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Giacomo Lanza
Teodor Gabriel Crainic
Mauro Passacantando
Maria Grazia Scutellà

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Bureau de Montréal
Université de Montréal
C.P. 6128, succ. Centre-Ville
Montréal (Québec) H3C 3J7
Tél : 1-514-343-7575
Télécopie : 1-514-343-7121

Bureau de Québec
Université Laval,
2325, rue de la Terrasse
Pavillon Palasis-Prince, local 2415
Québec (Québec) G1V 0A6
Tél : 1-418-656-2073
Télécopie : 1-418-656-2624

Continuous-Time Scheduled Service Network Design with Stochastic Travel Times

Giacomo Lanza[3], Teodor Gabriel Crainic[1,2]*, Mauro Passacantando[4], Maria Grazia Scutellà[3]

1. Interuniversity Research Centre on Enterprise Networks, Logistics and Transportation (CIRRELT)
2. School of Management, Université du Québec à Montréal
3. Department of Computer Science, University of Pisa, Italy
4. Department of Business and Law, University of Milano-Bicocca, Milano, Italy

Abstract. We address the Continuous-Time Scheduled Service Network Design problem with Stochastic Travel Times. The problem explicitly considers business-as-usual travel time fluctuations and quality targets for the on-time operation of services and the delivery of demand at destinations. We propose a two-stage mixed-integer stochastic programming model that prescribes service selection based on probability distributions (first stage) and recourse actions using updated travel time estimations (second stage), which include postponement of service departures, commodities itineraries, and outsourcing of demand transportation, with quality targets modeled through penalties. The formulation aims to mitigate delays, reduce operational costs, and guarantee the feasibility and profitability of the solutions obtained, even when travel time fluctuations occur. The formulation utilizes an innovative Stochastic-Aware Service-Leg Network to model time continuously, mitigating the increase in network size that would typically occur with traditional approaches. Through extensive experimentation on realistic small-to-medium-size instances, we assess the complexity and benefits of our stochastic formulation over the deterministic one, also highlighting specific features to hedge against time fluctuations that appear in stochastic solutions. The results clearly show that our approach effectively mitigates delays and improves operational costs.

Keywords: Transportation, Scheduled Service Network Design, stochastic travel time, continuous time, two-stage formulation.

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* Corresponding author: teodorgabriel.crainic@cirrelt.net

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1 Introduction

Consolidation-based freight transportation carriers group and dispatch within the same vehicle (or convoy) multiple shipments, each potentially associated with a different customer. These carriers operate within a network of terminals connected by infrastructure and provide transportation services along defined routes according to schedules. Carriers operate services to respond to the transportation demand between specific terminals raised by customers. Each shipment must be picked up from its origin terminal no earlier than its availability date and delivered to its destination terminal before the specified due date. Shipments are routed by services through itineraries, possibly visiting intermediate terminals where loading/unloading, shipment consolidation, and service-to-service transfer operations are performed. Jointly determining the itineraries that commodities must follow and selecting the scheduled services to operate efficiently is a complex problem faced at the tactical level by carriers, supported by the Service Network Design (SND) methodology. The scope of SND is to produce a transportation plan that will be operated for a medium-long period with the main objective of minimizing the total transportation cost. The problem is normally addressed for a given length of time, called schedule length, for which a certain demand regularity is observed over a longer planning horizon. The transportation plan is then repeated cyclically to cover the entire planning horizon (e.g., a weekly schedule to be repeated for six months). We refer to Crainic and Hewitt (2021) for a comprehensive overview of the problems and methodologies of SND in the context of consolidation-based transportation.

The formulations in the literature typically address the deterministic version of the problem, assuming that all the information necessary to define the plan is readily available and fixed to specific point estimations computed by suitable forecasting methods from historical data (see Crainic and Hewitt, 2021). However, the problem is inherently stochastic, since it requires carriers to make decisions in an environment with imperfect information on parameters such as seasonal demand, travel times, transportation costs, etc., complete information becoming available only after medium-term decisions have been made (Powell and Topaloglu, 2003; Wang et al., 2019; Hewitt et al., 2021). While the demand uncertainty has recently received some attention (e.g., Lium et al., 2009; Bai et al., 2014; Hewitt et al., 2019; Wang and Qi, 2020), the travel time uncertainty is a rarely addressed aspect in the stochastic SND literature (Demir et al., 2016; Lanza et al., 2021; Shu et al., 2023). Nevertheless, in applications such as just-in-time production systems, where timely and predictable deliveries are essential to avoid costly production halts, uncertainty primarily stems from travel time variability rather than demand fluctuations (Müllerklein and Fontaine, 2025).

This paper investigates a Scheduled Service Network Design (SSND) problem that explicitly considers the stochastic nature of *travel time*. Our focus is on fluctuations that occur during daily operations, the so-called randomness, foreseeable variations in travel times that can be modeled as random variables with known distributions (Klibi et al., 2010). These variations, which are caused by factors such as traffic congestion or adverse weather, can result in consequences such as disrupted service schedules, reduced consolidation efficiency, missed delivery deadlines, and decreased customer satisfaction. To address such an issue, we propose a two-stage stochastic programming model (Birge and Louveaux, 2011) prescribing the selection of services in the first stage, when only the travel time probability distributions are available. Depending on the selected services and on the realization of the travel time random variables, remaining decisions must be made. These decisions concern the postponement of some service departure times, the routing of commodities through the selected services, and outsourcing. Although these recourse actions have been individually addressed in previous models, their joint consideration within a stochastic SSND framework with uncertain travel times appears to be missing from the current literature. The expected costs associated with such decisions, the costs for delays related to the service operations, with respect to their schedule, and the commodity arrival times at the destination, with respect to the agreed-upon time of deliveries, are included in the objective function. The goal of the proposed formulation is to ensure robust and cost-efficient operations under travel time uncertainty.

A distinctive feature of the proposed stochastic formulation is its continuous time modeling, which results in a more compact network than the traditional time-space network often used in SSND formulations. To achieve such a time modeling, we extended to the stochastic setting the approach proposed in Lanza et al. (2024) for the deterministic case. This continuous time modeling approach adds to the body of existing methods aimed at overcoming the inconveniences due to the time-space network size growth as the level of detail in time representation increases. This body includes the method in Boland et al. (2017), where the Dynamic Discretization Discovery (DDD) algorithmic framework, still reliant on time-space networks, is presented, and the ones in Hewitt and Lehuédé (2023) and Shu et al. (2023), which present formulations depending on the enumeration of consolidation opportunities. In contrast, our approach utilizes a peculiar Service-Leg Network that does not depend on time-space networks or the enumeration of consolidation paths.

To summarize, the contributions of this paper are the following: i) we study a novel SSND setting that includes uncertainty in travel times; ii) we propose a two-stage mixed-integer stochastic formulation including recourse actions, which, to the best of our knowledge, have not been simultaneously incorporated in a unique model in the existing literature; iii) we use a new methodology to model time continuously through a compact Stochastic-Aware Service-Leg Network; iv) we present results from extensive computational experiments on small to medium-sized realistic instances, emphasizing the advantages of our formulation in contrast to a traditional deterministic approach.

This paper is structured as follows. In Section 2, we review the literature on stochastic SSND under travel time uncertainty and discuss alternative approaches to model time. Section 3 describes the stochastic SSND problem and introduces the notation relevant to its formulation. Section 4 is devoted to the mathematical formulation: Section 4.1 describes the Stochastic-Aware Service-Leg Network on which the formulation relies, while Section 4.2 presents the two-stage stochastic mathematical model. Section 5 focuses on the computational study: Section 5.1 describes the instances used in the study, Section 5.2 presents numerical results evaluating the performance of the stochastic model in terms of computational time and percentage optimality gap compared to a traditional deterministic approach; Section 5.3 quantifies the benefits of using the stochastic approach instead of a deterministic one in terms of costs of the respective solutions; strategies for reducing delays are discussed in Section 5.4, while Section 5.5 analyses how the network design configuration obtained from the stochastic model differs from that obtained from the deterministic model. Section 6 concludes the paper.

2 Literature Review

The literature review is structured around the two main contributions of this work: the consideration of uncertain travel times in SSND (Section 2.1), and the methodological framework for modeling time in SSND (Section 2.2).

2.1 Stochastic Travel Time in Scheduled Service Network Design

Many variants of SSND in the context of freight transportation have been studied in the literature within a deterministic setting. The most recent and complete survey on SSND in specific contexts and modes of transportation, including rail, truck, and maritime, can be found in Crainic et al. (2021). Other surveys on the topic can be found in Crainic (2000); Crainic and Kim (2007); Crainic (2024a,b); Crainic and Rei (2025).

SSND problems taking into account uncertainty have been studied to a lesser extent. Most available contributions focus on uncertainty in shipment volumes (Lium et al., 2009; Hoff et al., 2010; Bai et al., 2014; Wang et al., 2019; Wang and Qi, 2020; Müller et al., 2021b; Scherr et al., 2022), while travel time uncertainty is very rarely addressed. Among the few studies available, Demir et al. (2016) (and later Hrušovskỳ et al., 2018; Layeb et al., 2018) investigate an intermodal freight transportation planning problem whose goal is to route demands by both planning road services and selecting available rail and maritime transportation services, the latter operating according to fixed schedules. The proposed stochastic mixed integer linear programming approach incorporates probabilistic constraints ensuring that the probability of missing an available rail or maritime service for road services remains below a given threshold. The model does not explicitly address recourse actions or re-planning caused by missed connections at transshipment terminals. Similar intermodal SND problems have been treated in Zhao et al. (2018a,b); Sun (2020), and Müller et al. (2021a). Lanza et al. (2018, 2021) studied an SSND problem focusing on quality targets for the on-time operation of services according to a given schedule, and for the on-time delivery of commodities to destinations according to carrier-customer contracts under fluctuations in travel times. The authors propose a two-stage stochastic model defined over a time-space network, with quality targets modeled through penalties. The first stage involves selecting services and determining freight routes. A simple recourse strategy is employed in the second stage, where penalties are paid for lateness. Safe connections are only enforced through the penalties for late service arrival. In Shu et al. (2023), an SSND with uncertain travel times is addressed, focusing on delays resulting from the synchronization of multiple shipments that require consolidation at certain terminals before continuing their itineraries together. If one of these shipments is late, it can cause delays for all the others waiting for consolidation. In their setting, tactical decisions involve service selection and freight routing which are not modified at an operational level, and thus no vehicle may depart until all its shipments have arrived.

2.2 Modeling Time in Scheduled Service Network Design

Time-related features associated with demand and services are traditionally modeled in SSND using a time-space network, which is defined by explicitly discretizing time: given a schedule length T and a granularity Δ, a time-space network is constructed by partitioning the schedule length into T/Δ non-overlapping time intervals, replicating the physical nodes $T/\Delta + 1$ times and adding arcs to connect the timed-nodes appropriately. All events taking place within a time interval are modeled as occurring at the beginning of it. Moreover, to have a repeatable plan the activities that would end after T are modeled via arcs going back in time (Crainic and Hewitt, 2021). Choosing the appropriate granularity of the schedule length partitioning can be challenging as it has an impact on both solution quality and computational tractability (Boland et al., 2017, 2019). These challenges are mitigated by the DDD algorithmic framework presented in Boland et al. (2017) and further studied in Hewitt (2019); Marshall et al. (2021); Shu et al. (2024). DDD is an exact algorithm that utilizes an iterative process to adjust the level of discretization of an initial time-space network, without ever creating a fully time-space network (Shu et al., 2023). However, although DDD is computationally effective, especially on Less-Than-Truckload (LTL) applications, it can be challenging to implement, as discussed in Hewitt and Lehuédé (2023).

Recent formulations are based on more compact networks than time-space networks, particularly for problems requiring a detailed time representation. A consolidation-based formulation is proposed in Hewitt and Lehuédé (2023), which is not defined on a time-space network but rather on an a priori enumeration of all or part of the possible consolidation of OD demands on each arc. However, also the enumeration of the consolidation opportunities can be computationally challenging. Therefore, the authors proposed a hybrid formulation that combines elements of the consolidation-based formulation and the classical time-space network formulation. In Shu et al. (2023), a compact consolidation-indexed formulation for the deterministic SSND has been proposed, where consolidation indices rather than time indices are used to define decision variables and constraints. The formulation requires the construction of a consolidation-expanded network, which replicates each arc in the Service-Leg Network according to the number of commodities in the problem, to create consolidation traveling arcs. The formulation has been extended to a robust version of the problem with uncertain travel times. A different approach has been proposed in Lanza et al. (2024) for the deterministic SSND, relying neither on a time-space network nor on the enumeration of consolidation paths of commodities. This formulation relies on the construction of an Extended Service-Leg Network, which includes all arcs from the Service-Leg Network, plus additional arcs modeling different occurrences of some service legs at various times (in the past or in the future). The SSND is formulated as a capacitated multicommodity network design problem on this network. The proposed formulation has proved to be particularly effective when a fine granularity is required, compared to traditional time-space networks.

2.3 Positioning in the literature

The setting studied in Shu et al. (2023) is the most similar to the one studied in this paper. In fact, we consider consolidation delay propagation and service departure postponement as well. However, unlike Shu et al. (2023), we propose a more flexible setting in which service selection belongs to tactical decisions and the routing of commodities may vary across schedule repetitions based on observed travel times and delays, also including the possibility of outsourcing. Besides the problem settings, the proposed approaches are also different: Shu et al. (2023) proposes a robust formulation aimed at optimizing costs in a worst-case scenario, while this paper follows a stochastic optimization approach to optimize costs for average performance. To the best of our knowledge, this paper is the first approach to SSND in freight transportation with a combination of continuous time modeling, stochastic travel times, and recourse actions such as service postponement and outsourcing. From a methodological perspective, we extend the Service-Leg Network definition proposed in Lanza et al. (2024) for the deterministic SSND by introducing the Stochastic-Aware Service-Leg Network, which expands the set of temporal occurrences of certain service legs to account for travel time variability and potential postponement of service operations.

3 Problem Description

Let $(\mathcal{N}, \mathcal{A}^{PH})$ be the physical infrastructure network underlying the problem addressed: the set of nodes $\mathcal{N}$ represents physical terminals and $\mathcal{A}^{PH}$ denotes the set of directed links, that can be physical (highways or rail tracks) or conceptual (maritime or air corridors). Let T be the schedule length and $[0, T]$ the scheduling period. Each arc in $\mathcal{A}^{PH}$ has an associated travel time probability distribution and a representative point estimate. These distributions account for business-as-usual travel time fluctuations, thus excluding major disturbances or catastrophic events, as described in Klibi et al. (2010). The travel time distribution for each arc (i, j) ranges between a minimum $\tau^m_{(i,j)}$ and a maximum

$\tau^M_{(i,j)}$, while the point estimate, called *usual travel time*, represents the ideal travel time without delays and is derived statistically from the distribution, e.g., average or mode. The transportation demand is represented by a set of commodities $\mathcal{K}$. Each commodity $k \in \mathcal{K}$ requires the transport of a certain volume d^k from an origin terminal $O(k) \in \mathcal{N}$ to a destination terminal $D(k) \in \mathcal{N}$, according to its availability date $o(k) \in [0, T]$ at the origin and a given due date $d(k)$ at the destination. The flow of each commodity must follow a single directed path from origin to destination. To meet demand, the carrier operate a network of services selected out of a set Σ of potential services, which move all at the same speed. Each potential service $\sigma \in \Sigma$ is defined by a route in the physical network and by a schedule. The *route* of σ is defined through the set of its service legs $\mathcal{L}(\sigma)$. If there are no intermediate stops, $\mathcal{L}(\sigma)$ includes a single leg from origin $O(\sigma)$ to destination $D(\sigma)$. If there are intermediate stops, $\mathcal{L}(\sigma)$ contains multiple legs, $\mathcal{L}(\sigma) = \{l_i(\sigma) : \; i = 1, \ldots, |\mathcal{L}(\sigma)|\}$, each connecting consecutive terminals on the route, through a path in the physical network.

We define the *Service-Leg* Network $(\mathcal{N}, \mathcal{A})$, where $\mathcal{A} = \bigcup_{\sigma \in \Sigma} \mathcal{L}(\sigma)$ is the set of the service legs of the potential services in Σ. Note that $\mathcal{A}$ may contain parallel arcs, i.e., arcs having the same origin and destination, but being associated with different services. For each service leg $a \in \mathcal{A}$, let $\sigma_a \in \Sigma$ be the service associated with a and u_a its capacity. For each $a \in \mathcal{A}$, we define the travel time random variable τ_a as the convolution of the travel-time random variables of the physical arcs composing the service leg, and let $[\tau^m_a, \tau^M_a]$ be the finite support of τ_a. The *usual schedule* of a service σ gives the usual starting and ending times of each of its legs. Hence, for each arc a, $\hat{\phi}_a$ denotes the usual time instant when service σ_a begins to travel on arc a, while $\hat{\psi}_a$ denotes the usual time instant (i.e., computed using the usual travel times) when service σ_a arrives at the destination of the service leg. Finally, for those services including intermediate stops, we define $\theta_{l_i(\sigma)}$ the time that service σ must spend at the destination terminal of service leg $l_i(\sigma)$, $i = 1, \ldots, |\mathcal{L}(\sigma)| - 1$.

In the rest of the paper, we make the following assumptions on the potential services and transportation demand:

$$\hat{\phi}_{l_1(\sigma)} \in [0, T], \qquad \hat{\phi}_{l_{|\mathcal{L}(\sigma)|}(\sigma)} + \delta_{l_{|\mathcal{L}(\sigma)|}(\sigma)} + \tau^M_{l_{|\mathcal{L}(\sigma)|}(\sigma)} < 2T, \qquad \forall \; \sigma \in \Sigma, \tag{1}$$

$$o(k) \in [0, T], \qquad d(k) + \delta^k \leq o(k) + T, \qquad \forall \; k \in \mathcal{K}. \tag{2}$$

Conditions (1) ensure that the usual departure time of each service occurs within the scheduling period and the worst-case arrival time of each service at its destination occurs before $2T$. Conditions (2) ensure that the availability date of each commodity falls within the scheduling period and the worst-case duration of the itinerary of each commodity is at most T.

We consider the following information revelation and decision-making process. Service selection is made at the planning stage, before observing the actual travel time realizations, thus only taking into account their probability distributions and their costs. Specifically, f_σ is the cost to activate service σ. Once the service selection decisions are made, and just before service operations begin, i.e., at the beginning of each occurrence of the scheduling period, one obtains a more precise estimation of travel times based on system conditions. The new information refining travel time estimations is then used to adjust the plan to the revealed situation. Specifically, we consider the following adjustment decisions, or *recourse actions* as called in the literature: i) **Service early arrivals:** When the travel time on an arc a is shorter than the usual time, i.e., service σ_a arrives earlier at the head terminal, it must wait until the scheduled time $\hat{\psi}_a$ before proceeding (in order to comply with the strict terminal schedule). ii) **Service late arrivals:** If service σ_a arrives late due to a longer travel time, terminal operations start immediately, and the delay propagates to the next leg (if any) operated by the same service (terminal operations are assumed to take a deterministic amount of time). This delay may also impact other services, compromising the feasibility of subsequent connections. iii) **Consolidation delays:** If commodities are delayed due to service delays, the departure of a service σ_a may be postponed to ensure proper consolidation of multiple commodities on the service. Such a postponement is allowed up to a maximum δ_a. These delays may not only cause further disruptions across the network, but also result in late deliveries. A commodity k is allowed to arrive at its destination with a maximum delay of δ^k beyond its due date $d(k)$. Associated penalties are incurred as follows: q_a is the fixed penalty if service σ_a arrives later than its scheduled time $\hat{\psi}_a$, and q^k is the penalty per unit of commodity k delivered after $d(k)$. iv) **Commodity routing:** Updated information on travel times is used to determine the actual itineraries of the commodities through the selected services. Each routing decision incurs transportation and handling costs: c_a^k denotes the transportation cost per unit of commodity k on arc a, while c_i^k represents the unit cost for holding or handling commodity k at node i. v) **Outsourcing:** A commodity can be outsourced, meaning that its entire volume is transported directly from the origin to the destination by an external service provider. This choice incurs a cost C^k per unit of commodity k. Note that delays in service arrivals do not necessarily lead to late commodity deliveries, as subsequent legs with shorter travel times may compensate for early delays. Conversely, commodities may still arrive late even when services follow their scheduled times. The notation introduced in this section is summarized in the Supplementary Material.

4 Mathematical Formulation

We first describe the Stochastic-Aware Service-Leg Network, which enables us to model time continuously. Then, we present the two-stage stochastic model based on it.

4.1 Stochastic-Aware Service-Leg Network

The Stochastic-Aware Service-Leg Network $(\mathcal{N}, \mathcal{A}^{STT})$ accounts for the fact that not all the demand and services have their associated time attributes within the scheduling period.

The set $\mathcal{A}^{STT}$ includes all the service legs that can be used by the commodities to reach their destinations while accounting for possible delays.

Definition 1. *Let us define $\mathcal{A}^+ := \{a \in \mathcal{A} : \hat{\psi}_a \leq T\}$ and $\mathcal{A}^- := \{a \in \mathcal{A} : \hat{\phi}_a + \delta_a \geq T\}$. The set of arcs $\mathcal{A}^{STT}$ of the Stochastic-Aware Service-Leg Network contains: i) all the arcs in $\mathcal{A}$; ii) for each $a \in \mathcal{A}^+$, a parallel arc a^+, with the same head and tail of a, representing the next occurrence as a performed T time units ahead; iii) for each $a \in \mathcal{A}^-$, a parallel arc a^-, with the same head and tail as a, representing the previous occurrence of a performed T time units in the past. The parameters for the additional arcs are defined accordingly:*

$$\begin{array}{llllll} \hat{\phi}_{a^+} = \hat{\phi}_a + T, & \hat{\psi}_{a^+} = \hat{\psi}_a + T, & c^k_{a^+} = c^k_a, & q_{a^+} = q_a, & \delta_{a^+} = \delta_a, & \forall\ a \in \mathcal{A}^+, \\ \hat{\phi}_{a^-} = \hat{\phi}_a - T, & \hat{\psi}_{a^-} = \hat{\psi}_a - T, & c^k_{a^-} = c^k_a, & q_{a^-} = q_a, & \delta_{a^-} = \delta_a, & \forall\ a \in \mathcal{A}^-. \end{array}$$

The rationale underlying Definition 1 is the following: since the availability of each commodity is in $[0, T]$ and the commodity itinerary lasts at most T, the time interval $[0, 2T]$ must to be considered to plan the commodity itineraries. Hence, for each service leg in $\mathcal{A}$ whose usual ending time falls within the current scheduling period (subset $\mathcal{A}^+$), its occurrence starting in the next period can be used to move some commodities. Similarly, for each service leg whose worst case starting time falls in the next scheduling period (subset $\mathcal{A}^-$), its occurrence starting in the current period can be used to move some commodities. Notice that a service leg $a \in \mathcal{A}$ such that $\hat{\psi}_a > T$ and $\hat{\phi}_a + \delta_a < T$ does not belong to either $\mathcal{A}^+$ or $\mathcal{A}^-$, and therefore no additional arc associated with it is included in $\mathcal{A}^{STT}$. We illustrate Definition 1 through the following example.

Example 1. *Consider a schedule length $T = 10$, a network composed of three terminals, $\mathcal{N} = \{A, B, C\}$, and a transportation demand composed of four commodities, $\mathcal{K} = \{k_1, k_2, k_3, k_4\}$. Table 1a summarizes the commodity parameters. Three scheduled potential services are available for commodity transportation, $\Sigma = \{\sigma, \xi, \zeta\}$, where $\mathcal{L}(\sigma) = \{l_i(\sigma) : i = 1, 2\}$, $\mathcal{L}(\xi) = \{l(\xi)\}$ and $\mathcal{L}(\zeta) = \{l(\zeta)\}$. The route and schedule of services are shown in Table 1b.*

Figure 1 provides a graphical representation of the data in Table 1 through a time chart. The horizontal axis represents time, while the vertical axis represents the three terminals. Events indicating the starting and ending time of service legs are depicted with black round markers at the corresponding time instant and terminal. Each arrow represents a service leg. The plain arrow represents the travel time without perturbation. The possible maximum departure postponement of service ζ is highlighted by a dashed black line labeled $\delta_{l(\zeta)}$, extending from the usual departure time to the maximum allowable postponement time. Possible outcomes of the travel time for service ζ along its service

Table 1: Data of Example 1.

(a) Commodity requirement.

$\mathcal{K}$	$O(k)$	$D(k)$	$o(k)$	$d(k)$	d^k	δ^k
k_1	A	C	1	8	1	0
k_2	B	A	9	19	1	0
k_3	A	B	8	15	1	0
k_4	A	C	0	4	1	1

(b) Route and schedule of services.

$\mathcal{A}$	$\hat{\phi}_a$	$\hat{\psi}_a$	θ_a	u_a	δ_a
$l_1(\sigma): C \rightarrow A$	6	9	2	4	0
$l_2(\sigma): A \rightarrow B$	11	12	0	4	0
$l(\xi): B \rightarrow C$	3	5	0	4	0
$l(\zeta): B \rightarrow C$	9	11	0	4	2

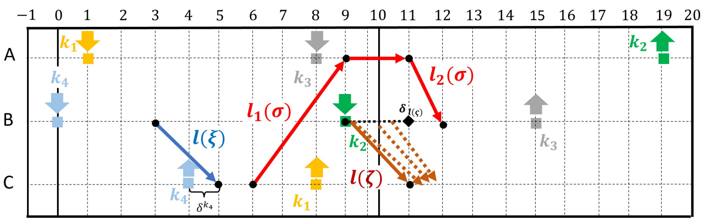


Figure 1: Set $\mathcal{A}$ associated with Example 1.

leg are depicted by dashed arrows. Events representing the availability and due dates of commodities are denoted by solid arrows pointing downwards or upwards, respectively. Moreover, the maximum allowed delay at the destination for commodity k_4 is displayed and denoted by δ^{k_4}. $\mathcal{A}^+ = \{l_1(\sigma), l(\xi)\}$ and $\mathcal{A}^- = \{l_2(\sigma), l(\zeta)\}$. Hence, the set $\mathcal{A}^{STT}$ includes all the service legs in $\mathcal{A}$, i.e., $\{l_1(\sigma), l_2(\sigma), l(\xi), l(\zeta)\}$, two additional service legs $l_1^+(\sigma)$ and $l^+(\xi)$, and two additional service legs $l_2^-(\sigma)$ and $l^-(\zeta)$. Figure 2 shows the set $\mathcal{A}^{STT}$ that the commodities may exploit to reach their destination.

4.2 Two-stage Stochastic SSND Model

Given the information revelation and the decision-making processes described in Section 3, the problem can be formulated as a two-stage stochastic programming model encompassing planning and recourse components. The goal is to minimize the overall cost of the system, i.e., the costs associated with activating services at the tactical level (first stage), as well as the expected costs at the operational level (second stage), i.e.,

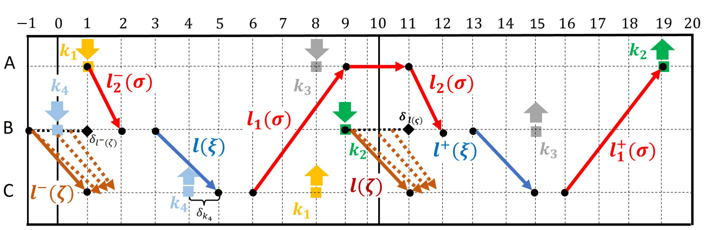


Figure 2: Set $\mathcal{A}^{STT}$ associated with Example 1.

commodity routing and holding costs, outsourcing cost, and costs related to delays of services and commodities.

As mentioned, the uncertain travel time of any service leg $a \in \mathcal{A}$ is represented by a random variable $\tau_a : \Omega \to \mathbb{R}$, where Ω is a probability space, which takes values within a finite support $[\tau_a^m, \tau_a^M]$. Given a specific outcome $\omega \in \Omega$, the realized travel time is denoted by $\tau_a(\omega)$.

The first-stage variables are $y_\sigma \in \{0,1\}$, $\sigma \in \Sigma$, which indicate whether the service σ is selected ($y_\sigma = 1$), or not ($y_\sigma = 0$).

The second stage variables are:
$x_a^k(\omega) \in \{0,1\}$, $k \in \mathcal{K}$, $a \in \mathcal{A}^{STT}$, indicates whether commodity k moves along arc a;
$\pi_a(\omega) \in [0, \delta_a]$, $a \in \mathcal{A}^{STT}$, represents the postponement with respect to the usual departure time of service σ_a on arc a;
$\psi_a(\omega) \geq 0$, $a \in \mathcal{A}^{STT}$, represents the arrival time of service σ_a at the end of arc a;
$\varepsilon_i^k(\omega) \geq 0$, $k \in \mathcal{K}$, $i \in \mathcal{N} \setminus \{D(k)\}$, represents the time instant at which commodity k begins its movement from terminal i;
$\eta_i^k(\omega) \geq 0$, $k \in \mathcal{K}$, $i \in \mathcal{N} \setminus \{O(k)\}$, represents the time instant at which commodity k ends its movement at terminal i;
$r_a(\omega) \geq 0$, $a \in \mathcal{A}$, represents the tardiness in completing travel on arc a relative to its usual ending time;
$r^k(\omega) \in [0, \delta^k]$, $k \in \mathcal{K}$, defines the tardiness of commodity k at its destination;
$z^k(\omega) \in \{0,1\}$, $k \in \mathcal{K}$, indicates whether commodity k is outsourced from $O(k)$ to $D(k)$.

The model is as follows:

$$\min \sum_{\sigma \in \Sigma} f_\sigma y_\sigma + \mathbb{E}_{\tau_a, a \in \mathcal{A}^{STT}} [Q(y; \tau_a(\omega))] \tag{3}$$

where

$$\begin{aligned} Q(y;\tau_a(\omega)) = \min \sum_{k \in \mathcal{K}} d^k \Bigg[& \sum_{a \in \mathcal{A}^{STT}} c_a^k x_a^k(\omega) + \sum_{\substack{i \in \mathcal{N}: \\ i \neq O(k), D(k)}} c_i^k \left(\varepsilon_i^k(\omega) - \eta_i^k(\omega)\right) \\ & + c_{O(k)}^k \left(\varepsilon_{O(k)}^k(\omega) - o(k)\right) + c_{D(k)}^k \left(d(k) - \eta_{D(k)}^k(\omega) + r^k(\omega)\right) \\ & + q^k r^k(\omega) + C^k z^k(\omega) \Bigg] + \sum_{a \in \mathcal{A}} q_a r_a(\omega) \end{aligned} \tag{4}$$

subject to the following constraints:

$$\sum_{a \in \mathcal{A}^{STT}: D(a) = i} x_a^k(\omega) - \sum_{a \in \mathcal{A}^{STT}: O(a) = i} x_a^k(\omega) = \begin{cases} -1 + z^k(\omega) & \text{if } i = O(k), \\ 1 - z^k(\omega) & \text{if } i = D(k), \\ 0 & \text{otherwise} \end{cases} \quad \forall\, k \in \mathcal{K}, i \in \mathcal{N} \tag{5}$$

$$\sum_{k \in \mathcal{K}} d^k \left[x_a^k(\omega) + x_{a^+}^k(\omega)\right] \leq u_a\, y_{\sigma_a} \quad \forall\ a \in \mathcal{A}^+ \tag{6}$$

$$\sum_{k \in \mathcal{K}} d^k \left[x_{a^-}^k(\omega) + x_a^k(\omega)\right] \leq u_a\, y_{\sigma_a} \quad \forall\ a \in \mathcal{A}^- \tag{7}$$

$$\sum_{k \in \mathcal{K}} d^k \, x_a^k(\omega) \leq u_a \, y_{\sigma_a} \qquad \forall \; a \in \mathcal{A} \setminus (\mathcal{A}^+ \cup \mathcal{A}^-) \tag{8}$$

$$\hat{\phi}_{l_{i+1}(\sigma)} + \pi_{l_{i+1}(\sigma)}(\omega) \geq \psi_{l_i(\sigma)}(\omega) + \theta_{l_i(\sigma)} \, y_\sigma \qquad \forall \, \sigma \in \Sigma, \; i = 1, \ldots, |\mathcal{L}(\sigma)| - 1 \tag{9}$$

$$\psi_a(\omega) = \hat{\phi}_a + \pi_a(\omega) + \tau_a(\omega) \, y_{\sigma_a} \qquad \forall \, a \in \mathcal{A} \tag{10}$$

$$\psi_a(\omega) \leq \hat{\psi}_a + r_a(\omega) \qquad \forall \, a \in \mathcal{A} \tag{11}$$

$$\pi_{a^+}(\omega) = \pi_a(\omega) \qquad \forall \; a \in \mathcal{A}^+ \tag{12}$$

$$\psi_{a^+}(\omega) = \psi_a(\omega) + T \qquad \forall \; a \in \mathcal{A}^+ \tag{13}$$

$$\pi_{a^-}(\omega) = \pi_a(\omega) \qquad \forall \; a \in \mathcal{A}^- \tag{14}$$

$$\psi_{a^-}(\omega) = \psi_a(\omega) - T \qquad \forall \; a \in \mathcal{A}^- \tag{15}$$

$$\varepsilon^k_{O(k)}(\omega) \geq o(k) \qquad \forall \, k \in \mathcal{K} \tag{16}$$

$$\varepsilon^k_i(\omega) \geq \eta^k_i(\omega) \qquad \forall \, k \in \mathcal{K}, \; i \in \mathcal{N} \setminus \{O(k), D(k)\} \tag{17}$$

$$\varepsilon^k_i(\omega) \leq 2T \sum_{a \in \mathcal{A}^{STT} : O(a) = i} x^k_a(\omega) \qquad \forall \, k \in \mathcal{K}, \; i \in \mathcal{N} \setminus \{O(k), D(k)\} \tag{18}$$

$$\varepsilon^k_{O(a)}(\omega) \geq \hat{\phi}_a + \pi_a(\omega) - 2T\left(1 - x^k_a(\omega)\right) \qquad \forall \, k \in \mathcal{K}, \; a \in \mathcal{A}^{STT} \tag{19}$$

$$\varepsilon^k_{O(a)}(\omega) \leq \hat{\phi}_a + \pi_a(\omega) + 2T\left(1 - x^k_a(\omega)\right) \qquad \forall \, k \in \mathcal{K}, \; a \in \mathcal{A}^{STT} \tag{20}$$

$$\eta^k_{D(a)}(\omega) \geq \psi_a(\omega) - 2T\left(1 - x^k_a(\omega)\right) \qquad \forall \, k \in \mathcal{K}, \; a \in \mathcal{A}^{STT} \tag{21}$$

$$\eta^k_{D(a)}(\omega) \leq \psi_a(\omega) + 2T\left(1 - x^k_a(\omega)\right) \qquad \forall \, k \in \mathcal{K}, \; a \in \mathcal{A}^{STT} \tag{22}$$

$$r^k(\omega) \geq \eta^k_{D(k)}(\omega) - d(k) \qquad \forall \, k \in \mathcal{K} \tag{23}$$

The objective function (3) includes the costs associated with the selection of the services (first term) plus the expectation, over the travel time probability distributions, of the minimum costs to be paid for adjusting the plan to the revealed time information and routing the demand. The latter is specified in (4) and is given by the routing costs of commodities (first term), holding and handling costs of commodities at intermediate stops (second term), origin (third term) and destination (fourth term). Moreover, it includes the costs for commodities arriving late at destinations (fifth term), the cost of outsourcing commodities (sixth term), and the costs for services arriving late at one of the stops along their route (seventh term).

Flow conservation constraints (5) ensure that each commodity k is routed from origin to destination through a single path when the commodity is not outsourced ($z^k(\omega) = 0$); otherwise a path is not determined since all the variables $x^k_a(\omega)$ equal 0 at the optimal solution level due to cost minimization. Inequalities (6)–(8) are linking-capacity constraints. They ensure that commodities can be routed on a service leg a only if the service σ_a operating on that leg is selected for activation. They also impose a limit on the total quantity of commodities that can be routed on the service leg. For each $a \in \mathcal{A}^+$, (6) ensure that the total volume of demand flowing on arcs a and a^+ does not exceed the capacity available on a; similarly, for each $a \in \mathcal{A}^-$, (7) enforce this limitation for arcs a and a^-; for each $a \in \mathcal{A} \setminus (\mathcal{A}^+ \cup \mathcal{A}^-)$, (8) guarantees that the volume of demand flowing on arc a is within the capacity available on a. The logic of these constraints is illustrated through an example (which extends Example 1) in the Supplementary Material.

Constraints (9)–(15) relate to the time management of services. Constraints (9) ensure that, for those services σ including an intermediate stop, the time instant when the service begins to travel on the service leg $l_{i+1}(\sigma)$ cannot be earlier than the time instant when the service has ended to travel on the previous leg $l_i(\sigma)$, plus the operation time $\theta_{l_i(\sigma)}$ required at the end of $l_i(\sigma)$. Constraints (10) define the time instant when service σ_a ends to travel on arc a as the time instant when the service σ_a begins to travel on arc a plus the travel time on arc a. Note that such a time instant depends on the realization of the random variable $\tau_a(\omega)$. Constraints (11) define the tardiness of arc a as the difference between the actual ending time of the movement of service σ_a and the usual ending time. Constraints (12)–(15) tie the time management of arc a with its past (12)–(13) or future (14)–(15) occurrences, if any.

Finally, constraints (16)–(23) relate to the time management for commodities. Constraints (16) ensure that each commodity departs from origin not before it becomes available. For those nodes i being neither the origin nor the destination of a commodity k, (17) guarantee that the leaving time of k from i is greater than or equal to the time at which k arrived at that node; otherwise, if commodity k does not pass through terminal i, then (18) force variables $\varepsilon_i^k(\omega)$ to 0. Constraints (19)–(20) ensure that the time instant $\epsilon^k_{O(a)}$ when commodity k begins executing arc a equals the time instant when the service associated with a begins to travel on it, if k is loaded on that service. Similarly, (21)–(22) ensure that the time instant $\eta^k_{D(a)}$ when commodity k ends executing arc a equals the time instant when the service associated with a, and on which k is loaded, ends to travel on it. Finally, (23) compute the tardiness of commodity k at destination.

We approximate the travel time probability distributions with a finite set of scenarios $\mathcal{S}$, each of which has dimension $|\mathcal{A}|$. Each scenario $s \in \mathcal{S}$ contains one realization of each travel time distribution and it has a probability p_s of occurrence. Given the set $\mathcal{S}$, the expectation in (3) can be expressed as a linear function of the decision variables and the stochastic program can be formulated as a deterministic mixed integer linear program. The scenario-based mathematical formulation is provided in the Supplementary Material.

5 Computational Results

This section presents the results of computational experiments conducted on a series of small to medium-sized instances utilizing commercial optimization software. Section 5.1 provides details on the instances used and discusses stability results related to the scenario generation procedure. Section 5.2 evaluates the efficiency of the proposed stochastic formulation in comparison to the deterministic formulation. Section 5.3 evaluates the benefits of employing the stochastic formulation over the deterministic one. Section 5.4 analyzes stochastic and deterministic solutions to identify strategies that can

(a) Network $\mathcal{G}_1$. (b) Network $\mathcal{G}_2$.

Figure 3: Physical networks used in the experimentation.

mitigate delays within the service network. Section 5.5 discusses how the network design configuration obtained from the stochastic model differs from that obtained from the deterministic model, by selecting two instances as a case study. The experiments were carried out on a computer equipped with Intel Xeon Gold 5120 CPU processors operating at 2.20 GHz and running the Ubuntu distribution of the Linux operating system. The formulation was implemented in AMPL and the optimization problems were solved with Gurobi Optimizer 10.0.2. Gurobi was configured to use the Network simplex algorithm, with a time limit of 2 hours.

5.1 Instances

Inspired by the dataset employed in Boland et al. (2017) and Hewitt et al. (2019) for a deterministic setting, we generated a set of test instances on two physical networks adapted to our problem setting. The network $\mathcal{G}_1$ consists of 5 terminals and 20 physical links, while $\mathcal{G}_2$ has 6 terminals and 17 physical links. Their topologies and the usual travel times associated with links are shown in Figure 3. We set a schedule length of 72 time units. Compared to the original dataset that contains only direct services, we added services with one intermediate stop, for approximately 15% of the total number of potential services. All services have the same capacity. Furthermore, all direct services incur the same activation cost, while each service with an intermediate stop has an activation cost which is 30% lower than the sum of the activation costs of the two directed services composing it. For those services having an intermediate stop, we set an idle time of 2 units of time between the end of the first leg and the beginning of the second leg. The transportation demand for $\mathcal{G}_1$ and $\mathcal{G}_2$ comprises 20 and 15 commodities, respectively. The volume of each commodity is randomly generated using a uniform distribution between 20% and 80% of a single service capacity. The availability dates are randomly generated over the scheduling period. We consider two classes of delivery-time windows for each network: tight (D-t) and loose (D-l). In both cases, the due date is sampled from a uniform distribution over an interval of length 28 following the availability date. The intervals are centered at 36 and 50 for D-t and D-l, respectively. The routing costs depend on the service travel times, while the unit holding costs are set to 1. The *Truncated*

Gamma (TG) class of probability distributions (Chapman, 1956; Coffey and Muller, 2000) is used to represent the stochastic travel times (Layeb et al., 2018). Each TG distribution varies between 0.95 μ and 1.35 μ, where μ is its mode. We consider two classes of scenarios, S-1 and S-2, where the standard deviation of TG distributions is set to 0.35 and 0.7, respectively. We consider two levels of allowed delays (AD-1 and AD-2) for service departure times and commodity delivery times, equal to 2 and 6 time units, respectively. Finally, we consider two levels (P-1 and P-2) of costs for service delays, commodity late arrivals at destinations, and outsourcing, where the values in P-2 double those in P-1. In P1, outsourcing costs are set higher than the maximum amount the carrier may pay to move a full-capacity load through the most expensive service, while delay costs are set at 15% of those outsourcing costs.

In summary, for each network topology ($\mathcal{G}_1$ and $\mathcal{G}_2$) we have generated 16 classes of instances based on the combination of two demand classes (D-t and D-l), two levels of variability of the TG distributions (S-1 and S-2), two levels of allowed delays (AD-1 and AD-2), and two levels of costs (P-1 and P-2). For each of these 16 classes of instances, we have generated 10 instances randomly. Thus, a total of 320 instances have been generated and solved.

We performed both in-sample and out-of-sample stability tests to assess the quality and representativeness of the generated scenarios, ensuring that no bias was introduced in the optimization results (see Kaut et al., 2007; King and Wallace, 2012, for additional details). Based on this analysis, whose results are reported in the Supplementary Material, we selected a scenario set of size 20 for the resolution of the stochastic model.

5.2 Efficiency of the stochastic approach

The performance of the stochastic formulation, referred to as STT, is evaluated in terms of average computational time (in seconds) and average percentage gap, compared to the deterministic counterpart of the formulation, called DET. Specifically, DET is characterized by a single scenario defined in terms of the usual travel times. We analyze the performance across all classes of instances and for the two network topologies. The results of the analysis are reported in Table 2. These results are averages over the 10 instances belonging to the same class of instances.

Gurobi solved all instances of DET in a very short time, requiring only a few seconds on average to obtain optimal solutions regardless of the network topology considered. However, it appears that looser delivery time windows render the problem slightly more challenging to solve, likely due to the increased number of feasible itineraries for the commodities. Conversely, and as expected, a significantly longer computational time is required to find solutions in the case of STT. The parameter combination D-t (tight delivery time window), AD-1 (2 time units allowed delay), and S-1 (low variability) defines instances that can still be solved to optimality by Gurobi no matter what penalty level or network topology is used. On the other hand, the other parameter combinations render

Table 2: Average computational time (in seconds) and average percentage gap of DET and STT formulations.

			Network $\mathcal{G}_1$				Network $\mathcal{G}_2$			
			P-1		P-2		P-1		P-2	
			Time	Gap	Time	Gap	Time	Gap	Time	Gap
D-t	AD-1	DET	1.93	0.00%	1.86	0.00%	1.20	0.00%	1.60	0.00%
		S-1	4301	0.00%	5293	0.00%	1287	0.00%	1404	0.00%
		S-2	5278	0.17%	4952	0.06%	2523	0.00%	3400	0.11%
	AD-2	DET	3.62	0.00%	3.46	0.00%	2.01	0.00%	2.46	0.00%
		S-1	6751	0.29%	7031	0.48%	3313	0.28%	4063	0.94%
		S-2	7014	0.68%	7200	1.75%	4429	0.55%	5719	2.09%
D-l	AD-1	DET	9.70	0.00%	8.72	0.00%	1.83	0.00%	1.79	0.00%
		S-1	7200	0.93%	7200	0.93%	2222	0.00%	2641	0.00%
		S-2	6743	0.36%	6678	0.83%	3502	0.01%	4348	0.26%
	AD-2	DET	13.20	0.00%	10.49	0.00%	2.58	0.00%	2.36	0.00%
		S-1	7200	1.91%	7200	1.94%	5757	0.25%	6343	0.46%
		S-2	7200	1.59%	7200	2.54%	6854	0.54%	6731	1.30%

the problem marginally harder to solve: Gurobi can compute near-optimal solutions, with generally low optimality gaps, by reaching in some cases the time limit of 2 hours imposed. The most challenging parameter combination for both topologies appears to be D-l (loose delivery time window), AD-2 (6 time units allowed delay), S-2 (high variability), and P2 (high penalty).

These findings suggest that some combinations of the input parameters, such as those identified above, can significantly increase the problem complexity.

5.3 Benefits of the Stochastic Formulation

This analysis aims to quantify the benefits of explicitly incorporating time-stochasticity into the formulation, rather than relying on a traditional deterministic approach. Such benefits are discussed in terms of two types of costs: the network setup costs (i.e., the costs incurred by selecting the services) and the estimated full costs (i.e., the sum of network setup costs, routing and holding costs, costs for delays in service operations and commodities deliveries, and outsourcing). Given a solution obtained through either DET or STT with a scenario set of size 20, the full costs are determined using a simulation-like procedure. This procedure involves solving the second stage of the mathematical model by fixing the services selected in DET or STT and using a scenario set of size 100, generated by the same scenario generation procedure used to construct the scenario sets for the optimization process of STT. This analysis covers all classes of instances and both network topologies. The results obtained are presented in Table 3. The table shows the percentage difference of the setup and full costs of the stochastic solutions with respect to the deterministic ones. Of particular interest are the setup costs of DET and STT for both network topologies. For $\mathcal{G}_1$, STT consistently yields lower setup costs compared to DET.

Table 3: Percentage difference in the setup and full costs between the STT and DET solutions.

			Network $\mathcal{G}_1$				Network $\mathcal{G}_2$			
			P-1		P-2		P-1		P-2	
			Setup Cost	Full Cost	Setup Cost	Full Cost	Setup Cost	Full Cost	Setup Cost	Full Cost
D-t	AD-1	S-1	-11.14%	-2.48%	-10.64%	-5.38%	+7.28%	-9.00%	+0.95%	-10.22%
		S-2	-11.57%	-11.08%	-10.67%	-18.27%	+14.76%	-21.77%	+6.71%	-23.99%
	AD-2	S-1	-10.34%	-2.56%	-10.47%	-4.62%	+3.83%	-6.91%	-3.00%	-13.76%
		S-2	-11.80%	-11.11%	-11.99%	-16.32%	+4.59%	-18.98%	-1.90%	-30.55%
D-l	AD-1	S-1	-3.75%	-4.51%	-9.73%	-6.48%	+1.96%	-18.41%	-1.01%	-23.79%
		S-2	-7.59%	-4.75%	-14.09%	-17.87%	+2.51%	-27.05%	-0.42%	-35.08%
	AD-2	S-1	-1.41%	-4.14%	-10.70%	-9.00%	-0.16%	-16.43%	-1.88%	-24.03%
		S-2	-7.94%	-9.11%	-16.33%	-21.99%	-2.91%	-24.57%	-4.55%	-33.81%

On the other hand, this is not always true for $\mathcal{G}_2$, i.e., for some classes of instances STT activates more services than DET to meet demand. This especially occurs when solving STT under the penalty cost P-1, showing a maximum increase in setup costs for the most volatile case with tight delivery time windows (D-t, AD-1, S-2, P-1). Nevertheless, across all classes of instances and topology networks, STT always exhibits lower full costs than DET. This underscores the advantage of explicitly incorporating the stochastic nature of travel times in tactical planning models, which can mitigate or reduce the economic impacts and consequences of uncertainty. For both network topologies, the greatest full cost reduction of STT with respect to DET is observed when the variability of travel time distributions is high and the reliability is maximally enforced in the formulation through the penalty cost P-2.

The analysis highlights that DET tends to underestimate the operational consequences of time variability, potentially leading to severe delays or high operational costs required to maintain the desired service quality.

5.4 Structural Analysis: Reducing Delay Risk

The purpose of this analysis is to identify the features that stochastic solutions exploit to hedge against time uncertainty.

Table 4 illustrates the composition of the service network given by the DET and STT solutions. In particular, for all parameter combinations, the table presents the percentage difference of the total number of activated services in DET and in STT, as well as disaggregated information in terms of the number of direct and non-direct activated services. In all instances with network $\mathcal{G}_1$, STT operates a lower total number of services compared to DET, sharing however some activated services. This trend is also observable in most classes of instances related to $\mathcal{G}_2$, with exceptions for D-t and P-1,

Table 4: Percentage difference between STT and DET solutions in the number of activated services (total, direct, and non-direct) for networks $\mathcal{G}_1$ and $\mathcal{G}_2$.

			P-1			P-2		
			Total	Direct	Non-dir.	Total	Direct	Non-dir.
Network $\mathcal{G}_1$								
D-t	AD-1	S-1	-11.09%	-11.31%	-10.75%	-11.50%	-7.69%	-22.87%
		S-2	-13.63%	-4.54%	-37.15%	-14.63%	+3.83%	-52.85%
	AD-2	S-1	-10.28%	-10.48%	-9.08%	-11.09%	-8.55%	-20.58%
		S-2	-13.49%	-5.81%	-32.27%	-15.35%	-0.13%	-50.24%
D-l	AD-1	S-1	-4.67%	-0.49%	-12.83%	-10.81%	-6.02%	-20.83%
		S-2	-9.92%	+0.38%	-34.00%	-17.17%	-3.60%	-49.67%
	AD-2	S-1	-1.92%	+0.38%	-6.43%	-12.01%	-5.66%	-22.00%
		S-2	-9.45%	-2.73%	-27.69%	-19.01%	-6.39%	-45.64%
Network $\mathcal{G}_2$								
D-t	AD-1	S-1	+6.34%	+10.26%	-5.00%	-0.57%	+5.85%	-19.33%
		S-2	+11.22%	+26.01%	-34.02%	+2.53%	+19.83%	-55.69%
	AD-2	S-1	+2.65%	+7.69%	-11.17%	-4.67%	+2.60%	-21.08%
		S-2	+1.67%	+14.10%	-33.02%	-5.08%	+8.33%	-47.75%
D-l	AD-1	S-1	+0.30%	+7.82%	-17.11%	-2.87%	+5.22%	-29.71%
		S-2	-1.02%	+14.55%	-43.54%	-5.06%	+14.99%	-61.60%
	AD-2	S-1	-1.94%	+6.11%	-22.25%	-4.09%	+5.35%	-35.74%
		S-2	-5.65%	+6.44%	-37.77%	-8.47%	+8.55%	-56.55%

where the number of activated services in STT is higher than in DET. The reduction in the total number of activated services may be primarily due to the fact that each active service entails the risk of delays and thus associated costs. On the other hand, the number of active direct services in $\mathcal{G}_2$ is always higher in STT than in DET. A notable remark is the consistent reduction in the activation of non-direct services in the STT solutions, no matter the network topology considered. When delays occur on the first leg of these services, they often lead to late arrivals at the second stop, unless the travel time for the second leg is significantly shorter than the usual one, thus mitigating the delay. Given the operational constraints of most transportation systems, where services cannot significantly accelerate due to legal and mechanical limitations, as reflected in the adopted probability distributions, complete absorption of delays is unlikely, resulting in higher risks and costs associated with operating one-stop services. Consequently, STT tends to prioritize more expensive direct connections to less costly multi-stop routes, to try to reduce the potential for additional costs during operations. This behavior may also explain the increase in the number of activated direct services observed in the classes of instances related to $\mathcal{G}_2$, as substituting a non-direct service requires activating two direct services. Note that in the latter cases, despite the increased number of active services, the operational costs associated with the STT solutions are lower compared to the DET ones, as discussed in the previous section. Generally, STT tends to design service networks composed only of direct services which are essential to meet the demand, by replacing non-direct services with direct services as necessary.

Table 5: Percentage difference in the total number of direct transports of commodities to destination, early arrivals of commodities at destination, and routing costs between STT and DET solutions.

			P-1			P-2		
			Direct Itiner.	Early Arrivals	Routing Costs	Direct Itiner.	Early Arrivals	Routing Costs
Network $\mathcal{G}_1$								
D-t	AD-1	S-1	+5.68%	+8.24%	+0.07%	+6.14%	+16.23%	-0.48%
		S-2	+17.65%	+15.97%	+1.95%	+23.33%	+28.71%	+3.13%
	AD-2	S-1	+5.65%	+5.39%	-0.09%	+5.85%	+7.04%	-0.10%
		S-2	+15.88%	+14.59%	+0.60%	+22.74%	+17.36%	+3.30%
D-l	AD-1	S-1	+6.64%	+19.08%	-1.06%	+3.77%	+5.49%	+0.74%
		S-2	+19.27%	+10.99%	+1.34%	+17.35%	+13.83%	+4.63%
	AD-2	S-1	+3.76%	+11.20%	-1.29%	+2.82%	+8.54%	-0.15%
		S-2	+16.53%	+15.71%	-0.45%	+19.18%	+28.63%	+2.13%
Network $\mathcal{G}_2$								
D-t	AD-1	S-1	-0.41%	+21.90%	-0.03%	+1.02%	+14.32%	0.00%
		S-2	+7.89%	+26.26%	+2.27%	+9.50%	+26.67%	+2.25%
	AD-2	S-1	-1.39%	+7.05%	+0.54%	-1.08%	+14.14%	+0.33%
		S-2	+6.93%	+14.96%	+0.97%	+6.52%	+19.06%	+0.95%
D-l	AD-1	S-1	+4.04%	+18.79%	+1.48%	+4.26%	+14.57%	+0.79%
		S-2	+7.09%	+17.16%	+4.14%	+3.95%	+23.55%	+4.85%
	AD-2	S-1	+3.98%	+2.27%	+1.24%	+5.82%	+6.95%	+1.70%
		S-2	+6.56%	+11.39%	+2.09%	+4.71%	+16.22%	+3.70%

Table 5 illustrates some features related to commodity routing in the DET and STT solutions. For all combinations of parameters, the table presents the percentage difference between the total number of direct transports of commodities to destination, the early arrivals of commodities at destination, and routing costs of STT solutions with respect to DET ones. In STT, commodity itineraries slightly deviate from the typical consolidation-based itineraries, where different commodities share the capacity of a single service for most of their journeys, often passing through several intermediate stops. In contrast, the STT solutions show an increase in the percentage of direct and dedicated itineraries for commodities compared to the deterministic solutions. There is also a noticeable decrease in just-in-time arrivals of commodities at destinations with respect to what happens in the deterministic solutions, with more commodities arriving well in advance with respect to their due dates. Despite the fact that different service networks have been designed, routing costs are nearly the same in the STT and DET solutions.

Table 6 illustrates the benefits in terms of reliability in service operations and punctuality of freight delivery at destinations in the solutions obtained through STT. For all parameter combinations, the table presents the percentage difference in delays in operating the service network, as well as the percentage difference in delivery delays for the STT solutions compared to the DET ones. Regarding the service network, delays are reduced in STT compared to DET by at least 12% and up to 45% for most of the classes

Table 6: Percentage difference in service and commodity delays between STT and DET solutions.

			Network $\mathcal{G}_1$				Network $\mathcal{G}_2$			
			P-1		P-2		P-1		P-2	
			Service	Cmdt	Service	Cmdt	Service	Cmdt	Service	Cmdt
D-t	AD-1	S-1	-16.48%	-59.57%	-25.28%	-59.56%	-12.57%	-72.99%	-25.47%	-51.79%
		S-2	-43.06%	-96.27%	-45.53%	-83.07%	-10.72%	-69.06%	-24.30%	-69.48%
	AD-2	S-1	-21.87%	-64.66%	-26.51%	-58.52%	+7.16%	-85.19%	+0.77%	-93.90%
		S-2	-37.97%	-95.49%	-42.47%	-45.93%	-10.32%	-90.90%	-17.76%	-94.12%
D-l	AD-1	S-1	-18.83%	-49.50%	-21.89%	-39.26%	-20.74%	-79.99%	-25.75%	-80.00%
		S-2	-34.25%	-79.75%	-39.94%	-95.99%	-32.89%	-79.83%	-41.23%	-78.20%
	AD-2	S-1	-14.67%	-66.39%	-27.57%	-66.41%	-22.59%	-87.78%	-30.80%	-79.99%
		S-2	-25.97%	-79.99%	-38.36%	-69.73%	-27.68%	-88.48%	-36.96%	-89.52%

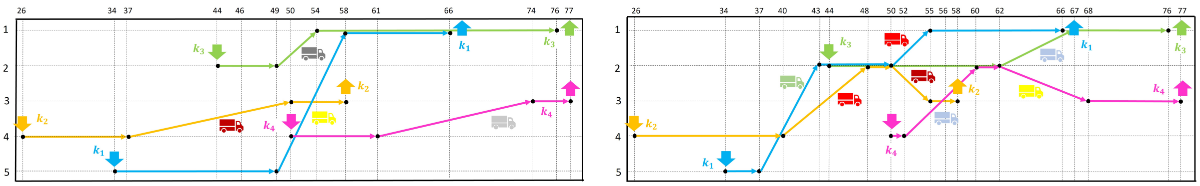
of instances. However, in $\mathcal{G}_2$ for two problem classes, namely D-t, AD-2, and S-1, the delays in service operations are slightly higher in STT compared to DET. In terms of deliveries, delays are at least halved in the STT solutions compared to the DET ones. Furthermore, in some cases, STT almost eliminates delivery delays, achieving reductions of over 90%. This is evident in the classes of instances such as D-t, P-1 and S-2 for $\mathcal{G}_1$ and D-t, AD-2, P-2 for $\mathcal{G}_2$. Finally, we note that increasing the penalty from P-1 to P-2 enhances the reliability of service operations. However, this increase in reliability comes at the expense of delivery punctuality, which decreases slightly.

The results above clearly emphasize that the stochastic formulation is essential to achieve the benefits of delay reductions. Such improvements in service reliability and delivery punctuality would not have been determined with a traditional time-deterministic formulation, underscoring the value of the stochastic approach when quality targets are of interest. Furthermore, enhancing the quality of the service network leads to a reduction in overall operating costs, as shown in Table 3.

5.5 Delay Distributions and Routes for two selected instances

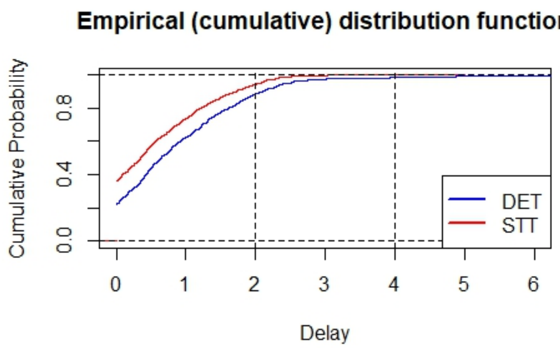
We now analyze how the STT and DET network design configurations differ on two selected instances: one instance of $\mathcal{G}_1$ with parameter combination AD-2, D-t, S-2 and P-2, and one instance of $\mathcal{G}_2$ with parameter combination AD-2, D-l, S-2 and P-1.

Figure 4 shows the STT and DET itineraries of four commodities in the considered instance of $\mathcal{G}_1$, based on a scenario randomly selected from the set of 100 scenarios used in Section 5.3. The comparison is illustrated using the same time chart as Example 1, where the horizontal axis represents time and the vertical axis represents terminals. In the STT solution, each commodity is moved directly from origin to destination through a direct service. The use of direct and dedicated services helps to reduce delays that arise from the need to synchronize multiple shipments, which require consolidation at specific



(a) STT itineraries. (b) DET itineraries.

Figure 4: STT and DET itineraries of four commodities in an instance of $\mathcal{G}_1$.



(a) Empirical cumulative distribution of the delay for an instance in $\mathcal{G}_1$.

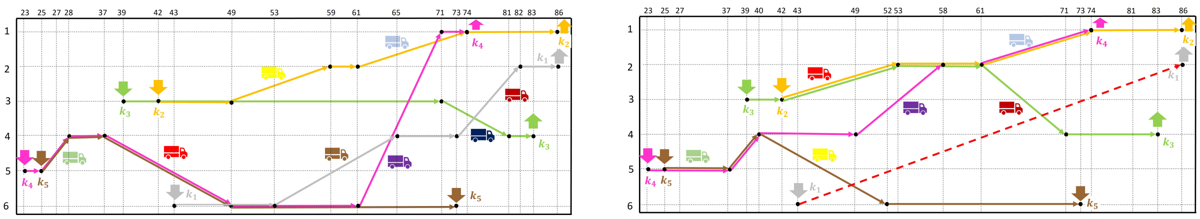
	STT	DET
Min	0.00	0.00
Max	129.32	640.32
Mean	18.03	27.73
Stdev	21.10	38.49
Median	10.40	19.03
Kurtosis	0.62	113.70
Skewness	1.13	7.95

(b) Descriptive statistics of the probability distribution of the delay for an instance in $\mathcal{G}_1$.

Figure 5: Characteristics of the probability distribution of delays for an instance in $\mathcal{G}_1$.

terminals before they can continue their journeys together. On the contrary, the DET solution activates five services, two of them with one intermediate stop, and three out of four commodities require transhipment to arrive at destination. DET solution displays characteristics typical of consolidation-based transportation networks, where different commodities share the capacity of single services for most of their journeys, passing through intermediate stops, before arriving at destination.

Figure 5 reports some statistics related to the total delay in service operations of the considered instance. Figure 5a shows the empirical cumulative (probability) distribution function (CDF) of the delays of the service network for STT and DET. Such a distribution has been estimated using a scenario set of size 100 and considering the delays observed by the whole service network. The CDF of STT exhibits a higher steepness towards its maximum value compared to the CDF of DET. Additionally, the CDF of STT consistently lies above the CDF of DET. This highlights a trend towards shorter delays for STT, which are thus more likely to be observed. In contrast, the broader support and extended tail of the CDF for DET indicate a significant proportion of delays extending beyond the range observed in the red CDF. Figure 5b reports some statistical indicators of the probability density function of STT and DET solutions (we refer to Moore



(a) STT itineraries.

(b) DET itineraries.

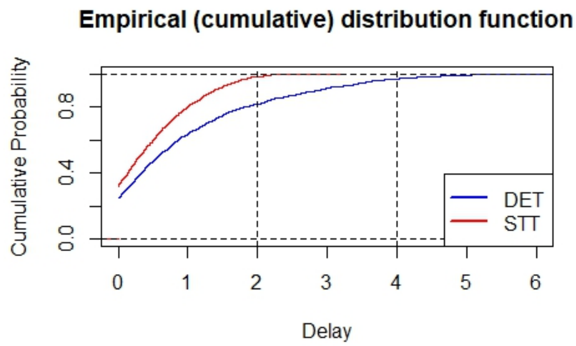
Figure 6: STT and DET itineraries of five commodities in an instance of $\mathcal{G}_2$.

and McCabe, 1989; Newbold et al., 2013, for a detailed explanantion of the mentioned indicators).

Figure 6 shows the STT and DET itineraries of five commodities in the considered instance of $\mathcal{G}_2$, again by randomly selecting one scenario. As mentioned in Section 5.1, in $\mathcal{G}_2$ there are no direct connections between each pair of terminals. This means that some commodities may be forced to move along certain routes despite probable delays. This occurs, for instance, with commodity k_2, where the physical links on which it is moved are the same in both STT and DET. Nevertheless, in DET, its first movement is performed by a non-direct service (red truck), which is more vulnerable to delays. This is replaced in STT by a direct service (yellow truck). Notice also that commodity k_3 is moved by one direct service in STT and two services in DET. Moreover, the solution in DET accounts for a just-in-time arrival of k_4 at destination, which is avoided in STT by defining a more reliable itinerary against time fluctuations. Finally, note that commodity k_1 is outsourced in DET, while it is moved by two services in STT. Figure 7a reports the CDF of the service network delays for STT and DET for the same instance. The higher steepness towards its maximum value of the CDF of STT and the broader support and extended tail of the CDF for DET are even more evident than in Figure 5a. Finally, Figure 7b presents some statistical indicators of the probability density function (PDF) for STT and DET solutions.

6 Conclusions

In this paper, we addressed the Continuous-Time Scheduled Service Network Design problem with Stochastic Travel Times, extending the classical SSND problem to consider business-as-usual fluctuations in travel times. In this setting, we assume service selection is made at the planning stage, before observing the actual travel-time realizations, only considering probability distributions. Once these decisions are made, we assume that more precise travel-time estimates based on system conditions are available to make additional decisions regarding service operations and commodity movements. Penalties for delays of services at stops with respect to the schedule and of commodities at destination with respect to due dates are also computed. The problem has been formulated as a



(a) Empirical cumulative distribution of the delay for an instance in $\mathcal{G}_2$.

	STT	DET
Min	0.00	0.00
Max	82.32	183.80
Mean	15.20	30.35
Stdev	17.23	36.03
Median	9.16	17.38
Kurtosis	0.40	1.79
Skewness	1.10	1.48

(b) Descriptive statistics of the probability distribution of the delay for an instance in $\mathcal{G}_2$.

Figure 7: Characteristics of the probability distribution of delays for an instance in $\mathcal{G}_2$.

two-stage stochastic programming model encompassing planning (i.e., service selection) and recourse (i.e., postponement of service departures, commodities itineraries, and outsourcing). We emphasize that such recourse actions have never been considered together in an SSND formulation with stochastic travel times. The proposed formulation utilizes a Stochastic-Aware Service-Leg Network, which models time continuously and compactly, mitigating the size issues associated with traditional time-space networks. Through extensive experimentation with realistic small-to-medium-size instances, we assessed the complexity and benefits of the stochastic formulation compared to the deterministic one. The results emphasize that our approach effectively mitigates delays and improves operational costs, highlighting specific features to hedge against time fluctuations appearing in stochastic solutions.

Future research directions include extending the current formulation to incorporate additional management aspects, such as resource allocation (vehicles or containers), or exploring other sources of uncertainty related to demand (volume or availability dates) or operation times at terminals. Finally, developing efficient solution methods is essential for handling larger instances.

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7 Supplementary material

7.1 Notation used in the mathematical models

In Table 7, we report the notation used in the mathematical models.

Table 7: Sets and parameters used in the mathematical models.

Sets	
$\mathcal{N}$	set of physical terminals
$\mathcal{A}^{PH}$	set of physical links between terminals
Σ	set of the potential services
$\mathcal{L}(\sigma)$	set of service legs of service $\sigma \in \Sigma$
$\mathcal{A}$	set of service legs of all the potential services in Σ
$\mathcal{A}^{STT}$	set of legs of the Stochastic-Aware Service-Leg Network
$\mathcal{K}$	set of commodities
Ω	set of possible outcomes of the travel time random variable
$\mathcal{S}$	set of scenarios
Parameters	
T	Schedule length
$O(k)$	origin of commodity $k \in \mathcal{K}$
$D(k)$	destination of commodity $k \in \mathcal{K}$
$o(k)$	availability date of commodity $k \in \mathcal{K}$
$d(k)$	due date of commodity $k \in \mathcal{K}$
d^k	volume of commodity $k \in \mathcal{K}$
δ^k	maximum allowed delay for commodity $k \in \mathcal{K}$
$O(\sigma)$	origin of service $\sigma \in \Sigma$
$D(\sigma)$	destination of service $\sigma \in \Sigma$
u_a	capacity of service leg $a \in \mathcal{A}$
τ_a	travel time random variable associated with service leg $a \in \mathcal{A}$, having support $[\tau_a^m, \tau_a^M]$
$\tau_a(\omega)$	travel time random variable realization of service leg $a \in \mathcal{A}$
τ_{as}	observed travel time of service leg $a \in \mathcal{A}$ in scenario $s \in \mathcal{S}$
$\theta_{l_i(\sigma)}$	time spent in terminal by service $\sigma \in \Sigma$ after traveling on service leg $l_i(\sigma) \in \mathcal{L}(\sigma)$
$\hat{\phi}_a$	usual time instant when service $\sigma_a \in \Sigma$ begins to travel on arc $a \in \mathcal{A}$
$\hat{\psi}_a$	usual time instant when service $\sigma_a \in \Sigma$ ends to travel on arc $a \in \mathcal{A}$
δ_a	maximum allowed postponement of the starting time of service σ_a on arc a
f_σ	activation cost of service $\sigma \in \Sigma$
c_a^k	unit transportation cost of commodity $k \in \mathcal{K}$ and arc $a \in \mathcal{A}$
c_i^k	unit holding and handling cost of commodity $k \in \mathcal{K}$ at node $i \in \mathcal{N}$
C^k	cost of outsourcing of commodity $k \in \mathcal{K}$
q^k	penalty for late arrival of commodity $k \in \mathcal{K}$ at destination
q_a	penalty for late arrival of service $\sigma_a \in \Sigma$ in ending traveling on its service leg $a \in \mathcal{A}$
p_s	probability of scenario $s \in \mathcal{S}$
$O(a)$	origin of arc a
$D(a)$	destination of arc a

7.2 Complement to Example 1

This section builds upon the Example 1 presented in Section 4.1.

Example 1 (continued)**.** *A possible solution to the instance considered in Example 1 is depicted in Figure 8. The itineraries of the four commodities k_1, k_2, k_3, and k_4 are shown using yellow, green, gray, and orange arrows, respectively. The figure shows that both k_1 and k_3 use the second leg of service σ. Specifically, within the Stochastic-Aware Service-Leg Network, k_1 is routed through $l_2^-(\sigma)$, while k_3 uses $l_2(\sigma)$. These two legs refer to the same service σ, with $l_2^-(\sigma)$ representing the same transportation segment* *as $l_2(\sigma)$ but occurring $T = 10$ time units earlier. Given the regularity of transportation demand — that is, its repetition every $T = 10$ time units (e.g., k_1 requires transportation between $o(k_1)+T$ and $d(k_1)+T$) — we must ensure that the combined volume of commodities k_1 and k_3 does not exceed the capacity of service leg $l_2(\sigma)$. A similar consideration applies to the itineraries of k_1 and k_2, particularly for service legs $l(\xi)$ and $l^+(\xi)$. Finally, observe that two options are available for transporting commodity k_4. The first involves using service leg $l(\xi)$, which delivers k_4 after its due date, but still within its maximum allowable delay δ^{k_4}. The second option entails postponing the departure of service leg $l^-(\zeta)$ to a time between 0 and 1, as illustrated in the figure, allowing k_4 to reach its destination before the due date. The service leg $l^-(\zeta)$ exemplifies an additional occurrence of the original leg $l(\zeta)$, which would not have been included in the Extended Service-Leg Network under a deterministic setting.*

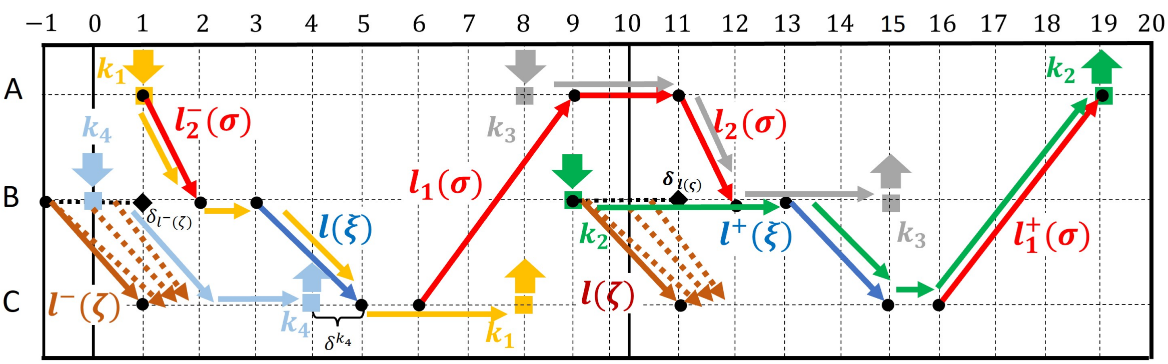

Figure 8: Set $\mathcal{A}^{STT}$ associated with Example 1.

7.3 Scenario-based Mathematical Formulation

In the following, we report a scenario-based approximation of the stochastic two-stage model described in Section 4.2, where we approximate the random travel time probability distributions through a set of scenarios. Specifically, each scenario represents a possible realization of the travel time for each service leg of each potential service, and has a probability to be observed.

Let $\mathcal{S}$ be the considered set of scenarios. Each scenario $s \in \mathcal{S}$ has dimension $|\mathcal{A}|$. A probability p_s is assigned to each scenario $s \in \mathcal{S}$, with $0 \leq p_s \leq 1$ and $\sum_{s \in \mathcal{S}} p_s = 1$. Lastly, we define as τ_{as} the travel time realization for service $\sigma_a \in \Sigma$ along its service leg $a \in \mathcal{A}$ in scenario $s \in \mathcal{S}$. We define nine sets of variables.

The first stage variables are:

- $y_\sigma \in \{0,1\}$, $\sigma \in \Sigma$, which indicate whether the service σ is selected ($y_\sigma = 1$), or not ($y_\sigma = 0$).

The second stage variables are:

- $x^k_{as} \in \{0,1\}$, $k \in \mathcal{K}$, $a \in \mathcal{A}^{STT}$, indicates whether commodity k moves along arc a in scenario $s \in \mathcal{S}$;
- $\pi_{as} \in [0, \delta_a]$, $a \in \mathcal{A}^{STT}$, represents the postponement with respect to the usual departure time of service σ_a on arc a in scenario $s \in \mathcal{S}$;
- $\psi_{as} \geq 0$, $a \in \mathcal{A}^{STT}$, represents the arrival time of service σ_a at the end of arc a in scenario $s \in \mathcal{S}$;
- $\varepsilon^k_{is} \geq 0$, $k \in \mathcal{K}$, $i \in \mathcal{N} \setminus \{D(k)\}$, represents the time instant at which commodity k begins its movement from terminal i in scenario $s \in \mathcal{S}$;

- $\eta^k_{is} \geq 0$, $k \in \mathcal{K}$, $i \in \mathcal{N} \setminus \{O(k)\}$, represents the time instant at which commodity k ends its movement at terminal i in scenario $s \in \mathcal{S}$;
- $r_{as} \geq 0$, $a \in \mathcal{A}$, represents the tardiness in completing travel on arc a relative to its usual ending time in scenario $s \in \mathcal{S}$;
- $r^k_s \in [0, \delta^k]$, $k \in \mathcal{K}$, defines the tardiness of commodity k at destination in scenario $s \in \mathcal{S}$;
- $z^k_s \in \{0,1\}$, $k \in \mathcal{K}$, indicates whether commodity k is outsourced from $O(k)$ to $D(k)$ in scenario $s \in \mathcal{S}$.

Mathematical formulation

$$\begin{aligned} \min \sum_{\sigma \in \Sigma} f_\sigma y_\sigma + \sum_{s \in \mathcal{S}} p_s \Bigg[\sum_{k \in \mathcal{K}} d^k \Big(& \sum_{a \in \mathcal{A}^{STT}} c^k_a x^k_{as} + \sum_{\substack{i \in \mathcal{N}: \\ i \neq O(k), D(k)}} c^k_i \left(\varepsilon^k_{is} - \eta^k_{is}\right) \\ & + c^k_{O(k)} \left(\varepsilon^k_{O(k)s} - o(k)\right) + c^k_{D(k)} \left(d(k) - \eta^k_{D(k)s} + r^k_s\right) \\ & + q^k r^k_s + C^k z^k_s \Big) \Bigg] + \sum_{a \in \mathcal{A}} q_a r_{as} \end{aligned} \tag{24}$$

Commodity routing management

$$\sum_{a \in \mathcal{A}^{STT}: D(a) = i} x^k_{as} - \sum_{a \in \mathcal{A}^{STT}: O(a) = i} x^k_{as} = \begin{cases} -1 + z^k_s & \text{if } i = O(k), \\ 1 - z^k_s & \text{if } i = D(k), \\ 0 & \text{otherwise} \end{cases} \qquad \begin{aligned} &\forall\, k \in \mathcal{K}, \\ &\forall\, i \in \mathcal{N}, \\ &\forall\, s \in \mathcal{S} \end{aligned} \tag{25}$$

$$\sum_{k \in \mathcal{K}} d^k \left[x^k_{as} + x^k_{a^+ s}\right] \leq u_a\, y_{\sigma_a} \qquad \forall\ a \in \mathcal{A}^+,\ \forall\, s \in \mathcal{S} \tag{26}$$

$$\sum_{k \in \mathcal{K}} d^k \left[x^k_{a^- s} + x^k_{as}\right] \leq u_a\, y_{\sigma_a} \qquad \forall\ a \in \mathcal{A}^-,\ \forall\, s \in \mathcal{S} \tag{27}$$

$$\sum_{k \in \mathcal{K}} d^k\, x^k_{as} \leq u_a\, y_{\sigma_a} \qquad \forall\ a \in \mathcal{A} \setminus (\mathcal{A}^+ \cup \mathcal{A}^-),\ \forall\, s \in \mathcal{S} \tag{28}$$

Service time management

$$\hat{\phi}_{l_{i+1}(\sigma)} + \pi_{l_{i+1}(\sigma)s} \geq \psi_{l_i(\sigma)s} + \theta_{l_i(\sigma)}\, y_\sigma \qquad \forall\, \sigma \in \Sigma,\ i = 1, \ldots, |\mathcal{L}(\sigma)| - 1,\ \forall\, s \in \mathcal{S} \tag{29}$$

$$\psi_{as} = \hat{\phi}_a + \pi_{as} + \tau_{as} \cdot y_{\sigma_a} \qquad \forall\, a \in \mathcal{A},\ \forall\, s \in \mathcal{S} \tag{30}$$

$$\psi_{as} \leq \hat{\psi}_a + r_{as} \qquad \forall\, a \in \mathcal{A},\ \forall\, s \in \mathcal{S} \tag{31}$$

$$\pi_{a^+ s} = \pi_{as} \qquad \forall\ a \in \mathcal{A}^+,\ \forall\, s \in \mathcal{S} \tag{32}$$

$$\psi_{a^+ s} = \psi_{as} + T \qquad \forall\ a \in \mathcal{A}^+,\ \forall\, s \in \mathcal{S} \tag{33}$$

$$\pi_{a^- s} = \pi_{as} \qquad \forall\ a \in \mathcal{A}^-,\ \forall\, s \in \mathcal{S} \tag{34}$$

$$\psi_{a^- s} = \psi_{as} - T \qquad \forall\ a \in \mathcal{A}^-,\ \forall\, s \in \mathcal{S} \tag{35}$$

Commodity time management

$$\varepsilon^k_{O(k)s} \geq o(k) \qquad \forall\, k \in \mathcal{K},\ \forall\, s \in \mathcal{S} \tag{36}$$

$$\varepsilon^k_{is} \geq \eta^k_{is} \qquad \forall\, k \in \mathcal{K},\ \forall\, i \in \mathcal{N} \setminus \{O(k), D(k)\},\ \forall\, s \in \mathcal{S} \tag{37}$$

$$\varepsilon^k_{is} \leq 2T \sum_{a \in \mathcal{A}^{STT}: O(a) = i} x^k_{as} \qquad \forall\, k \in \mathcal{K},\ \forall\, i \in \mathcal{N} \setminus \{O(k), D(k)\},\ \forall\, s \in \mathcal{S} \tag{38}$$

$$\varepsilon^k_{O(a)s} \geq \hat{\phi}_a + \pi_{as} - 2T\left(1 - x^k_{as}\right) \qquad \forall\, k \in \mathcal{K},\ \forall\, a \in \mathcal{A}^{STT},\ \forall\, s \in \mathcal{S} \tag{39}$$

$$\varepsilon^k_{O(a)s} \leq \hat{\phi}_a + \pi_{as} + 2T\left(1 - x^k_{a}\right) \qquad \forall\, k \in \mathcal{K},\ \forall\, a \in \mathcal{A}^{STT},\ \forall\, s \in \mathcal{S} \tag{40}$$

$$\eta^k_{D(a)s} \geq \psi_{as} - 2T\left(1 - x^k_{as}\right) \qquad \forall\, k \in \mathcal{K},\ \forall\, a \in \mathcal{A}^{STT},\ \forall\, s \in \mathcal{S} \tag{41}$$

$$\eta^k_{D(a)s} \leq \psi_{as} + 2T\left(1 - x^k_{as}\right) \qquad \forall\, k \in \mathcal{K},\ \forall\, a \in \mathcal{A}^{STT},\ \forall\, s \in \mathcal{S} \tag{42}$$

$$r^k_s \geq \eta^k_{D(k)s} - d(k) \qquad \forall\, k \in \mathcal{K},\ \forall\, s \in \mathcal{S} \tag{43}$$

7.4 In-sample and out-of-sample stability

We conducted in-sample and out-of-sample stability verification to evaluate the accuracy of our scenario generation procedure and the representativeness of the generated scenarios, and, thus, avoid introducing bias in the results of the optimization model. In fact, once both in-sample and out-of-sample stabilities are verified, we can confidently ensure that the solutions obtained by solving the stochastic formulation are unaffected by any bias introduced by the specific scenario set used in the resolution process (see Kaut et al., 2007; King and Wallace, 2012, for additional details).

We conducted stability tests for both network topologies, i.e., $\mathcal{G}_1$ and $\mathcal{G}_2$, by considering instances belonging to S-2 (the highest variability level), D-t and D-l (tight and loose delivery-times), AD-1 and AD-2 (2 and 6 units of time of delay for service departure times and commodity delivery time), and P-2 (the highest level of penalty). Approximating a probability distribution using a set of scenarios introduces noise, which decreases as the number of scenarios increases. However, this also raises the complexity of obtaining solutions. Thus, there is a trade-off between stability and problem size. Preliminary tests indicated that scenario sets of size 20 yielded satisfactory results, meeting our goals for solution accuracy and acceptable computational time.

Regarding in-sample stability, each instance from the aforementioned classes was solved 10 times, each with a different scenario set of size 20. The best objective function values, obtained within a 2-hour time limit, were collected and analyzed. For each instance, three statistical indicators were used for the analysis: the relative range, i.e., the difference between the highest and the lowest objective function values, the relative mean deviation, and the relative median deviation (note that, although the mean is a more classical indicator, it may not represent the value of a possible solution as the median does). Table 8 reports average aggregated results related to three mentioned statistical indicators, namely the average range (column 1), the average relative mean deviation (column 2), and the average relative median deviation (column 3) across instances of the same classes, for $\mathcal{G}_1$ and $\mathcal{G}_2$, respectively.

Regarding out-of-sample stability, the solutions obtained during the in-sample testing were evaluated by approximating the "true" travel time probability distributions resorting to a 100-scenario-sized set, generated from the same TG distribution used to construct the scenario sets for the optimization process. Table 9 shows average aggregated results related to the three above-mentioned statistical indicators across instances of the same classes, for $\mathcal{G}_1$ and $\mathcal{G}_2$, respectively.

It is worth noting that the reported results pertain to the highest variability level, representing the most volatile situations. In less variable cases, the obtained values are even lower. Consequently, all the results discussed in the next sessions are obtained by solving stochastic formulations with a scenario size of 20.

Table 8: In-sample stability results for the scenario-generation procedure.

Network $\mathcal{G}_1$						
	Avg. Relative Range		Avg. Relative Mean Deviation		Avg. Relative Median Deviation	
	AD-1	AD-2	AD-1	AD-2	AD-1	AD-2
D-t	4.88%	3.90%	1.57%	1.21%	0.81%	0.71%
D-l	5.99%	3.30%	1.94%	1.02%	1.33%	0.50%
Network $\mathcal{G}_2$						
	Avg. Relative Range		Avg. Relative Mean Deviation		Avg. Relative Median Deviation	
	AD-1	AD-2	AD-1	AD-2	AD-1	AD-2
D-t	9.64%	5.43%	3.60%	1.92%	3.46%	1.77%
D-l	6.43%	3.84%	2.27%	1.31%	2.41%	1.32%

Table 9: Out-of-sample stability results for the scenario-generation procedure.

Network $\mathcal{G}_1$						
	Avg. Relative Range		Avg. Relative Mean Deviation		Avg. Relative Median Deviation	
	AD-1	AD-2	AD-1	AD-2	AD-1	AD-2
D-t	3.59%	1.85%	1.38%	0.74%	1.32%	0.85%
D-l	4.57%	1.03%	1.71%	0.34%	1.30%	1.12%
Network $\mathcal{G}_2$						
	Avg. Relative Range		Avg. Relative Mean Deviation		Avg. Relative Median Deviation	
	AD-1	AD-2	AD-1	AD-2	AD-1	AD-2
D-t	5.43%	1.98%	2.41%	0.74%	4.04%	0.60%
D-l	4.04%	0.69%	1.65%	0.24%	2.15%	0.35%