shipper request is defined by a pair (l, d_l), where l is an origin-destination pair (a lane) and d_l is the volume to be transported on l. The set of all lanes required by the shipper is denoted L. The shipper submits its requests to the participating carriers. The latter make offers in forms of combinatorial bids in order to win the shipments in which they are interested.

Each carrier's bid gathers the set of lanes it offers to serve, the price asked for shipping one volume unit on each lane, and some bounds on the minimum and maximum volume quantities that the carrier offers to transport. A minimum volume condition represents the guarantee that the shipper gives to the carrier a minimum amount of business. Commonly, if during operations, the actual demand of a lane is lower than that predicted by the shipper, the latter is obliged to pay the carrier a price equivalent to the minimum volume requested by the carrier in its winning bid. The maximum volume condition reflects the carrier capacity.

Formally, each carrier $t \in T$ submits a set of bids B_t. A bid $b \in B_t$ is defined by a tuple $(\mathcal{L}_{tb},\ [\mathrm{LB}_{tb}, \mathrm{UB}_{tb}],\ c_{tb})$, where $\mathcal{L}_{tb}$ is the set of lanes that carrier t offers to serve in bid b, LB_{tb} is the minimum volume that the shipper guarantees to the carrier if bid b wins, UB_{tb} is the maximum volume that the carrier can ship if bid b wins, and c_{tb} is the price asked by carrier t in bid b for transporting one unit volume on each lane $l \in \mathcal{L}_{tb}$. For example, consider a bid b_1 submitted by carrier t_1 and defined by $(\{1, 2\},\ [1, 3],\ 4)$. This bid implies that t_1 offers to ship a volume varying between 1 and 3 units on each of the lanes 1 and 2 with a price of 4 for each unit volume shipped.

We consider a context in which a carrier can submit any number of bids it wants. However, in the final allocation, each carrier can be awarded at most one bid. In other words, XOR bids are assumed (Nisan, 2006). Moreover, if a carrier wins a bid, it must ensure the service for all the lanes covered by this bid. It is also assumed that each lane can be served by at most one participating carrier.

In the problem considered, bids are given data and their construction are not addressed. The paper rather focuses on the winner determination problem. The objective is to choose bids and associated volumes that minimize the shipper transportation costs and satisfy the transportation demand on shipper's lanes. In case carriers' bids are not able to satisfy all lanes' demands, the shipper has the possibility to call a carrier from the spot market to ensure the shipment of the remaining unsatisfied demands. We assume that it is always more expensive for the shipper to satisfy the demand by the spot carrier than by negotiating contracts with the carriers participating in the auction . In the following, we denote by ce_l the cost of shipping one unit volume on lane l by a spot carrier.

Moreover, based on historical data, the shipper may promote or penalize participating carriers depending on the quality of services they provided in the past. Formally, the shipper associates to each carrier $t \in T$ a performance factor, denoted p_t, based on its service quality. Parameters p_t take values within the interval $[-1, 1]$. The higher is this value, the less reliable is the carrier.

We also consider a context where the shipper sets minimum and maximum volumes to win for each carrier $t \in T$, denoted q_t and Q_t, respectively. As reported by Caplice and Sheffi (2006), a maximum volume allocation avoids a shipper to rely too heavily on a single carrier, where as a minimum volume allocation permits a shipper to remain a significant customer for the carrier.

Another commonly used constraint in transportation auctions, that we consider here, is the restriction on the total number of winning carriers (Caplice and Sheffi, 2006). In the following, we denote N_{min}, respectively, N_{max}, the minimum, respectively, the maximum number of winning carriers in the final allocation. Setting such restrictions avoids one carrier winning the lion's share.

2.2. Mathematical formulation

To model the WDP described in Section 2.1, we use three sets of decision variables: a set of binary variables x_{tb}, a set of continuous variables y_{tb}, defined both for each carrier t and each bid b submitted by this carrier, and a set of e_l variables defined for each lane $l \in L$. Variable $x_{tb} = 1$ if bid b offered by carrier t wins and $x_{tb} = 0$, otherwise. The continuous variable y_{tb} represents the volume assigned to carrier t on each lane covered by bid b, if bid b wins. In the previous example, if carrier t_1 wins bid b_1, then, $x_{t_1 b_1} = 1$, $y_{t_1 b_1} \in [1, 3]$, and the shipper has to pay the carrier t_1 a price equal to $4y_{t_1 b_1}$.

Variable e_l represents the demand of lane l uncovered by winning bids. That is, the volume assigned to spot carriers on lane l.

We also define a constant parameter a^l_{tb}, for each carrier $t \in T$, each bid $b \in B^t$, and each lane $l \in L$. Parameter a^l_{tb} equals 1 if lane l belongs to the set of lanes $\mathcal{L}_{tb}$ submitted by carrier t in its bid b, and 0 otherwise.

The deterministic winner determination problem can be formulated using model (W), as follows:

$$
(\mathrm{W})\begin{cases}
\min & \sum_{t\in T}\sum_{b\in B_t}(1+p_t)c_{tb}\,y_{tb}+\sum_{l\in L}ce_l\,e_l & (1)\\
\text{s.t.} & \sum_{t\in T}\sum_{b\in B_t}a^l_{tb}\,y_{tb}+e_l\geq d_l,\quad l\in L & (2)\\
& \mathrm{LB}_{tb}\,x_{tb}\leq y_{tb}\leq \mathrm{UB}_{tb}\,x_{tb},\; t\in T,\; b\in B_t & (3)\\
& \sum_{b\in B_t}x_{tb}\leq 1,\quad t\in T & (4)\\
& \sum_{t\in T}\sum_{b\in B_t}a^l_{tb}\,x_{tb}\leq 1,\quad l\in L & (5)\\
& \mathrm{N_{min}}\leq\sum_{t\in T}\sum_{b\in B_t}x_{tb}\leq \mathrm{N_{max}} & (6)\\
& \mathrm{q}_t\leq\sum_{b\in B_t}y_{tb}\leq \mathrm{Q}_t,\quad t\in T & (7)\\
& x_{tb}\in\{0,1\},\; y_{tb}\geq 0,\quad t\in T,\; b\in B_t & (8)\\
& e_l\geq 0,\quad l\in L &
\end{cases}
$$

The objective function (1) minimizes the total transportation cost of the shipper. This cost includes that paid to the winning carriers of the auction and that paid to spot carriers. Constraints (2) ensure that the volume requested by the shipper on each lane is satisfied. Constraints (3) impose the minimum and maximum volume restrictions on the volume allocated to carriers if the corresponding bids win. These constraints also link x_{tb} to y_{tb} variables assigning a null volume to each losing bid b (i.e., if $x_{tb} = 0$ then $y_{tb} = 0$). Constraints (4) limit the number of bids won by a carrier to one maximum (XOR bids). Constraints (5) make sure that each lane l is affected to one carrier at most. Constraints (6) set bounds on the number of winning carriers in the final allocation. Constraints (7) specify the minimum and maximum volume that each carrier $t \in T$ is allowed to ship. Constraints (8) are binary, respectively, non-negative, constraints on x_{tb}, respectively, y_{tb} and e_l variables.

It is common, in practice, that the volume shipments requested on the lanes of the network are not known with certainty when carriers bids have to be selected. Only estimations are available at this step. Such uncertainty has a considerable impact on the behaviour of the shipper, especially if the effective demands are larger than those expected (due to the extra cost induced by spot carriers). Thus, a robust solution is of great interest for the shipper to avoid bad surprises. In the next section, we describe the uncertainty on the lanes demand d_l in problem (W) and we propose a 2-stage robust formulation for it.

3. The 2-stage robust formulation

As no probability distribution is available on the uncertain demands, we choose to represent the uncertainties using interval numbers. Namely, each demand d_l on lane $l \in L$ is known to belong to an interval $[\overline{d}_l - \hat{d}_l, \overline{d}_l + \hat{d}_l]$, where $\overline{d}_l$ is the nominal demand and $\hat{d}_l \geq 0$ is the maximum deviation. In the following, we denote (W^d) the winner determination problem for a fixed $d \in [\overline{d} - \hat{d}, \overline{d} + \hat{d}]$ and $opt(\mathrm{W}^d)$ the corresponding optimal value. It is assumed that (W^d) is feasible for all $d \in [\overline{d} - \hat{d}, \overline{d} + \hat{d}]$.

Applying a robust methodology on the uncertain WDP leads to consider the worst scenario demands (the larger ones). Nevertheless, to be closer to reality and avoid uncommon worst case demands, we define the budget of uncertainty Γ representing the range of deviation of the demands from the nominal values. The idea is to restrict the total number of uncertain parameters that deviate from their nominal values. This approach was initially introduced by Bertsimas and Sim (2003, 2004), which consider uncertainties on the costs and the left hand sides of the LP constraints. This approach was then extended by Thiele et al. (2009), and Gabrel and Murat (2010) to problems with uncertain right hand sides. As the uncertainties are on lanes demands in (W^d) (precisely in the right hand sides), we assume that Γ can take any value in the range $[0, |L|]$ (observe that $\Gamma = |L|$ implies that the demand deviates from its nominal value for all the lanes). Indeed, this approach gives a column-wise description of the uncertainty that is concentrated on the right hand side of the problem. The parameter Γ is set depending on the preferences of the decision maker (the shipper) and the level of conservatism it wants to achieve. The extreme value of $\Gamma = 0$ corresponds to the nominal problem (no uncertainty in the model), and $\Gamma = |L|$ describes the worst case problem with the greatest demands.

In this paper, we use a 2-stage robust framework to build a robust formulation for the uncertain problem (W^d). In the proposed formulation, the decisions have to be taken in two steps: lanes have to be allocated to carriers (through their winning bids) then, once the demand is revealed, the volumes of shipments associated to each wining bid (and consequently, a carrier) are determined. Thus, we consider x_{tb} as first stage variables and $y_{tb},\ e_l$ as second stage, also called recourse, variables. Under the assumptions above, the robust winner determination problem, denoted $\mathrm{W}_{rob}(\Gamma)$, consists in selecting the winning bids and the associated volumes at the minimum cost, such that the *worst demand* -lying in the uncertainty set- is satisfied. The robust WDP problem is formulated as follows:

$$\mathrm{W}_{rob}(\Gamma) \begin{cases} \min & opt(R(x,\Gamma)) \\ \text{s.t.} & \sum\limits_{b \in B_t} x_{tb} \leq 1, \quad t \in T \\ & \sum\limits_{t \in T} \sum\limits_{b \in B_t} a_{tb}^l\, x_{tb} \leq 1, \quad l \in L \\ & \mathrm{N}_{\min} \leq \sum\limits_{t \in T} \sum\limits_{b \in B_t} x_{tb} \leq \mathrm{N}_{\max} \\ & x_{tb} \in \{0,1\}, \quad t \in T,\ b \in B_t \end{cases}$$

where $opt(R(x,\Gamma))$ represents the optimum value of the recourse problem:

$$R(x,\Gamma)\left\{\max_{d\in\mathcal{U}(\Gamma)}\min_{(y,e)\in\mathcal{Y}(x)}\sum_{t\in T}\sum_{b\in B_t}(1+p_t)c_{tb}y_{tb}+\sum_{l\in L}ce_l e_l\right.$$

The uncertainty set $\mathcal{U}(\Gamma)$ is defined by:

$$\mathcal{U}(\Gamma)=\{d\in\mathbb{R}^{|L|}:d_l=\overline{d}_l+z_l\,\hat{d}_l,\ l\in L,\ z\in\mathcal{Z}(\Gamma)\}\tag{9}$$

where

$$\mathcal{Z}(\Gamma)=\{z\in\mathbb{R}^{|L|}:\sum_{l\in L}z_l\leq\Gamma,\ 0\leq z_l\leq 1,\ l\in L\}\tag{10}$$

The feasible set $\mathcal{Y}(x)$ includes all vectors (y,e) satisfying the following constraints:

$$\sum_{t\in T}\sum_{b\in B_t}a^l_{tb}\,y_{tb}+e_l\geq d_l,\quad l\in L\tag{11}$$

$$y_{tb}\geq \mathrm{LB}_{tb}\,x_{tb},\ t\in T,\ b\in B_t\tag{12}$$

$$y_{tb}\leq \mathrm{UB}_{tb}\,x_{tb},\ t\in T,\ b\in B_t\tag{13}$$

$$\sum_{b\in B_t}y_{tb}\geq \mathrm{q}_t,\quad t\in T\tag{14}$$

$$\sum_{b\in B_t}y_{tb}\leq \mathrm{Q}_t,\quad t\in T\tag{15}$$

$$y_{tb}\geq 0,\quad t\in T,\ b\in B_t;\quad e_l\geq 0,\quad l\in L\tag{16}$$

The 2-stage robust problem $\mathrm{W}_{rob}(\Gamma)$ is a min-max-min problem, that is difficult to handle in this shape. In the following, we propose a generation algorithm to solve it, but first let's focus on the recourse problem.

3.1. Linearization of the recourse problem

At optimality, $opt(R(x,\Gamma))$ represents the transportation cost over the whole network for the winning bids x and the associated volumes that satisfy the Γ worst demands. As the problem is feasible for all uncertain $d \in \mathcal{U}(\Gamma)$ by assumption, the optimal solution of the recourse problem can be obtained by considering its dual (using the strong duality theorem). The dual of the inner minimization problem in $R(x,\Gamma)$ is written as follows:

$$Q(x,\Gamma)\begin{cases} \max & \sum_{l\in L}\overline{d}_l u_l + \sum_{l\in L}\hat{d}_l u_l z_l + \sum_{t\in T}\sum_{b\in B_t}\mathrm{LB}_{tb}x_{tb}v_{tb} - \\ & \sum_{t\in T}\sum_{b\in B_t}\mathrm{UB}_{tb}\,x_{tb}\,w_{tb} + \sum_{t\in T}\mathrm{q}_t\,g_t - \sum_{t\in T}\mathrm{Q}_t\,h_t \\ \text{s.t.} & \\ & \sum_{l\in L}a^l_{tb}u_l + v_{tb} - w_{tb} + g_t - h_t \le (1+p_t)c_{tb}, \quad t\in T,\ b\in B_t \\ & u_l \le ce_l, \quad l\in L \\ & \sum_{l\in L} z_l \le \Gamma \\ & 0\le z_l \le 1, \quad l\in L \\ & u_l \ge 0,\ l\in L;\quad v_{tb},\ w_{tb},\ g_t,\ h_t \ge 0, \quad t\in T,\ b\in B_t \end{cases}$$

where $u_l,\ v_{tb},\ w_{tb}, g_t$, and h_t are the dual variables of the minimization problem associated with constraints (11), (12), (13), (14) and (15), respectively. Notice that problem $Q(x,\Gamma)$ is bilinear and thus NP-hard and non convex (see Floudas and Parlados (1995) and Vavasis (1991)). Nevertheless, following the same methodology as in Gabrel et al. (2011), we propose a linearization of this problem, by considering Γ as an integer number in $\{0,\ldots,L\}$. This implies that z_l variables can be considered as $0-1$ variables (from a property of bilinear problems, for details see Gabrel et al. (2011)). This allows us to replace the products $u_l z_l$ by new variables s_l and add constraints that enforce s_l to be equal to u_l if $z_l = 1$ and 0 otherwise (see Glover and Woolsey (1974)). Hence, in the optimal solution, either d_l will be equal to $\overline{d}_l$ or $\overline{d}_l + \hat{d}_l$. Recall that Γ represents the number of constraints for which the right hand sides, namely the shipment demands, deviate from their nominal values.

The recourse problem is thus reduced to the following mixed integer program:

$$Q'(x,\Gamma)\begin{cases} \max & \sum_{l\in L}\bar{d}_l u_l + \sum_{l\in L}\hat{d}_l s_l + \sum_{t\in T}\sum_{b\in B_t}\mathrm{LB}_{tb}x_{tb}v_{tb} - \\ & \sum_{t\in T}\sum_{b\in B_t}\mathrm{UB}_{tb}x_{tb}w_{tb} + \sum_{t\in T}\mathrm{q}_t g_t - \sum_{t\in T}\mathrm{Q}_t h_t \\ \text{s.t.} & \\ & \sum_{l\in L} a^l_{tb}u_l + v_{tb} - w_{tb} + g_t - h_t \le (1+p_t)c_{tb}, \quad t\in T,\ b\in B_t \\ & u_l \le ce_l, \quad l\in L \\ & \sum_{l\in L} z_l \le \Gamma \\ & s_l \le \mathrm{M}\, z_l, \quad l\in L \\ & s_l \le u_l, \quad l\in L \\ & z_l\in\{0,1\};\ s_l,\ u_l\ge 0, \quad l\in L \\ & v_{tb},\ w_{tb},\ g_t,\ h_t \ge 0, \quad t\in T,\ b\in B_t \end{cases}$$

where M is a large constant. To reduce the integrality gap when solving $Q'(x,\Gamma)$, the constants M have to be as small as possible. Generally, setting these values can be a hard task, but here, as u_l variables are bounded, one can assign the value of ce_l to M.

3.2. Constraint generation algorithm

Now, we are able to solve the robust problem $\mathrm{W}_{rob}(\Gamma)$. Indeed, we observe that the optimum solution of the recourse problem $Q'(x,\Gamma)$ is reached at one extreme point of its feasible set. This allows us to rewrite the robust problem as:

$$
\mathrm{W}rob(\Gamma)' \begin{cases} \min & A \\ \text{s.t.} & A \geq \sum_{l \in L} \bar{d}_l u_l^\sigma + \sum_{l \in L} \hat{d}_l s_l^\sigma + \sum_{t \in T} \mathrm{q}_t g_t^\sigma - \\ & \sum_{t \in T} \mathrm{Q}_t h_t^\sigma + \sum_{t \in T} \sum_{b \in B_t} \mathrm{LB}_{tb} x_{tb} v_{tb}^\sigma - \\ & \sum_{t \in T} \sum_{b \in B_t} \mathrm{UB}_{tb} x_{tb} w_{tb}^\sigma, \quad \sigma \in \mathcal{S} \\ & \sum_{b \in B_t} x_{tb} \leq 1, \quad t \in T \\ & \sum_{t \in T} \sum_{b \in B_t} a_{tb}^l x_{tb} \leq 1, \quad l \in L \\ & \mathrm{N}_{\min} \leq \sum_{t \in T} \sum_{b \in B_t} x_{tb} \leq \mathrm{N}_{\max} \\ & A \geq 0, \ x_{tb} \in \{0,1\}, \quad t \in T, \ b \in B_t \end{cases} \tag{17}
$$

where $(u_l^\sigma,\ s_l^\sigma,\ v_{tb}^\sigma,\ w_{tb}^\sigma,\ g_t^\sigma,\ h_t^\sigma)$ with $\sigma \in \mathcal{S}$ are the extreme points of the recourse problem $Q'(x, \Gamma)$ for fixed x and Γ. Note that the number of constraints in this formulation is exponential. We propose thus the constraint generation Algorithm 1 to solve it (Kelley, 1960).

The basic idea of Algorithm 1 is to start with a relaxation of the problem $\mathrm{W}_{rob}(\Gamma)'$ that contains none of constraints (17), then iteratively generate a new extreme point of $\mathcal{S}$, by solving the recourse problem (the slave problem), and add the corresponding constraint to the master problem until an optimal solution is found. The algorithm is exact and ends when a lower bound LB, defined by the value of the problem relaxation, equals to an upper bound UB, that corresponds to the value of a feasible solution of $\mathrm{W}_{rob}(\Gamma)'$ (for a complete proof of convergence, see Kelley (1960)).

Algorithm 1 Constraint generation algorithm to solve $\mathrm{W}_{rob}(\Gamma)$

Step 0: Initialization
Define and solve the problem $\mathrm{W}(\Gamma)'^0$ containing no extreme point of the recourse problem (we suppose that $u^0 = v^0 = w^0 = g^0 = h^0 = z^0 = 0$).
Set $LB \leftarrow -\infty$, $UB \leftarrow +\infty$ and $r \leftarrow 1$. Go to Step 1.

Step 1: Solve the master problem

$$\mathrm{W}(\Gamma)'^r \begin{cases} \min & A \\ \text{s.t.} & A \geq \sum_{l\in L} \overline{d}_l u_l^i + \sum_{l\in L} \hat{d}_l\, u_l^r\, z_l^i + \sum_{t\in T} \mathrm{q}_t g_t^i - \\ & \sum_{t\in T} \mathrm{Q}_t h_t^i + \sum_{t\in T}\sum_{b\in B_t} \mathrm{LB}_{tb} x_{tb} v_{tb}^i - \\ & \sum_{t\in T}\sum_{b\in B_t} \mathrm{UB}_{tb} x_{tb} w_{tb}^i, \quad i = 0\dots r-1 \\ & \sum_{b\in B_t} x_{tb} \leq 1, \quad t \in T \\ & \sum_{t\in T}\sum_{b\in B_t} a_{tb}^l\, x_{tb} \leq 1, \quad l \in L \\ & \mathrm{N}_{min} \leq \sum_{t\in T}\sum_{b\in B_t} x_{tb} \leq \mathrm{N}_{max} \\ & A \geq 0,\ x_{tb} \in \{0,1\}, \quad t \in T,\ b \in B_t \end{cases}$$

and denote (x^r, A^r) its optimal solution. Update $LB \leftarrow A^r$, and go to Step 2.

Step 2: For the fixed assignments x^r, solve the recourse problem $Q'(x^r, \Gamma)$ and denote $(u^r, v^r, w^r, g^r, h^r, z^r)$ its optimal solution. Set

$$UB \leftarrow \min\{UB, \sum_{l\in L} \overline{d}_l u_l^r + \sum_{l\in L} \hat{d}_l u_l^r z_l^r + \sum_{t\in T} \mathrm{q}_t g_t^r - \sum_{t\in T} \mathrm{Q}_t h_t^r + \sum_{t\in T}\sum_{b\in B_t} \mathrm{LB}_{tb}\, x_{tb}^r v_{tb}^r - \sum_{t\in T}\sum_{b\in B_t} \mathrm{UB}_{tb}\, x_{tb}^r\, w_{tb}^r\}$$

if $UB = LB$ **then**
  return (x^r, A^r) as an optimal solution to the problem $\mathrm{W}_{rob}(\Gamma)'$;
else
  go to Step 3
end if

Step 3: Add the constraint

$$A \geq \sum_{l\in L} \overline{d}_l u_l^r + \sum_{l\in L} \hat{d}_l u_l^r z_l^r + \sum_{t\in T} \mathrm{q}_t g_t^r - \sum_{t\in T} \mathrm{Q}_t h_t^r + \sum_{t\in T}\sum_{b\in B_t} \mathrm{LB}_{tb} x_{tb} v_{tb}^r - \sum_{t\in T}\sum_{b\in B_t} \mathrm{UB}_{tb} x_{tb} w_{tb}^r$$

to the master problem, $r \leftarrow r+1$ and go to Step 1.

4. Experimental analysis

The goal of this section is twofold. First, we want to assess the computational performance of the proposed robust approach to deal with the uncertainty on shipment volumes for the winner determination problem. To this end, we evaluate the computing times required by Algorithm 1 for different sets of generated instances. Second, we want to study the relevance of the robust approach and the profit it generates for the shipper when compared to a deterministic approach. This is done by comparing the cost incurred by the shipper with the proposed robust approach and that resulting from applying the deterministic WDP with nominal demands.

4.1. Problem tests

Several tests were realized for various sizes and parameter settings of the WDP. The problem tests considered are grouped in twelve instances sets. Each instance set gathers five instances that are randomly generated. An instance set is defined by the number of lanes $|L|$, the number of carriers $|T|$ and the number of bids submitted by each carrier $t \in T$, $|B_t|$. We assume here that all carriers submit the same number B of bids (that is $\forall t, t' \in T, |B_t| = |B_{t'}| = B$). In the following, an instance set is described by a 2-tuple $(N^o,\ |L| - |T| - B)$, where N^o represents the number of the set. We consider three different values for the number of lanes: $|L| = 60$, $|L| = 120$, and $|L| = 240$. The number of carriers $|T|$ varies between 10 and 40 and the number of bids submitted by each carrier takes the values $B = 10$ and $B = 20$. Instances are classified into three categories: small, medium and large. Small instances correspond to instances sets with $|L| = 60$. Medium, respectively, large, instances are those belonging to instances sets with $|L| = 120$, respectively, $|L| = 240$.

When generating instances, we consider the following data and restrictions for all instances sets:

- The packages of lanes covered by bids ($\mathcal{L}_{tb}$) are generated such that some of them cover several lanes and others few lanes.This is done in order to ensure as much as possible that all lanes are covered by the participating carriers in the final allocation,
- the nominal demand $\overline{d}_l$ on each lane $l \in L$ is generated in the interval $[10, 50]$. The deviations $\hat{d}_l$ are equal to some percentage of the nominal demands. More precisely, $\hat{d}_l = \alpha \times \overline{d}_l$ where α is randomly generated within $[0.10, 0.50]$,
- the lower bounds LB_{tb} and upper bounds UB_{tb} on the volume to be transported by carrier $t \in T$ in bid b are generated in the intervals $[0, 5]$ and $[40, 75]$, respectively,
- the lower bounds q_t and the upper bounds Q_t on the total volume of shipments assigned to each carrier $t \in T$ are uniformly distributed in the intervals $[0, 5]$ and $[70, 100]$, respectively. To avoid infeasibility in the recourse problem, we assume that $q_t \leq \text{LB}_{tb}$ for all $t \in T$ and $b \in B_t$,
- the unit costs c_{tb} associated with carriers t and bids b are generated in $[10, 50]$,
- N_{min} is set to 1 and N_{max} is set to 5,
- the extra costs ce_l associated with lanes $l \in L$ are uniformly distributed in $[50, 200]$,
- the penalty/performance factor of a carrier $t \in T$ is generated in the interval $[-0.05, 0.05]$.

For each generated instance, we consider 11 different values of the budget of uncertainty Γ ranging from 0% to 100% with a step of 10% ($\Gamma = 100\%$ implies that the demand of all lanes are deviated where as $\Gamma = 50\%$ means that half of these are deviated). Hence, a total of 660 instances were solved. The results obtained for these instances are reported in next section.

4.2. Computational performance

All models were solved by CPLEX 12.1 (with its default parameters) on a 3.00 GHz Intel Core 2 Duo PC with a 4.00 Go RAM. A time limit of ten hours was fixed for solving each instance by algorithm 1.

Table 1, Table 2 and Table 3 summarize the numerical results for small, medium and large instances, respectively. They give for each value of the budget of uncertainty Γ, the average number of iterations of Algorithm 1 (#*iter.*), the average running time in seconds (Time), the average percentage of time spent for solving the master problem (Master (%)), and the slave problem (Slave (%)), these averages being computed for the 5 generated instances of each set. A dash ($'-'$) in a column indicates that no optimal solution was identified within the time limit of 10 hours.

The results obtained in Table 1 show that Algorithm 1 converges relatively quickly to the optimal solution, for small instances, for any value of the parameter Γ. The number of iterations required varies between 22 and 221 and computing times between 2 and 748 seconds. The number of iterations increases for medium instances, as reported in Table 2, but remains reasonable (it does not exceed 226 iterations). Computing times also increase but do not exceed 1903 seconds (instance set N^o = 8). For large instances (Table 3), the algorithm was unable to converge to an optimal solution (within the time limit of ten hours) for instance set $(12, 240 - 80 - 20)$ when the budget of uncertainty Γ takes values 0%, 10%, 20%, 30% and 40%. For the remaining instances, an optimal solution was identified within a computing time varying between 283 and 35631 seconds.

N°	$\|L\|-\|T\|-B$	Γ(%)	# iter.	Time (s)	Master(%)	Slave (%)
1	60-10-10	0 %	38.80	4.04	80.90	19.10
		10 %	36.80	4.93	80.19	19.81
		20 %	31.80	4.36	80.70	19.30
		30 %	28.60	3.46	79.34	20.66
		40 %	24.40	2.68	79.33	20.67
		50 %	23.80	2.48	79.18	20.82
		60 %	23.40	2.36	79.13	20.87
		70 %	23.60	2.46	80.77	19.23
		80 %	23.60	2.44	79.50	20.50
		90 %	23.80	2.41	79.73	20.27
		100 %	22.80	2.27	80.21	19.79
2	60-10-20	0 %	66.40	31.66	91.95	8.05
		10 %	50.80	18.32	87.99	12.01
		20 %	42.20	12.73	86.54	13.46
		30 %	37.80	10.68	87.55	12.45
		40 %	35.80	9.87	85.99	14.01
		50 %	35.40	10.01	86.50	13.50
		60 %	34.60	9.79	87.86	12.14
		70 %	36.00	9.92	88.69	11.31
		80 %	35.00	9.59	88.68	11.32
		90 %	35.00	9.62	89.06	10.94
		100 %	35.80	10.05	89.21	10.79
3	60-20-10	0 %	84.60	41.98	94.26	5.74
		10 %	69.80	33.98	92.53	7.47
		20 %	58.80	22.71	88.92	11.08
		30 %	52.60	18.35	89.51	10.49
		40 %	50.40	16.83	90.25	9.75
		50 %	48.20	15.45	90.68	9.32
		60 %	48.00	15.31	90.45	9.55
		70 %	47.80	15.54	90.91	9.09
		80 %	48.00	14.97	91.18	8.82
		90 %	48.40	15.86	90.59	9.41
		100 %	47.00	14.27	91.79	8.21
4	60-40-10	0 %	221.80	701.18	98.61	1.39
		10 %	187.80	747.99	98.92	1.08
		20 %	146.80	495.24	98.52	1.48
		30 %	121.20	353.39	98.30	1.70
		40 %	104.00	281.82	98.31	1.69
		50 %	102.00	257.07	98.35	1.65
		60 %	100.80	252.05	98.36	1.64
		70 %	101.60	261.88	98.46	1.54
		80 %	101.60	260.23	98.34	1.66
		90 %	102.40	264.01	98.45	1.55
		100 %	99.60	267.09	98.46	1.54

Table 1: Results of Algorithm 1 for small instances ($|L| = 60$)

N°	$\lvert L\rvert - \lvert T\rvert - B$	Γ(%)	# iter.	Time (s)	Master(%)	Slave (%)
5	120-10-20	0 %	63.80	25.91	82.96	17.04
		10 %	59.00	22.32	75.90	24.10
		20 %	54.00	19.31	83.88	16.12
		30 %	48.00	16.57	76.80	23.20
		40 %	43.40	13.18	75.74	24.26
		50 %	41.00	11.04	81.69	18.31
		60 %	40.20	11.17	78.90	21.10
		70 %	40.40	12.16	73.99	26.01
		80 %	40.40	10.84	81.63	18.37
		90 %	40.20	11.83	76.47	23.53
		100 %	40.00	10.52	82.14	17.86
6	120-20-10	0 %	67.20	25.64	87.55	12.45
		10 %	60.00	25.19	86.20	13.80
		20 %	53.80	22.56	83.78	16.22
		30 %	47.00	17.59	85.09	14.91
		40 %	41.80	14.28	84.64	15.36
		50 %	39.60	12.77	83.78	16.22
		60 %	39.40	12.55	84.63	15.37
		70 %	39.40	12.64	85.05	14.95
		80 %	39.40	13.17	81.72	18.28
		90 %	39.40	12.68	84.52	15.48
		100 %	38.60	11.78	85.98	14.02
7	120-40-10	0 %	148.20	365.80	97.40	2.60
		10 %	121.60	357.17	97.46	2.54
		20 %	102.60	293.10	97.29	2.71
		30 %	86.20	223.92	95.52	4.48
		40 %	77.60	176.62	97.02	2.98
		50 %	72.00	150.64	96.86	3.14
		60 %	72.20	151.10	96.78	3.22
		70 %	71.80	150.17	96.87	3.13
		80 %	72.00	150.99	96.85	3.15
		90 %	72.00	151.58	96.90	3.10
		100 %	71.00	146.55	97.06	2.94
8	120-40-20	0 %	225.80	1 902.72	98.14	1.86
		10 %	200.80	1 921.70	98.52	1.48
		20 %	131.40	1 131.70	96.51	3.49
		30 %	134.00	1 137.55	98.19	1.81
		40 %	121.00	936.03	98.15	1.85
		50 %	120.20	927.71	98.29	1.71
		60 %	119.80	911.79	98.22	1.78
		70 %	120.20	925.44	98.20	1.80
		80 %	120.20	927.84	98.05	1.95
		90 %	120.20	919.85	98.36	1.64
		100 %	118.00	884.08	97.55	2.45

Table 2: Results of Algorithm 1 for medium instances ($\lvert L\rvert = 120$)

N^o	$\|L\| - \|T\| - B$	Γ(%)	# iter.	Time (s)	Master(%)	Slave (%)
9	240-40-10	0 %	135.40	430.29	92.44	7.56
		10 %	125.60	501.13	92.40	7.60
		20 %	116.60	461.34	92.16	7.84
		30 %	102.40	372.45	92.38	7.62
		40 %	94.00	310.79	91.52	8.48
		50 %	89.80	290.80	90.17	9.83
		60 %	89.20	295.58	89.55	10.45
		70 %	89.20	291.97	90.17	9.83
		80 %	89.20	294.34	89.44	10.56
		90 %	89.20	283.42	90.40	9.60
		100 %	87.40	282.91	91.06	8.94
10	240-40-20	0 %	235.20	3 367.67	97.32	2.68
		10 %	212.20	4 100.87	97.81	2.19
		20 %	186.40	3 794.54	97.95	2.05
		30 %	136.20	2 519.00	95.69	4.31
		40 %	146.60	2 584.58	98.22	1.78
		50 %	142.60	2 408.78	92.99	7.01
		60 %	142.40	2 390.35	93.64	6.36
		70 %	142.40	2 385.33	92.84	7.16
		80 %	142.40	2 329.27	93.16	6.84
		90 %	142.40	1 976.13	93.47	6.53
		100 %	140.20	2 300.20	98.29	1.71
11	240-80-10	0 %	263.20	4 554.69	98.86	1.14
		10 %	243.40	5 480.67	99.03	0.97
		20 %	215.60	5 388.22	99.08	0.92
		30 %	193.00	4 530.50	99.07	0.93
		40 %	171.60	3 647.93	98.98	1.02
		50 %	161.60	3 203.22	98.93	1.07
		60 %	159.00	3 174.25	98.97	1.03
		70 %	158.80	3 144.73	98.81	1.19
		80 %	158.80	3 028.56	98.70	1.30
		90 %	158.80	2 967.69	98.68	1.32
		100 %	156.40	2 923.44	98.61	1.39
12	240-80-20	0 %	-	-	-	-
		10 %	-	-	-	-
		20 %	-	-	-	-
		30 %	-	-	-	-
		40 %	-	-	-	-
		50 %	307.50	33 298.10	99.66	0.34
		60 %	306.00	33 298.15	99.17	0.83
		70 %	308.00	33 406.80	99.66	0.34
		80 %	308.00	35 139.05	99.61	0.39
		90 %	308.00	35 522.25	99.34	0.66
		100 %	302.00	35 631.00	99.34	0.66

Table 3: Results of Algorithm 1 for large instances ($|L| = 240$)

One can also note from Tables 1, 2, and 3 that even if both the master and the slave problems are NP-hard, most of the computing time is dedicated to solve the master problem. This time represents in fact between 75% and 99%

of the total time and increases with the problem size. Hence, the recourse problem is well addressed and the bounds M$(= ce_l)$ are very tight. Recall that this problem is originally bilinear and is generally difficult to solve.

4.3. Relevance of the robust approach

This section studies the relevance of the proposed robust approach with respect to deterministic approaches. For each instance, we first determine the so-called nominal solution by solving the deterministic WDP described in Section 2.2 with a demand equal to the nominal demand for each lane $(d_l = \overline{d}_l, \forall l \in L)$. Then, we compute the total cost incurred by the shipper in the worst case (for each value of the budget of uncertainty Γ) when the nominal first-stage allocation solutions are considered. This is done simply by solving the recourse problem $Q'(\overline{x}, \Gamma)$, where $\overline{x}$ denotes the nominal first stage solution. These costs, called in the following nominal costs, are compared to the costs incurred by the shipper with the robust solution (called robust costs). It is noteworthy that, the worst scenario demands for the nominal solution is not necessarily the same worst scenario for the robust solution.

Table 4 reports the average percentage (over the five instances of each set) of monetary losses (given by (nominal cost - robust cost)/robust cost) that could be incurred by the shipper when using a nominal rather than a robust solution for the 12 instances sets considered in our study. Table 4 shows that as the value of the budget of uncertainty increases, the difference between both costs becomes larger. This implies that the shipper can avoid significant losses choosing robust solutions, in particular if it expects large deviations in shipment volumes. In our experiments, losses reach 41.67% (Instance set N$^{\text{o}}$ = 2). This is mostly due to the extra costs the shipper is obliged to pay to spot carriers to satisfy an unexpected increase of demand on lanes in case nominal solutions are considered rather than robust ones.

N°	$\lvert L\rvert - \lvert T\rvert - B$	$\Gamma(\%)$	% losses	N^o	$\lvert L\rvert - \lvert T\rvert - B$	$\Gamma(\%)$	% losses
1	60-10-10	0 %	0.00	2	60-10-20	0 %	0.00
		10 %	0.80			10 %	7.72
		20 %	4.49			20 %	21.24
		30 %	9.79			30 %	31.40
		40 %	13.37			40 %	37.81
		50 %	15.34			50 %	41.11
		60 %	16.07			60 %	41.67
		70 %	16.34			70 %	41.67
		80 %	16.34			80 %	41.67
		90 %	16.34			90 %	41.67
		100 %	16.34			100 %	41.67
3	60-20-10	0 %	0.00	4	60-40-10	0 %	0.00
		10 %	2.24			10 %	3.23
		20 %	5.11			20 %	10.74
		30 %	9.44			30 %	17.12
		40 %	11.86			40 %	21.90
		50 %	13.12			50 %	22.19
		60 %	13.39			60 %	22.19
		70 %	13.39			70 %	22.19
		80 %	13.39			80 %	22.19
		90 %	13.39			90 %	22.19
		100 %	13.39			100 %	22.19
5	120-10-20	0 %	0.00	6	120-20-10	0 %	0.00
		10 %	1.55			10 %	0.42
		20 %	4.70			20 %	5.00
		30 %	8.93			30 %	8.90
		40 %	11.58			40 %	14.50
		50 %	12.31			50 %	17.97
		60 %	12.44			60 %	18.14
		70 %	12.44			70 %	18.14
		80 %	12.44			80 %	18.14
		90 %	12.44			90 %	18.14
		100 %	12.44			100 %	18.14
7	120-40-10	0 %	0.00	8	120-40-20	0 %	0.00
		10 %	3.00			10 %	4.75
		20 %	8.65			20 %	13.56
		30 %	15.21			30 %	20.20
		40 %	18.84			40 %	21.23
		50 %	20.36			50 %	21.23
		60 %	20.39			60 %	21.23
		70 %	20.39			70 %	21.23
		80 %	20.39			80 %	21.23
		90 %	20.39			90 %	21.23
		100 %	20.39			100 %	21.23
9	240-40-10	0 %	0.00	10	240-40-20	0 %	0.00
		10 %	0.00			10 %	3.85
		20 %	8.00			20 %	14.25
		30 %	16.74			30 %	23.35
		40 %	22.69			40 %	27.04
		50 %	25.57			50 %	27.85
		60 %	25.80			60 %	27.85
		70 %	25.80			70 %	27.85
		80 %	25.80			80 %	27.85
		90 %	25.80			90 %	27.85
		100 %	25.80			100 %	27.85
11	240-80-10	0 %	0.00	12	240-80-20	0 %	-
		10 %	0.00			10 %	-
		20 %	8.00			20 %	-
		30 %	16.74			30 %	-
		40 %	22.69			40 %	-
		50 %	25.57			50 %	10.84
		60 %	25.80			60 %	10.84
		70 %	25.80			70 %	10.84
		80 %	25.80			80 %	10.84
		90 %	25.80			90 %	10.84
		100 %	25.80			100 %	10.84

Table 4: Percentage of monetary losses with nominal solutions compared to robust solutions

To give more details on the behaviour of both approaches, we illustrate in Figure 1 the nominal costs and the robust costs for an instance including

120 lanes, 40 carriers and 10 bids per carrier.

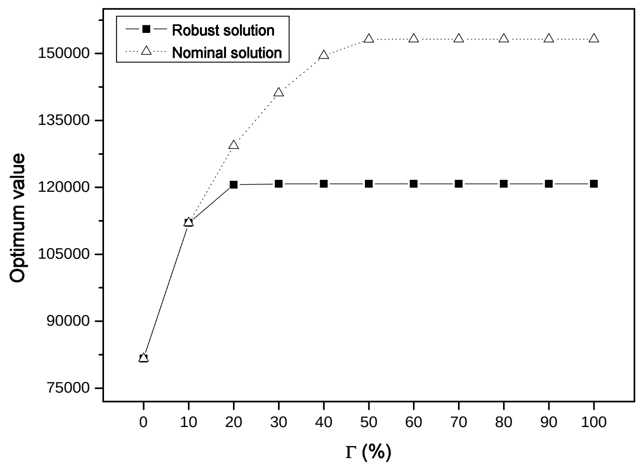

Figure 1: Variation of the nominal and robust costs with Γ for an instance with $|L| = 12$, $|T| = 40$ and B=10

When examining the evolution of both nominal and robust costs with the budget of uncertainty Γ (Figure 1), one can see that both curves are concave and that costs increase rapidly for small values of Γ and slowly for large values of Γ. In fact, when the value of Γ exceeds a certain threshold ($\Gamma \geq 30\%$ and $\Gamma \geq 50\%$ for the robust and the nominal solutions, respectively) the cost remains the same. In other words, we reach the worst-case solution. This is due to the restrictive constraints of the WDP (XOR constraints (4), allocation constraints (5), and volume restriction constraints). Indeed, it may occur that for a given combination of lanes $\tilde{L}$ corresponding to a given value of Γ (= 60%, for example), winning bids and corresponding volumes are still optimal for larger values of Γ (= 70%, for example) since they also satisfy the new combination of lanes $\tilde{L}'$ (because lanes in $\tilde{L}' \setminus \tilde{L}$ have deviated demands that are smaller than the deviated demands of lanes in $\tilde{L}$).

5. Conclusion

In this paper, we address the winner determination problem, an important decision problem in combinatorial transportation procurement auctions, under uncertain shipment volumes. The objective is to select robust solutions before knowing the actual realization of the demands on lanes volumes. To do so, we apply a 2-stage robust approach. We first define the uncertainty model then we write and simplify the robust formulation. A constraint generation algorithm is proposed to exactly solve the problem. To the best of our knowledge, this is the first work that investigates robust optimization framework to handle uncertainty in solving WDP. The proposed approach shows good computational performances for instances including up to 280 lanes, 80 carriers and 800 bids. We also investigate the relevance of our robust solutions and show that monetary losses reaching 41% could be avoided when robust solutions are chosen rather than deterministic ones.

To go further, many tracks of research can be followed. Shippers may consider the case where uncertainty on bids must be handled together with uncertainty on lanes demands when solving the WDP. Other extensions can also be investigated concerning the uncertainty modelling. For instance, if all the demands are homogeneous, a budget of uncertainty may consider a global amount of deviation, where Γ represents a quantity of deviated demand rather than a number of deviated demands. In this case, the recourse problem keeps its bilinear shape and has to be solved by an adequate method. This will be the subject of future research.

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