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## **An Integrated Modeling Approach for Day-Before Planning in Two-Tier Multi-Modal City Logistics with Satellite Synchronization**

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**Abstract***.* In designing and managing city logistics systems, the consolidation of freight, the coordination of transport activities and a resource-efficient use of urban infrastructure is of upmost importance. The considered day-before planning problem consists of shipping inbound, outbound and inner-city demands through a two-tiered network of available public and private transportation services. Rarely considered in the literature, we deal with flexible freight handover locations without storage capacity and address the waiting activities of vehicles. This requires an exact synchronization of both tiers and a special focus on temporal aspects in the mathematical description. We present a path-based model with continuous time variables involving concepts to synchronize multi-directional demand handovers and estimate waiting durations of vehicles. A numerical study is conducted on a vast new instance set featuring road- and rail-based outer-tier transportation and a variety of instance parameter settings. The results validate the mathematical description and provide valuable methodological and managerial insights. We propose and successfully apply three strategies to improve the performance of general mixed-integer programming methods. Further, the effects of different problem characteristics are discussed.

**Keywords**: Transportation, City Logistics, day-before planning, satellite synchronization, waiting times

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## **1 Introduction**

In supporting a vast majority of economic and social activities, commodity flows play a significant role in urban areas. To keep cities livable, the economic stability must be maintained while the environmental and social nuisances of shipping must be reduced. This goal is pursued by designing and managing efficient freight transportation systems in metropolitan areas, which is a key aspect of *City Logistics (CL)* (Taniguchi et al., 2001; Crainic et al., 2009). Deliveries and pickups of commodities requested by private and business customers need to be performed by transportation services. While logistics service providers (LSPs) mainly aim at minimizing the operating costs, governmental actors care for lowering carbon emissions and increasing people's well-being. To limit traffic and congestion, the main strategies are (a) the consolidation of freight, (b) an effective coordination of transport activities and (c) the adaption of vehicles types to the environment (Bektaş et al., 2017; Crainic et al., 2020). Environmental and social effects are additionally bound through legal regulations, such as maximum vehicle sizes, access time windows and congestion fees (Muñuzuri et al., 2005).

Various studies identify actions to implement CL strategies on the strategic, tactical and operational planning level: (i) multi-tier and multi-modal transportation systems, where commodities travel step-wise in heterogeneous vehicles changed at handover locations (Savelsbergh and Van Woensel, 2016; Ben Mohamed et al., 2017; Boysen et al., 2020), (ii) the consolidation of multi-directional freight (Crainic et al., 2012; Ben Mohamed et al., 2017), (iii) the integration of freight into public transportation (Masson et al., 2017; Elbert and Rentschler, 2022; Li et al., 2023), and (iv) collaborations of LSPs (Taniguchi, 2014; Savelsbergh and Van Woensel, 2016; Boysen et al., 2020). All of them induce a more efficient use of idle capacities of transportation resources like delivery vehicles, but complicate planning. While (i) and (ii) require a greater focus on temporal aspects to synchronize activities, (iii) and (iv) involve limitations like predefined routes, schedules and capacities of transportation services. Modeling and planning approaches that implement and evaluate these actions and their beneficial effects have been developed in the last years. However, a general mathematical description that allows the integration of all of them in a tactical planning approach is still missing (Bektaş et al., 2017).

Based on the modeling framework of Crainic et al. (2009), this work provides such a mathematical model for a *two-tier multi-modal CL (2TM-CL) system* with multi-directional commodity flows and predefined transportation services, which may represent public transport lines, spare capacities of LSPs or a result of a more aggregate planning. While in general transportation logistics, the fulfillment of estimated demands in a given transportation network is considered on tactical level, day-to-day variability in resource availability and demand of various commodities must be managed in CL. The resulting planning problems are denoted *day-before planning* (Crainic et al., 2009), where detailed individual demand data like availability times and time windows (TWs), transportation services with predefined routes and schedules, and satellite locations with spatial features are given. In addition, our model combines the goals of all stakeholders. It involves the selection of transportation services to operate, the assignment of demand flows and the scheduling of travel and handling activities so that the operating costs of the system and the environmental and social effects are minimal.

As shown in Figure 1a, the physical 2TM-CL network considered follows a two-tier setting often applied in metropolitan areas, see (Savelsbergh and Van Woensel, 2016; Bektaş et al., 2017; Boysen et al., 2020). *City distribution centers* (CDCs) are located in the cities' outskirts or rural areas. They act as hubs for trans-regional and international, inbound and outbound freight, where commodities are handled and transferred between interurban and urban transport modes. *Satellite* locations, also known as urban hubs (Ben Mohamed et al., 2017) or micro-hubs (Boysen et al., 2020), are located in the urban area as a second level of (de-)consolidation facilities. They enable the use of environmentally friendly vehicles in the inner city, where customers are settled and the desire for traffic reduction is highest. Transportation on the outer tier between CDCs and satellites is operated road- or rail-based with small trucks, trams, or tram compartments, named *urban vehicles*, while a fleet of smaller *city freighters* like electric cars and cargo bikes travels on the inner tier between satellites and customer locations. Note that the mostly circular geography of urban areas particularly allows the consolidation of commodities flowing into, out of and inside the city. However, this kind of consolidation is often neglected.

Another important aspect in day-before planning of 2TM-CL systems is their demand for traffic and building infrastructure. In urban areas, spatial resources are sparse so that an efficient use and sharing of existing infrastructure is essential. So far, this is poorly addressed with regard to freight handling, storage and parking areas in multi-tier transportation. Typical assumptions are sufficient storage capacity at satellites and the possibility for vehicles to wait for unlimited amounts of time at any location. In this work, we investigate the opportunities and challenges created by using existing areas like parking lots of supermarkets and gas stations or tram stations, which so far show an uneven usage over time. While such satellite locations provide flexibility in location decisions, the lack of storage necessitates meetings of vehicles for handovers. Restrictions of an exact synchronization of the two tiers at the satellites are implied. We refer to this requirement as *satellite synchronization*, which represents simultaneous operation synchronization according to Drexl (2012). When waiting is allowed without restrictions, a first study on satellite synchronization by Grangier et al. (2016) has shown that urban vehicles tend to park at satellites and act as storage facilities. Therefore, the *minimization of the waiting activities* of delivery vehicles is explicitly addressed here. To do so, the mathematical model uses continuous time variables to track vehicle activities finely detailed.

The day-before planning problem considers a typical delivery period limited to a few hours, in which transport vehicles have access and parking rights in the inner city area. Customer locations may be individual households, apartment buildings or business facilities with different restrictions on the maximum waiting duration of vehicles. Note that we use the term *duration* to indicate the lengths of a lasting activity, like waiting and handovers, while the term *time* describes moments at which a certain action is happening, like start of travel. *Demands* constitute flows of individual commodities with given origin, destination, volume and time restrictions. Next to the predominant inbound demands traveling from external locations to customers  $(e2c)$ , we consider customer-to-external (c2e) and customer-to-customer (c2c) flows. On the outer tier, a homogeneous fleets of trucks or trams with given routes, schedules, capacities and fixed cost is operated, while a fleet of identical cargo bikes each with a predetermined route and fixed cost ships demands on the inner tier. Dependent on the outer-tier transportation mode, satellites are formed by parking lots or trams stations given with a maximum number of vehicles allowed to be present at the same time, a maximum waiting duration and no storage capacity.



Figure 1: Representations of the 2TM-CL System

On a two-tier network as shown in Figure 1b, we define the *two-tier multi-modal city logistics planning problem with satellite synchronization* (2TM-CLP-SS) as selecting transportation services for both tiers, including a schedule for each of them, and allocating all demands so that capacity and timing restrictions are met at minimal distance-based operating costs and waiting durations. Note that the existence of a transportation link between two locations in the network is defined by the set of given services.

The 2TM-CLP-SS shares common characteristics with planning problems in scheduled service network design (SSND) and the two-echelon vehicle routing problem with time windows (2E-VRPTW), both constituting NP-hard problems in the area of transportation and logistics. Even if it is not surprising that general mixed-integer programming (MIP) methods struggle in solving 2TM-CLP-SS instances, there exists no indication which problem characteristics or parameters increase planning complexity and which research directions are promising to development dedicated solution methods. With a comprehensive computational study on a vast new set of real world-inspired instances, this study fills this gap. A state-of-the-art MIP solver is used to validate the proposed mathematical model and provide valuable insights on the price of planning flexibility in several dimensions. Effects of multi-directional demand consolidation, the number of satellite locations given, the widths of demand TWs and different wait permission policies for customer locations on solution quality and structure are discussed. In addition, simple techniques that improve the performance of general MIP methods by predetermining some selection and assignment decisions based on the instance data or the CL goals described above are proposed. The results reveal the appearance of feasibility issues that need to be minded when taking real-world design decisions and developing future solution methodologies.

The contributions of this work are:

1. a general mathematical formulation combining SSND and 2E-VRPTW theory and focusing on temporal aspects like satellite synchronization and waiting duration minimization,

- 2. an enhanced estimation scheme for vehicle waiting durations in scenarios with multidirectional demands and satellite synchronization,
- 3. techniques to facilitate the usage of general MIP methods in CL planning and methodological insights on future planning methods,
- 4. managerial insights on the effect of instance characteristics on planning complexity and feasibility.

The remainder of the paper is organized as follows. Section 2 reviews the modeling approaches applied to 2TM-CL systems in the literature. In Section 4, the 2TM-CLP-SS and its notation are introduced in detail. The fundamental basics of modeling satellite synchronization and determining waiting durations are presented in Section 4 followed by the full mathematical formulation in Section 5. Section 6 contains model enhancements and three problem-specific facilitating methods for general MIP solving. In Section 7, the computational study is reported, and the managerial and methodological insights are discussed. We conclude in Section 8.

## **2 Related Work**

Next to existing works on deterministic day-before planning, this section summarize studies on SSND and 2E-VRPTW with a special emphasis on the CL actions our modeling approach is focused on.

Crainic et al. (2009) propose the first general path-based formulation for inbound transportation planning in a 2TM-CL system using SSND. A time-expanded network is used and, for each demand, a set of possible itineraries is predetermined. Consequently, a selection of itineraries with minimal operating costs must be found. Since the demand itineraries are fully defined from origin to destination, the complicating aspect of synchronization between tiers is hidden in a non-trivial preprocessing step. Satellite capacity restrictions are approximately implemented on a discrete-time basis. Following this modeling approach, Crainic and Sgalambro (2013) show by means of two SSND formulations that fixed and flexible demand-satellite assignments can be considered in outer-tier planning. Fontaine et al. (2021) develop an extended discrete-time SSND formulation for the 2TM-CL setting including c2e demands and resource management decisions. The problem consists in finding a cost-minimal selection of services and determining demand itineraries through CDC and satellite assignment. Thereby, limited storage capacities at satellites are respected and the distribution on the inner tier is approximated. Hewitt and Lehuédé (2023) propose an alternative modeling approach for SSND based on arc-wise demand consolidation options. Even if working with continuous time variables is advantageous, similar to predefining demand itineraries, the determination of all possible consolidation options becomes challenging if the number of demands and the size of the transportation network increase.

Li et al. (2023) model and plan shared freight and passenger transportation in a high-speed railway network with 6 train lines and storage spaces at three satellite stations. A time-discrete path-based SSND model forms the basis for a Benders' decomposition algorithm. Due to the structure of the considered railway network, reverse commodity flows appear but can never be consolidated. This highlights local transportation services, such as tram or bus lines and freight delivery in inner cities, as our main application areas. Satici and Dayarian (2024) present a matheuristic decision pipeline for an intra-city express package service in a single-tier network. The authors use a time-discrete multi-commodity service network design formulation to model the tactical planning level. Based on estimated demands, commodity paths and cyclic transportation services between 48 locations are determined in a three-step procedure. To meet the actual demands on the operational level, a VRPTW is solved by tabu search to provide additional shipping options. This shows that day-before planning problems considering detailed demand information and given transportation services are highly relevant.

Cattaruzza et al. (2017) provide a survey on applications of vehicle routing models in CL and highlight multi-tier transportation systems as a significant challenge. Consequently, some studies approximate parts of the multi-tier setting. Masson et al. (2017) plan e2c deliveries in a 2TM-CL system involving public buses on the outer tier. While a fixed bus schedule is considered and no storage exists at the bus stations as satellites, the inner tier planning problem is described as a VRPTW with minimization of the number of city freighters used and the total distance traveled. Ben Mohamed et al. (2017) present a hierarchical decision structure for a physical internet-enabled two-tier urban transportation network with e2c and c2e demands. Based on predetermined assignments of customers and city freighters to satellites, the authors consider a day-before planning problem for the inner tier with time-dependent congestion levels. The time of the day is discretized and, for each period, the set of demands to be served and corresponding city freighter routes are determined. The objective consists of minimizing the transportation costs, the total travel distance and a penalty for postponed demands. Since synchronization between tiers is not explicitly considered, storage capacity at satellites is assumed.

Grangier et al. (2016) consider a 2E-VRPTW with satellite synchronization for e2c demand only. The goal is to determine routes and continuous start time sequences for the vehicles on both tiers so that the fleet sizes and travel costs are minimal. City freighters can perform multiple trips, while urban vehicles are not allowed to visit customers. Anderluh et al. (2021) deal with the same setting, but allow demands to be delivered by urban vehicles. In addition, the authors consider multiple objectives including time- and distance-related total transportation cost, the total greenhouse gas emissions and external factors like disturbances of inhabitants. Therein, waiting durations of vehicles at satellites, which are easily determined, since they only occur prior to handovers, are minimized. Dumez et al. (2023) generalize the work of Grangier et al. (2016) by involving c2e demands, multiple trips for urban vehicles and limited storage capacity at satellites. The problem is modeled path-based with discrete time variables, where paths represent trips of urban vehicles or city freighters including schedule and load plan. The goal is to find a cost-minimal set of trips so that demands are satisfied, the maximum number of vehicles is not exceeded and satellite synchronization is fulfilled. While the time-discrete path-based model enables the consideration of externally given transportation services with routes and schedules, it significantly simplifies synchronization and predetermining load plans requires a solution approach with iterative generation.

The literature shows that approximating parts of the system and aggregating time are commonly applied modeling strategies for integrated transportation planning in 2TM-CL. The requirement for parking areas at satellite and customer locations is almost completely neglected. Waiting is commonly allowed everywhere, at any time and for unlimited duration. Except for the work by Anderluh et al. (2021), the proposed models do not account for external effects and disturbances and mainly take the economic perspective of an LSP operating the whole system. A few studies present first attempts to tackling limited up to no storage at satellites, while taking advantage of a unidirectional demand flow, at most one urban vehicle visiting a satellite, time discretization and predetermined scheduled trips.

This paper extends the literature in providing a general modeling approach focused on temporal aspects. We precisely synchronize multi-directional demand handovers with arbitrarily many vehicles and integrate the minimization of waiting durations, determined by an enhanced estimation scheme, as an objective. Furthermore, while the existing studies are performed on either randomly generated or single-case real-world data, our computational experiments are conducted on a vast real world-inspired set of instances with different parameters. Thus, the results reveal which problem characteristics and instance parameter complicate the design of a 2TM-CL system and the methodological solving process.

# **3 The City Logistics Planning Problem with Satellite Synchronization**

In the 2TM-CLP-SS, a set of demands  $\mathcal{D} = \{d \mid d = 1, \ldots, n\}$  must be shipped through a transportation network as shown in Figure 1b. This is split into an outer tier with a set of CDCs  $\mathcal{E} = \{e \mid e = 1, \ldots, E\}$  and an inner tier with customer locations  $\mathcal{I} = \{i \mid i = 1, \ldots, I\}$ . Both tiers are connected by a set of satellites  $\mathcal{Z} = \{z \mid z = 1, \ldots, Z\}$ . Transportation links on the outer tier are taken from  $\{\mathcal{E} \times \mathcal{Z}\} \cup \{\mathcal{Z} \times \mathcal{Z}\}\,$ , while transportation on the inner tier is possible on links in  $\{\mathcal{Z} \times \mathcal{I}\} \cup \{\mathcal{I} \times \mathcal{I}\}\$ . Without loss of generality, we assume that customers are not directly served from CDCs, while this can easily be realized by establishing a satellite right next to a CDC if required.

To satisfy the demands, two homogeneous fleets of vehicles, namely urban vehicles on the outer tier and city freighters on the inner tier, are operated. Satellites can be used for demand handovers between tiers while respecting their capacities. These are given for each satellite z by a maximum number of vehicles allowed to be present at the same time and denoted as  $N_z^{\text{urb}}$  for urban vehicles and  $N_z^{\text{cit}}$  for city freighters. To further limit the presence of vehicles in the urban area, maximum waiting durations are defined by  $w_z^{\text{sat}} \geq 0$  for satellites z and by  $w_i^{\text{cust}} \geq 0$  for customer locations *i*. Nonetheless, waiting is allowed at satellites and customer locations within the given bounds and without consuming capacity even if no handover activity takes place to provide sufficient planning flexibility. We further assume that waiting without handover is equally acceptable for all locations so that the introduction of dedicated penalty factors is omitted.

The 2TM-CLP-SS integrates three types of commodity flows. e2c demands represent shipments from the *external zone* X to the inner city area, requiring delivery to a certain customer location. c2e demands need to be picked up at a customer location for transport towards the external zone, while c2c demands require pickup and delivery at different customer locations. Accordingly, the set of demands can be divided into three disjoint subsets  $\mathcal{D}^{e2c}$  for inbound,  $\mathcal{D}^{\text{c2e}}$  for outbound and  $\mathcal{D}^{\text{c2c}}$  for inner-city flows. Every demand d features an origin  $O(d)$  and a destination  $D(d)$  defining its type, and a handover duration  $h_d$  applicable at satellites and customer locations. The handover duration defines the prescribed allotted time to perform loading, unloading, handling and customer interaction. Demand volumes are measured in required portions of vehicle capacity given as  $vol_d^{\text{cit}}, vol_d^{\text{urb}} \in (0,1]$  for city freighters and urban vehicles, respectively. An availability time  $\tilde{a}_d$  is given for each demand  $d \in \mathcal{D}^{e2c}$  indicating the moment the demand is available at any CDC. Note that the selection of a CDC and a satellite for a demand is part of the assignment decisions here, while given assignments can easily be integrated in the problem setting. As common in classical VRPs, hard demand time windows  $[b_{d}^{\rm{pick}}]$  $\bar{b}_d^{\text{pick}}, \bar{b}_d^{\text{pick}}$  $\mu_d^{\text{pick}}$  and  $[b_d^{\text{deli}}, \bar{b}_d^{\text{deli}}]$  are given for the start of service of pickups and deliveries. Waiting is allowed outside the time windows for location-specific limited amounts.

On the outer tier, a set of rail- or road-based *services*  $\mathcal{R} = \{r \mid r = 1, ..., R\}$  can be operated. Every service has a given capacity, a starting time interval  $[a_r, \bar{a}_r]$  and, if used, entails a fixed cost  $c_r^{\text{urb}} \geq 0$ . Its predefined physical route is an open path in the transportation network described by an origin  $O(r)$  and a destination  $D(r)$  with  $O(r) \neq D(r)$ , both contained in a set of satellites  $\mathcal{Z}(r) \subseteq \mathcal{Z}$  and a set of CDCs  $\mathcal{E}(r) \subseteq \mathcal{E}$  visited. Note that each service must visit at least one CDC, and CDCs can only be visited at the beginning and at the end of a service.

To precisely schedule all activities of urban vehicles, services are divided into a set of segments  $\mathcal{G}(r) = \{g \mid g = 0, \ldots, m_r^{\text{seg}} + 1\}.$  Travel and handover activities are summarizes in  $\mathcal{G}^{\text{travel}}(r)$  and  $\mathcal{G}^{\text{hand}}(r)$ , respectively, while the end of the service is represented by the dummy segment  $g = m_r^{\text{seg}} + 1$ . For each segment  $g \in \mathcal{G}^{\text{travel}}(r)$ , a travel duration  $t^{\text{urb}}_{r,g} > 0$  is given. Every segment  $g \in \mathcal{G}^{\text{hand}}(r)$  describes the urban vehicle visiting a location in  $\mathcal{Z}(r) \cup \mathcal{E}(r)$  and a unique assignment  $g^r(z) \in \mathcal{G}^{\text{hand}}(r)$  is defined for each satellite  $z \in \mathcal{Z}(r)$ . Figure 2a depicts an exemplary inbound service starting at CDC 1 and visiting satellites 3 and 5. Travel durations are defined for segments 1 and 3, while handovers may take place in segments 2 and 4. Waiting can only appear in handover segments as indicated by purple hashed areas in the figure. Note that demand handling activities in CDCs are not explicitly considered in this work so that stopping durations are set to zero for these segments. However, the segment-wise representation of the services facilitate the integration of handling activities and waiting durations at CDCs as a straightforward extension.

On the inner tier, a set of road-based *tours*  $\mathcal{K} = \{k \mid k = 1, ..., K\}$  can be operated with fixed cost  $c_k^{\text{cit}} \geq 0$  and flexible starting times. Every tour's physical route consists of an origin  $O(k) \in \mathcal{Z}$ , a destination  $D(k) \in \mathcal{Z}$ , and a set of customers visited  $\mathcal{I}(k) \subseteq \mathcal{I}$ , where customers are pairwise different and starting and terminal satellites may be equal or different. Thus, it may be an open or a closed path in the transportation network. Similar to the service division, every tour is divided by a set of legs  $\mathcal{L}(k) = \{l \mid l = 0, \ldots, m_k^{\text{leg}} + 1\}$ . As indicated in Figure 2b for an acyclic tour, legs represent demand handling at satellites (legs 0 and 6), travel activities



Figure 2: Elements of Services and Tours

(legs 1, 3 and 5), pickups and deliveries at customer locations (legs 2 and 4), and the end of the tour (leg 7). Travel legs are summarizes in  $\mathcal{L}^{\text{travel}}(k)$ , while  $\mathcal{L}^{\text{pd}}(k)$  denotes the set of pickup and delivery legs. Travel durations  $tt_{k,l}^{\text{cit}} > 0$  are given for all  $l \in \mathcal{L}^{\text{travel}}(k)$  and a unique assignment  $l^k(i)$  exists for each customer location  $i \in \mathcal{I}(k)$ . Waiting durations may occur with every demand handover shown for customer location 9 and satellite 5 in the figure.

The main decisions in solving the 2TM-CLP-SS are (i) selecting appropriate services and tours, (ii) assigning demands to them, and (iii) scheduling and synchronizing service segments and tour legs. All demands need to be shipped, while demand time windows and availability times, waiting duration restrictions at satellites and customer locations, and vehicle and satellite capacities are respected. In line with the idea of reducing traffic in urban areas, the 2TM-CLP-SS aims at finding a solution which minimizes the total operating costs and the total waiting duration of all services and tours.

The following *assumptions* are additionally taken:

- 1. Every urban vehicle or city freighter performs at most one service or tour during the planning horizon.
- 2. There are sufficiently many vehicles of both types available.
- 3. For each tour, a city freighter can be made available at the starting satellite whenever necessary.
- 4. For each service, an urban vehicle can be made available at a starting location at any time prior to its start time window to perform handover activities.
- 5. For each e2c and c2e demand, the itinerary consists of exactly one service and one tour. Demand handovers between two services or two tours are not allowed.
- 6. Demand splitting is not allowed.
- 7. Considering demand handling at satellites, unloading tasks are always performed prior to loading tasks.
- 8. If more than one demand is handed over between one service and one tour, handovers are performed consecutively and handover durations are summed up.

9. If several demands are handed over between one service and several tours or one tour and several services, parallel demand handling is possible and handover durations may overlap.

Assumptions 1 - 4 and 7 - 9 simplify the modeling process without limiting the general validity of the results of this study. With 5, we support highly streamlined distribution processes desirable in the CL setting, while 6 emphasizes that the planning problem focuses on private customers and small-volume business demands.

## **4 Modeling Fundamentals and Challenges**

This section summarizes fundamental aspects of the mathematical model. Data preprocessing steps are explained together with the basic logic of synchronization, waiting duration estimation and satellite occupancy tracking. Appendix B contains a full overview of the notation used.

## **4.1 Preliminaries**

The 2TM-CLP-SS is modeled on a two-tier transportation network as shown in Figure 1b, where nodes represent CDCs, satellites, and customer locations, while edges describe bidirectional transportation links. Services and tours feature given routes in this network and involve at least two nodes and one link. Since a major decision level is the selection of services and tours, the mathematical formulation follows a path-based modeling approach. In addition, this work focuses on the exact temporal synchronization between the two tiers of the transportation network. Accordingly, time is modeled by continuous variables without any discretization and freight management activities in the external zone are disregarded. Nonetheless, the CDCs are fully represented as parts of each service's route so that the model can easily be extended by additional handover durations and other restrictions on transport actions in the external zone.

## **4.2 Feasible Connections**

Since the itineraries of e2c and c2e demands consist of one service r and one tour k with handover at a certain satellite z, feasible connections  $(r, k, z)$  are determined in a preprocessing step. A connection  $(r, k, z) \in \mathcal{C}$  with  $\mathcal{C} \subset \mathcal{R} \times \mathcal{K} \times \mathcal{Z}$  is feasible, if considering the physical routes, there exists a demand  $d \in \mathcal{D}^{e2c} \cup \mathcal{D}^{c2e}$  that can use this connection.  $\mathcal C$  can be divided into two subsets  $\mathcal{C}^{\text{in}}$  and  $\mathcal{C}^{\text{out}}$ , where

•  $\mathcal{C}^{\text{in}} = \{(r, k, z) \in \mathcal{C} \mid O(k) \in \mathcal{Z}(r), O(r) \in \mathcal{E}\}\)$  describes all feasible inbound (e2c) connections and

•  $\mathcal{C}^{\text{out}} = \{(r, k, z) \in \mathcal{C} \mid D(k) \in \mathcal{Z}(r), D(r) \in \mathcal{E}\}\)$  describes all feasible outbound (c2e) connections.

To simplify the notation, we use  $\mathcal{C}^{\text{in}}(\cdot)$  and  $\mathcal{C}^{\text{out}}(\cdot)$  to indicate sets of feasible connections including a certain service, tour, satellite, or combination of those given in brackets, e.g.,  $\mathcal{C}^{\text{in}}(k') = \{(r, k, z) \in \mathcal{C}^{\text{in}} \mid k = k'\}\$ and  $\mathcal{C}^{\text{out}}(r', z') = \{(r, k, z) \in \mathcal{C}^{\text{out}} \mid (r = r') \land (z = z')\}.$ 

Considering a demand d, sets of related connections, services and tours are similarly defined as:

•  $\mathcal{C}^{\text{in}}(d) = \{(r, k, z) \in \mathcal{C}^{\text{in}} \mid (D(d) \in \mathcal{I}(k)) \wedge \left( \left[ b_d^{\text{deli}}, \overline{b}_d^{\text{deli}} \right] \right] \text{ reachable} \} \}$  for  $d \in \mathcal{D}^{\text{e2c}}$ ,

• 
$$
\mathcal{C}^{\text{out}}(d) = \{(r, k, z) \in \mathcal{C}^{\text{out}} \mid O(d) \in \mathcal{I}(k)\}\)
$$
for  $d \in \mathcal{D}^{\text{c2e}}$ ,

•  $\mathcal{R}(d) = \{r \in \mathcal{R} \mid \exists (r, k, z) \in \mathcal{C}^{\text{in}}(d) \cup \mathcal{C}^{\text{out}}(d)\}\$ for  $d \in \mathcal{D}^{\text{e2c}} \cup \mathcal{D}^{\text{c2e}}$ ,

• 
$$
\mathcal{K}(d) = \begin{cases} \{k \in \mathcal{K} \mid \exists (r, k, z) \in \mathcal{C}^{\text{in}}(d) \cup \mathcal{C}^{\text{out}}(d)\} & \text{for } d \in \mathcal{D}^{\text{e2c}} \cup \mathcal{D}^{\text{c2e}} \\ \{k \in \mathcal{K} \mid (O(d) \in \mathcal{I}(k)) \land (D(d) \in \mathcal{I}(k)) \land (l^k(O(d)) < l^k(D(d)))\} & \text{for } d \in \mathcal{D}^{\text{c2c}} \end{cases}
$$

Note that for  $\mathcal{C}^{\text{in}}(d)$  and  $\mathcal{C}^{\text{out}}(d)$ , it is sufficient to assure that tour k visits the required customer location, since the assignment of demands to satellites and CDCs is a part of the decision process. To deliver demand 1 in Figure 1b, two inbound connections can be used, i.e.,  $\mathcal{C}^{\text{in}}(1) = \{(1,5,3), (2,2,5)\}.$  In addition, temporal reachability is regarded for  $\mathcal{C}^{\text{in}}(d)$ . An inbound connection is only feasible for an e2c demand, if the earliest possible schedule of service and tour allows a delivery before the end of the customer time window. Tour- and service-specific sets of demands can be defined in the same fashion. They are denoted by  $\mathcal{D}(k)$ ,  $\mathcal{D}^{\text{c2c}}(k)$ ,  $\mathcal{D}^{\text{e2c}}(k)$ , and  $\mathcal{D}^{\text{c2e}}(k)$  for tours and  $\mathcal{D}(r)$ ,  $\mathcal{D}^{\text{e2c}}(r)$ , and  $\mathcal{D}^{\text{c2e}}(r)$  for services.

#### **4.3 Satellite Synchronization**

Satellite synchronization between connected services and tours is realized through a commonly applied modeling approach using task-related time variables (Drexl, 2012). A handover interval  $[\gamma_{r,k,z}^{\text{sat}}, \bar{\gamma}_{r,k,z}^{\text{sat}}]$  is introduced for each connection  $(r, k, z) \in \mathcal{C}$ . The total handover duration of all assigned demands defines the width of the interval as indicated by assumption 8 in Section 3. Assume that demand 1 is assigned to connection  $(2, 2, 5)$  in Figure 1b. Then,  $\bar{\gamma}_{2,2,5}^{\text{sat}} - \gamma_{2,2,5}^{\text{sat}} = h_1$ and  $\gamma_{1,5,3}^{\text{sat}} = \bar{\gamma}_{1,5,3}^{\text{sat}}$  hold. To ensure that the vehicles meet at the satellite, the starting times of the corresponding service segments and tour legs are related to  $[\gamma_{r,k,z}^{\text{sat}}, \bar{\gamma}_{r,k,z}^{\text{sat}}]$  as indicated in Figure 3. If more than two vehicles meet for demand handovers at a satellite, handovers can be performed consecutively, in parallel, or with intermediate idle time following assumption 9. Detailed explanation on resulting difficulties in the waiting duration calculation are given in Section 4.4.



Figure 3: Satellite Synchronization for Connections with Assigned Demands



Figure 4: Waiting Duration Determination at Satellites

#### **4.4 Waiting Durations at Satellites**

Generally, we apply a waiting duration calculation approach of deducting the actual demand handling duration from the time passed between arrival and departure of a vehicle. However, the determination of the length of the demand handling duration is challenging for vehicles taking part in several connections and handovers at the same time, see for instance service 1 in Figure 4. The cases of single, consecutive, overlapping, and spread handovers require different calculation schemes, while the identification of these cases is complex when using continuous starting time variables  $\alpha_{r,q}$  for service segments and  $\beta_{k,l}$  for tour legs.

To assure reasonable accuracy and cover as many cases as possible, we introduce two types of waiting duration estimates. They involve different determinations of the length of the demand handling duration and are either accurate or underestimating for the different cases shown in Figure 4. Since an overestimation can never appear and the minimization of the total waiting duration is a key objective, the estimates can be used as lower bounds on the actual waiting durations  $\omega_k^{\text{in}}$  and  $\omega_k^{\text{out}}$  for a tour k at its starting and terminal satellite, and  $\omega_{r,z}^{\text{urb}}$  for a service  $r$  at a satellite  $z$ .

The first type of estimates determines the length of the demand handling duration as the total handover duration of all demands involved. For a service r and satellite  $z$ , it is given in Equation (4.1), where the binary variables  $\pi_{d,r,k,z}^{\text{in}}$  and  $\pi_{d,r,k,z}^{\text{out}}$  indicate the assignment of a demand d to an inbound or outbound connection  $(r, k, z)$ . The estimates  $\tilde{\omega}_k^{\text{in}}$  and  $\tilde{\omega}_k^{\text{out}}$  are



Figure 5: Limited Underestimation of Waiting Durations at Satellites

equivalently set up for tours and their starting and terminal satellite.

$$
\tilde{\omega}_{r,z}^{\text{urb}} = \alpha_{r,g^r(z)+1} - \alpha_{r,g^r(z)} - \sum_{\substack{(r',k',z')\\ \in \mathcal{C}^{\text{in}}(r,z)}} \sum_{d \in \mathcal{D}^{\text{e2c}}(k')} h_d \cdot \pi_{d,r',k',z'}^{\text{in}} - \sum_{\substack{(r',k',z')\\ \in \mathcal{C}^{\text{out}}(r,z)}} \sum_{d \in \mathcal{D}^{\text{c2e}}(k')} h_d \cdot \pi_{d,r',k',z'}^{\text{out}} \tag{4.1}
$$

For the second type of estimates, activity intervals that represent the time between the start of the first handover activity and the end of the last handover activity related to a certain vehicle are introduced. The width of these intervals is minimized as part of the objective function to avoid unnecessarily spread handovers. The estimates are then determined by deducting the width of the activity interval from the time between arrival and departure of a vehicle. For a service r at a satellite z, this is shown in Equation (4.2), where  $[\lambda_{r,z}^{\text{urb}}, \bar{\lambda}_{r,z}^{\text{urb}}]$  denotes the activity interval. Correspondingly, the activity intervals for tours are described as  $[\lambda_k^{\text{in}}, \bar{\lambda}_k^{\text{in}}]$  and  $[\lambda_k^{\text{out}}, \bar{\lambda}_k^{\text{out}}]$  and used in the calculation of the estimates  $\hat{\omega}_k^{\text{in}}$  and  $\hat{\omega}_k^{\text{out}}$ .

$$
\hat{\omega}_{r,z}^{\text{urb}} = \alpha_{r,g^r(z)+1} - \alpha_{r,g^r(z)} - \left(\bar{\lambda}_{r,z}^{\text{urb}} - \lambda_{r,z}^{\text{urb}}\right)
$$
\n(4.2)

Figure 4 illustrates the different cases for two demand handovers, marked by the bars, using connections  $(1, 5, 3)$  and  $(1, 4, 3)$ . For consecutive handovers in Figure 4a, the total handover duration equals the width of the activity interval  $[\lambda_{1,3}^{\text{urb}}, \overline{\lambda}_{1,3}^{\text{urb}}]$ , and both estimates are accurate for the waiting duration of the service marked as the hashed area. For overlapping handovers in Figure 4b, the width of the activity interval is smaller than the total handover duration. Thus, the first measure underestimates the actual waiting duration with  $\tilde{\omega}_{1,3}^{\text{urb}} = 0$ , while the second measure gives the actual value  $\hat{\omega}_{1,3}^{\text{urb}} = \omega_{1,3}^{\text{urb}} = 0.5$ . The opposite situation occurs when looking at spread handovers with intermediate waiting in Figure 4c, where  $\hat{\omega}_{1,3}^{\text{urb}} = 0.5 < \tilde{\omega}_{1,3}^{\text{urb}} = \omega_{1,3}^{\text{urb}} = 1$ .

Even if the implementation of the two estimates is accurately bounding the actual waiting durations in the vast majority of scenarios, Figure 5 shows a situation in which an intermediate

waiting is compensated by overlapping handovers. In this case, both measures  $\tilde{\omega}_{r,z}^{\text{urb}}$  and  $\hat{\omega}_{r,z}^{\text{urb}}$ underestimate the actual waiting duration of the service. However, this scenario is expected to rarely appear, since it requires at least 3 demands with corresponding time windows and a total vehicle capacity of the satellite of at least 4. Furthermore, the value of underestimation of the actual waiting duration is bounded by the maximum total overlap of  $|\mathcal{D}^{e2c}|+|\mathcal{D}^{c2e}|-1$  handover activities. Note additionally, that the incorporation of the widths of the activity intervals in the objective function to avoid unnecessary handover spreading opposes the idea of minimizing vehicle waiting durations. These two conflicting objectives require appropriate weighting.

#### **4.5 Satellite Capacities**

To implement satellite capacity restrictions, the satellite location is interpreted as a discrete set of resources.  $\mathcal{U}_z^{\text{cit}} = \{1, \ldots, N_z^{\text{cit}}\}$  denotes the set of units u available for city freighters to stop at satellite z, while  $\mathcal{U}_z^{\text{urb}} = \{1, \ldots, N_z^{\text{urb}}\}\$  describes the available units v for urban vehicles. For each demand handover, the vehicles involved are assigned to a corresponding satellite unit, and pairs of vehicles assigned to the same unit get a proper sequencing. A set  $S_z^{\text{urb}}$  of pairs of potentially competing services  $(r, r')$  is defined for each satellite z. Since tours may visit satellites twice, assignment and sequencing is done leg-based with a set  $\mathcal{S}_z^{\text{cit}}$  of pairs of potentially competing tour legs  $(k, l, k', l')$ . Consider again the handover situation in Figure 4 and assume that the capacity of the satellite is  $N_3^{\text{cit}} = N_3^{\text{urb}} = 1$ . Then, overlapping handovers as shown in 4b are not feasible. Both city freighters have to be assigned to the same unit and  $\mathcal{S}_3^{\text{cit}} = \{(4,0,5,0)\}\$ is given. Proper sequencing forces either  $\beta_{4,1} \leq \beta_{5,0}$  or  $\beta_{5,1} \leq \beta_{4,0}$  to hold, where the consecutive handovers depicted in Figure 4a implement the former. In addition to the implementation of capacity restrictions, the explicit assignment of vehicles to satellite units can enable the consideration of opening hours for satellites in future research.

## **5 Mathematical Model**

Table 1 summarizes the decision variables which implement assignment, selection and sequencing, as well as starting times, handover and waiting durations. Binary variables  $\pi_{d,r,k,z}^{\text{in}}$  and  $\pi_{d,r,k,z}^{\text{out}}$  indicate the assignment of an e2c or c2e demand d to an inbound or outbound connection  $(r, k, z)$ , respectively. For c2c demands d, binary variables  $\pi_{d,k}^{cc} = 1$  indicate the assignment to a fitting tour k. Directly linked to the assignments, the binary variables  $\rho_k^{\text{cit}}$  and  $\rho_r^{\text{urb}}$  describe the selection of a tour  $k$  or a service  $r$ , where a value of 1 defines the route to be operated. To satisfy satellite capacities, binary variables  $\phi_{k,l,z,u}^{\text{cit}}$  and  $\phi_{r,z,v}^{\text{urb}}$  take a value of 1, if a tour leg or a service is assigned to a certain satellite unit, while  $\chi_{k,l,k',l'}^{\text{cit}}$  and  $\chi_{r,r',z}^{\text{urb}}$  indicate by a value of 1 that leg l of tour k precedes leg l' of tour k' and service r precedes service r' at satellite z, respectively. All continuous variables naturally represent points in time with non-negative values.

Binary Variables	
$\pi_{d,r,k,z}^{\rm in}, \pi_{d,r,k,z}^{\rm out}$	assignment of demand d to inbound/outbound connection $(r, k, z)$
$\pi_{d,k}^{\text{cc}}$ $\rho_{r}^{\text{urb}}, \rho_{k}^{\text{cit}}$	assignment of c2c demand $d$ to tour $k$
	selection of service $r/\text{tour } k$
$\phi_{k,l,z,u}^{\rm cit}$	assignment of leg $l$ of tour $k$ to city freighter unit $u$ of satellite $z$
$\phi^{\text{urb}}_{r,z,v}$	assignment of service $r$ to urban vehicle unit $v$ of satellite $z$
$\chi_{k,l,k',l'}^{\text{cit}}$	sequencing of leg l of tour k and leg l' of tour $k'$
$\chi_{r,r',z}^{\rm{urb}}$	sequencing of service r and service r' at satellite z
Continuous Variables	
$\alpha_{r,q}, \beta_{k,l}$	starting time of segment g of service $r/\log l$ of tour k
$\begin{array}{l} [\gamma^\text{sat}_{r,k,z},\bar{\gamma}^\text{sat}_{r,k,z}]\\ [\gamma^\text{pick}_{d,k},\bar{\gamma}^\text{pick}_{d,k}], [\gamma^\text{deli}_{d,k},\bar{\gamma}^\text{deli}_{d,k}] \end{array}$	handover interval between service $r$ and tour $k$ at satellite $z$
	pickup/delivery interval of a demand d on tour $k$
$[\lambda_k^{\text{in}}, \bar{\lambda}_k^{\text{in}}], [\lambda_k^{\text{out}}, \bar{\lambda}_k^{\text{out}}]$	activity interval of tour $k$ at starting/terminal satellite
$[\lambda_{r,z}^{\rm urb}, \bar{\lambda}_{r,z}^{\rm urb}]$	activity interval of service $r$ at satellite $z$
$\omega_{r,z}^{\rm urb}, \omega_{k,i}^{\rm cust}$	waiting duration of service r at satellite $z$ /tour k at customer lo-
	$\cot$ cation $i$
$\omega_k^{\text{in}}, \omega_k^{\text{out}}$	waiting duration of tour k at the starting $\binom{m}{t}$ terminal (out) satel-
	lite

Table 1: Overview of Decision Variables

The **objective function** of the 2TM-CLP-SS is described as follows:

$$
\begin{aligned}\n\min \quad & q_1 \cdot \left[ \sum_{r \in \mathcal{R}} c_r^{\text{urb}} \cdot \rho_r^{\text{urb}} + \sum_{k \in \mathcal{K}} c_k^{\text{cit}} \cdot \rho_k^{\text{cit}} \right] \\
& + q_2 \cdot \left[ \sum_{r \in \mathcal{R}} \sum_{z \in \mathcal{Z}(r)} (\bar{\lambda}_{r,z}^{\text{urb}} - \lambda_{r,z}^{\text{urb}}) + \sum_{k \in \mathcal{K}} (\bar{\lambda}_k^{\text{in}} - \lambda_k^{\text{in}} + \bar{\lambda}_k^{\text{out}} - \lambda_k^{\text{out}}) \right] \\
& + q_3 \cdot \left[ \sum_{r \in \mathcal{R}} \sum_{z \in \mathcal{Z}(r)} \omega_{r,z}^{\text{urb}} + \sum_{k \in \mathcal{K}} (\omega_k^{\text{in}} + \omega_k^{\text{out}}) + \sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{I}(k)} \omega_{k,i}^{\text{cust}} \right]\n\end{aligned} \tag{5.1}
$$

It is formed as a linear combination of the fixed costs for selected services and tours, the total width of all activity intervals and the total waiting duration of all vehicles traveling. Weights  $q_1, q_2$  and  $q_3$  are used to calibrate the model regarding the conflicting objectives. The objective function is minimized subject to the following **constraints**:

*Demand Assignment*

$$
\sum_{(r,k,z)\in\mathcal{C}^{\text{in}}(d)} \pi^{\text{in}}_{d,r,k,z} = 1 \quad \forall \ d \in \mathcal{D}^{\text{e2c}} \tag{5.2}
$$

$$
\sum_{(r,k,z)\in \mathcal{C}^{\text{out}}(d)} \pi_{d,r,k,z}^{\text{out}} = 1 \quad \forall \ d \in \mathcal{D}^{\text{c2e}} \tag{5.3}
$$



Figure 6: Determining the Load of a Vehicle

$$
\sum_{k \in \mathcal{K}(d)} \pi_{d,k}^{\text{cc}} = 1 \quad \forall \ d \in \mathcal{D}^{\text{c2c}} \tag{5.4}
$$

Constraints (5.2) and (5.3) assure that exactly one fitting inbound or outbound connection is assigned to every e2c and c2e demand, respectively. Similarly, constraints (5.4) enforce the assignment of every c2c demand to exactly one tour. With this, every demand is fulfilled.

*Assignment-Selection Linkage and Vehicle Capacities*

$$
\sum_{(r,k,z)\in\mathcal{C}^{\text{in}}(r')} \sum_{d\in\mathcal{D}^{\text{e2c}}(r)} vol_{d}^{\text{urb}} \cdot \pi_{d,r,k,z}^{\text{in}} \n+ \sum_{(r,k,z)\in\mathcal{C}^{\text{out}}(r')} \sum_{d\in\mathcal{D}^{\text{e2c}}(r)} vol_{d}^{\text{urb}} \cdot \pi_{d,r,k,z}^{\text{out}} \leq \rho_{r'}^{\text{urb}} \quad \forall g' \in \mathcal{G}^{\text{travel}}(r'), r' \in \mathcal{R} \quad (5.5) \n+ \sum_{\substack{(r,k,z)\in\mathcal{C}^{\text{in}}(k')\\ \text{with } g^{r}(z) < g'}} \sum_{d\in\mathcal{D}^{\text{e2c}}(k) \text{ with} \atop l^{k}(D(d)) > l'} vol_{d}^{\text{cit}} \cdot \pi_{d,r,k,z}^{\text{in}} \n+ \sum_{d\in\mathcal{D}^{\text{e2c}}(k') \text{ with} \atop l^{k}(O(d)) \leq l' \text{ and } l^{k}(D(d)) > l'} vol_{d}^{\text{cit}} \cdot \pi_{d,r,k,z}^{\text{cut}} \leq \rho_{k'}^{\text{cit}} \quad \forall l' \in \mathcal{L}^{\text{travel}}(k'), k' \in \mathcal{K} \quad (5.6)
$$

Constraints (5.5) and (5.6) combine the purposes of satisfying vehicle capacities and linking assignment and selection decisions. Since continuous time variables are used to indicate the start of each service segment and tour leg, vehicle loads are tracked segment- and leg-wise. Following assumption 7, capacity restrictions can focus on the travel segments and legs as shown in Figure 6. For each travel segment  $g'$  in (5.5) and travel leg l' in (5.6), the total volume of demands previously loaded and not yet unloaded is considered. To do so, the current segment or leg is related to handling segments  $g^r(z)$  and handling legs  $l^k(i)$  visited before and after, respectively. Since the parameters  $vol_d^{\text{cit}}$  and  $vol_d^{\text{urb}}$  are given in proportions of city freighter's and urban

vehicle's capacity, the binary variables  $\rho_k^{\text{cit}}$  and  $\rho_r^{\text{urb}}$  represent the full volume of a selected vehicle. Thus, the selection of the corresponding tour and service is forced, whenever a demand is assigned to a connection. Note that volume-dependent capacity restrictions of satellites can easily be integrated in a similar way.

*Linking Selection and Scheduling*

$$
\alpha_{r,g} - M \cdot \rho_r^{\text{urb}} \le 0 \quad \forall \ g \in \mathcal{G}(r), r \in \mathcal{R} \tag{5.7}
$$

$$
\beta_{k,l} - M \cdot \rho_k^{\text{cit}} \le 0 \quad \forall \ l \in \mathcal{L}(k), k \in \mathcal{K} \tag{5.8}
$$

$$
\lambda_{r,z}^{\text{urb}} - M \cdot \rho_r^{\text{urb}} \le 0 \quad \forall \ z \in \mathcal{Z}(r), r \in \mathcal{R}
$$
\n
$$
(5.9)
$$

$$
\bar{\lambda}_{r,z}^{\text{urb}} - M \cdot \rho_r^{\text{urb}} \le 0 \quad \forall \ z \in \mathcal{Z}(r), r \in \mathcal{R}
$$
\n(5.10)

$$
\lambda_k^{\text{in}} - M \cdot \rho_k^{\text{cit}} \le 0 \quad \forall \ k \in \mathcal{K}
$$
\n(5.11)

$$
\bar{\lambda}_k^{\text{in}} - M \cdot \rho_k^{\text{cit}} \le 0 \quad \forall \ k \in \mathcal{K}
$$
\n(5.12)

$$
\lambda_k^{\text{out}} - M \cdot \rho_k^{\text{cit}} \le 0 \quad \forall \ k \in \mathcal{K} \tag{5.13}
$$

$$
\bar{\lambda}_k^{\text{out}} - M \cdot \rho_k^{\text{cit}} \le 0 \quad \forall \ k \in \mathcal{K} \tag{5.14}
$$

Constraints (5.7) and (5.8) link selection and scheduling by assuring that positive-valued starting times are only determined for selected services and tours. Similarly, constraints (5.9) to (5.14) only allow positive-valued activity interval bounds for selected services and tours. Even though constraints  $(5.9)$  and  $(5.10)$ , and constraints  $(5.11)$  to  $(5.14)$  can be combined without loss of functionality, they are implemented in the given disaggregated manner to allow the determination of tighter values for M.

*Interval Widths for Handovers, Pickups and Deliveries*

$$
\gamma_{r,k,z}^{\text{sat}} + \sum_{d \in \mathcal{D}^{\text{e2c}}(k)} h_d \cdot \pi_{d,r,k,z}^{\text{in}} + \sum_{d \in \mathcal{D}^{\text{e2e}}(k)} h_d \cdot \pi_{d,r,k,z}^{\text{out}} = \bar{\gamma}_{r,k,z}^{\text{sat}} \quad \forall (r,k,z) \in \mathcal{C}
$$
\n(5.15)

$$
\gamma_{d,k}^{\text{pick}} + h_d \cdot \sum_{(r',k',z') \in \mathcal{C}^{\text{out}}(d,k)} \pi_{d,r',k',z'}^{\text{out}} = \bar{\gamma}_{d,k}^{\text{pick}} \quad \forall \ k \in \mathcal{K}(d), d \in \mathcal{D}^{\text{c2e}} \tag{5.16}
$$

$$
\gamma_{d,k}^{\text{deli}} + h_d \cdot \sum_{(r',k',z') \in \mathcal{C}^{\text{in}}(d,k)} \pi_{d,r',k',z'}^{\text{in}} = \bar{\gamma}_{d,k}^{\text{deli}} \quad \forall \ k \in \mathcal{K}(d), d \in \mathcal{D}^{\text{e2c}} \tag{5.17}
$$

An appropriate handover interval width is defined by constraints (5.15) for each connection. Pickup and delivery intervals  $[\gamma_{d,k}^{\text{pick}}, \bar{\gamma}_{d,k}^{\text{pick}}]$  and  $[\gamma_{d,k}^{\text{deli}}, \bar{\gamma}_{d,k}^{\text{deli}}]$  are introduced for each demand d and fitting tour k. Similar to demand handovers at satellites, constraints  $(5.16)$  and  $(5.17)$ determine their widths. For c2c demands, which feature both pickup and delivery, interval widths are defined in an analog fashion, see constraints (C.18) and (C.19) in Appendix C.

*Customer Time Windows*

$$
b_d^{\text{pick}} \le \gamma_{d,k}^{\text{pick}} + M \cdot \left(1 - \pi_{d,r,k,z}^{\text{out}}\right) \quad \forall \ (r,k,z) \in \mathcal{C}^{\text{out}}(d), d \in \mathcal{D}^{\text{c2e}} \tag{5.18}
$$

$$
\gamma_{d,k}^{\text{pick}} \le \bar{b}_d^{\text{pick}} + M \cdot \left(1 - \pi_{d,r,k,z}^{\text{out}}\right) \quad \forall \ (r,k,z) \in \mathcal{C}^{\text{out}}(d), d \in \mathcal{D}^{\text{c2e}} \tag{5.19}
$$

Pickup and delivery intervals link tour scheduling and customer requirements. For c2e demands, constraints (5.18) and (5.19) relate the start of the pickup interval of the assigned tour to the given demand time window. The same restrictions are implemented for deliveries of e2c demands and handover activities of c2c demands, see constraints  $(C.22)$  to  $(C.27)$  in Appendix C.

*Demand Availability Times*

$$
\tilde{a}_d \le \alpha_{r,0} + M \cdot \left(1 - \pi^{\text{in}}_{d,r',k,z}\right) \quad \forall \ (r,k,z) \in \mathcal{C}^{\text{in}}(d), d \in \mathcal{D}^{\text{e2c}} \tag{5.20}
$$

In addition to an indirect linkage through tour schedules, service scheduling and e2c demand assignment are also linked through the demand availability times. Constraints (5.20) restrict the starting time of the assigned service correspondingly.

*Services: Scheduling and Waiting Times*

$$
\alpha_{r,0} \ge 0 \qquad \forall \ r \in \mathcal{R} \tag{5.21}
$$

$$
\alpha_{r,1} + M \cdot (1 - \rho_r^{\text{urb}}) \ge a_r \qquad \forall \ r \in \mathcal{R} \tag{5.22}
$$

$$
\alpha_{r,1} \le \bar{a}_r \qquad \forall \ r \in \mathcal{R} \tag{5.23}
$$

$$
\alpha_{r,g} + \ t t_{r,g}^{\text{urb}} \cdot \rho_r^{\text{urb}} = \alpha_{r,g+1} \qquad \forall \ g \in \mathcal{G}^{\text{travel}}(r), r \in \mathcal{R} \tag{5.24}
$$

$$
\alpha_{r,g^r(z)} \le \lambda_{r,z}^{\text{urb}} \qquad \forall \ z \in \mathcal{Z}(r), r \in \mathcal{R}
$$
\n
$$
\mathcal{Z}(r) \tag{5.25}
$$

$$
\bar{\lambda}_{r,z}^{\text{urb}} \le \alpha_{r,g^r(z)+1} \quad \forall \ z \in \mathcal{Z}(r), r \in \mathcal{R}
$$
\n(5.26)

$$
\lambda_{r,z}^{\text{urb}} \le \gamma_{r,k,z}^{\text{sat}} \qquad \forall (r,k,z) \in \mathcal{C}
$$
\n(5.27)

$$
\bar{\gamma}_{r,k,z}^{\text{sat}} \le \bar{\lambda}_{r,z}^{\text{urb}} \qquad \forall (r,k,z) \in \mathcal{C}
$$
\n(5.28)

$$
\lambda_{r,z}^{\text{urb}} \le \bar{\lambda}_{r,z}^{\text{urb}} \qquad \forall \ z \in \mathcal{Z}(r), r \in \mathcal{R}
$$
\n(5.29)

$$
\alpha_{r,g'} = \alpha_{r,g'+1} \qquad \forall \ g' \in \mathcal{G}^{\text{hand}}(r) : \exists \ e \in \mathcal{E} \text{ with } g^r(e) = g', r \in \mathcal{R} \qquad (5.30)
$$
\n
$$
(4.1) \text{ and } (4.2)
$$

$$
\hat{\omega}_{r,z}^{\text{urb}} \le \omega_{r,z}^{\text{urb}} \qquad \forall \ z \in \mathcal{Z}(r), r \in \mathcal{R} \tag{5.31}
$$

$$
\tilde{\omega}_{r,z}^{\text{urb}} \le \omega_{r,z}^{\text{urb}} \qquad \forall \ z \in \mathcal{Z}(r), r \in \mathcal{R}
$$
\n(5.32)

$$
\omega_{r,z}^{\text{urb}} \le w_z^{\text{sat}} \qquad \forall \ z \in \mathcal{Z}(r), r \in \mathcal{R} \tag{5.33}
$$

The scheduling process of service segments is described by constraints (5.21) to (5.33). While (5.21) defines the initial starting time of a service to be non-negative, constraints (5.22) and  $(5.23)$  limit the starting time of the first travel segment of service r to meet its start time window  $[a_r, \bar{a}_r]$ . Travel durations are respected for all travel segments with constraints (5.24). Activity intervals, which facilitate correct waiting duration estimation, are related to the corresponding segment starting times by constraints  $(5.25)$  and  $(5.26)$ , while constraints  $(5.27)$  and  $(5.28)$ implement on-time synchronization during handover intervals. Note that start and end of all related handover intervals are set to zero, if a service is not selected, see (5.7), (5.9) and (5.10). To cover the special case of visited satellites, for which no feasible connection exists, a proper sequence of start and end of activity intervals is assured by constraints (5.29). The dwell time of handling segments representing stops at CDCs is set to zero in constraints (5.30). To complete the scheduling process of services, the waiting duration estimates (4.1) and (4.2) are integrated into the model and the considered waiting duration  $\omega_{r,z}^{\text{urb}}$  of service r at satellite z is determined through constraints (5.31) and (5.32). The waiting duration bounds of satellites are enforced by constraints (5.33).

*Tours: Scheduling and Waiting Times*

 $(r,k,z) \in \mathcal{C}^{\text{in}}(k') d \in \mathcal{D}^{\text{e2c}}(k)$ 

$$
\beta_{k,0} \ge 0 \qquad \qquad \forall \ k \in \mathcal{K} \tag{5.34}
$$

$$
\beta_{k,0} \le \lambda_k^{\text{in}} \qquad \forall \ k \in \mathcal{K} \tag{5.35}
$$

$$
\bar{\lambda}_k^{\text{in}} \le \beta_{k,1} \qquad \forall \ k \in \mathcal{K} \tag{5.36}
$$

$$
\lambda_k^{\text{in}} - M \cdot \left(1 - \pi_{d,r,k,z}^{\text{in}}\right) \le \gamma_{r,k,z}^{\text{sat}} \qquad \forall (r,k,z) \in \mathcal{C}^{\text{in}}(d), d \in \mathcal{D}^{\text{e2c}} \qquad (5.37)
$$
  

$$
\bar{\gamma}_{r,k,z}^{\text{sat}} - M \cdot \left(1 - \pi_{d,r,k,z}^{\text{in}}\right) \le \bar{\lambda}_k^{\text{in}} \qquad \forall (r,k,z) \in \mathcal{C}^{\text{in}}(d), d \in \mathcal{D}^{\text{e2c}} \qquad (5.38)
$$

$$
\lambda_k^{\text{in}} \le \bar{\lambda}_k^{\text{in}} \qquad \forall \ k \in \mathcal{K} \tag{5.39}
$$

$$
(\beta_{k,1} - \beta_{k,0}) - (\bar{\lambda}_k^{\text{in}} - \lambda_k^{\text{in}}) = \hat{\omega}_k^{\text{in}} \qquad \forall k \in \mathcal{K}
$$
\n
$$
\hat{\omega}_k^{\text{in}} \leq \omega_k^{\text{in}} \qquad \forall k \in \mathcal{K}
$$
\n(5.40)\n
$$
(5.41)
$$

$$
\sum (\beta_{k,1} - \beta_{k,0}) - \sum h_d \cdot \pi_{d,r,k,z}^{\text{in}} = \tilde{\omega}_k^{\text{in}} \qquad \forall k' \in \mathcal{K}
$$
\n(5.42)

$$
\tilde{\omega}_k^{\text{in}} \le \omega_k^{\text{in}} \qquad \forall \ k \in \mathcal{K} \tag{5.43}
$$

$$
\omega_k^{\text{in}} \le w_{O(k)}^{\text{sat}} \qquad \forall \ k \in \mathcal{K} \tag{5.44}
$$

$$
\beta_{k,l} + \; tt_{k,l}^{\text{cit}} \cdot \rho_k^{\text{cit}} = \beta_{k,l+1} \qquad \forall \; l \in \mathcal{L}^{\text{travel}}(k), k \in \mathcal{K} \tag{5.45}
$$

$$
\beta_{k,l^k(O(d))} \le \gamma_{d,k}^{\text{pick}} \qquad \forall \ k \in \mathcal{K}(d), d \in \mathcal{D}^{\text{c2e}} \cup \mathcal{D}^{\text{c2c}} \tag{5.46}
$$

$$
\bar{\gamma}_{d,k}^{\text{pick}} \le \beta_{k,l^k(O(d))+1} \ \forall \ k \in \mathcal{K}(d), d \in \mathcal{D}^{\text{c2e}} \cup \mathcal{D}^{\text{c2c}} \tag{5.47}
$$

$$
\beta_{k,l^k(D(d))} \le \gamma_{d,k}^{\text{deli}} \qquad \forall \ k \in \mathcal{K}(d), d \in \mathcal{D}^{\text{e2c}} \cup \mathcal{D}^{\text{c2c}} \qquad (5.48)
$$

$$
\bar{\gamma}_{d,k}^{\text{deli}} \le \beta_{k,l^k(D(d))+1} \ \forall \ k \in \mathcal{K}(d), d \in \mathcal{D}^{\text{e2c}} \cup \mathcal{D}^{\text{c2c}} \tag{5.49}
$$

$$
-\sum_{\substack{d \in \mathcal{D}^{e2c}(k) \\ \text{with } D(d)=i}} \sum_{\substack{(r',k',z') \\ (r',k',z')}} h_d \cdot \pi_{d,r',k',z'}^{\text{in}} \n-\sum_{\substack{d \in \mathcal{D}^{e2e}(k) \\ \text{with } O(d)=i}} \sum_{\substack{(r',k',z') \\ \text{with } O(d)=i \in \mathcal{C}^{\text{out}}(k)}} h_d \cdot \pi_{d,r',k',z'}^{\text{out}} \n-\sum_{\substack{d \in \mathcal{D}^{e2e}(k) \\ \text{with } O(d)=i \in \mathcal{C}^{\text{out}}(k)}} h_d \cdot \pi_{d,k}^{\text{cc}} = \omega_{k,i}^{\text{cust}} \qquad \forall i \in \mathcal{I}(k), k \in \mathcal{K}
$$
\n(5.50)

$$
\omega_{k,i}^{\text{cust}} \le w_i^{\text{cust}} \qquad \forall \ i \in \mathcal{I}(k), k \in \mathcal{K} \tag{5.51}
$$

The key aspects of the scheduling process of tour legs are described by constraints (5.34) to (5.51). Constraints (5.34) define non-negative starting times for all legs. The starting

times of the first two tour legs are related to the inbound activity interval  $[\lambda_k^{\text{in}}, \overline{\lambda}_k^{\text{in}}]$  of each tour k by constraints  $(5.35)$  and  $(5.36)$ . If e2c demands are assigned to a tour, their activity intervals are related to the corresponding handover intervals of the inbound connections in constraints (5.37) and (5.38). To properly cover the case, where no e2c demands are assigned to a tour, start and end of the activity interval are sequenced by constraints (5.39). Applying the measures explained in Section 4.1, a tour's inbound waiting duration is estimated using the activity interval and the total handover duration, see (5.40) and (5.42), while the considered waiting duration is determined as the maximum by constraints (5.41) and (5.43). Constraints (5.44) assure that the maximum waiting duration at a tour's initial satellite is not exceeded. Constraints (5.35) to (5.44) similarly exist for the terminal satellite of each tour as shown in  $(C.55)$  to  $(C.64)$  in Appendix C.

Constraints (5.45) assure that the travel durations are respected. Similar to demand handling at satellites, pickup or delivery of a demand  $d$  at a customer location is performed by a tour k during an interval  $[\gamma_{d,k}^{\text{pick}}, \bar{\gamma}_{d,k}^{\text{pick}}]$  or  $[\gamma_{d,k}^{\text{deli}}, \bar{\gamma}_{d,k}^{\text{deli}}]$ . The starting times of the corresponding tour legs are related to the interval by constraints (5.46) and (5.47) for pickups and (5.48) and  $(5.49)$  for deliveries. Note that if a tour k visiting customer location i is operated, but demand d with  $D(d) = i$  is delivered via another tour k', constraints (5.17) define a zero-width delivery interval for  $(d, k)$  and a proper sequence of the related legs of tour k is assured. The same holds for pickups in constraints (5.16). Constraints (5.50) determine the waiting duration  $\omega_{k,i}^{\text{cut}}$  of each tour  $k$  at each customer location  $i$  and constraints (5.51) guarantee that the maximum waiting duration is respected.

*Satellite Capacities*

$$
\sum_{\substack{(r,k,z)\\ \in \mathcal{C}^{\text{in}}(r',z')}} \sum_{d \in \mathcal{D}^{\text{e2c}}(k)} \pi^{\text{in}}_{d,r,k,z} + \sum_{\substack{(r,k,z)\\ \in \mathcal{C}^{\text{out}}(r',z')}} \sum_{d \in \mathcal{D}^{\text{c2e}}(k)} \pi^{\text{out}}_{d,r,k,z} \n- \mid \mathcal{D}(r') \mid \sum_{v \in \mathcal{U}^{\text{urb}}_{z'}} \phi^{\text{urb}}_{r',z',v} \leq 0 \quad \forall z' \in \mathcal{Z}(r'), r' \in \mathcal{R}
$$
\n(5.52)

$$
\sum_{v \in \mathcal{U}_z^{\text{urb}}} \phi_{r,z,v}^{\text{urb}} \le 1 \quad \forall \ z \in \mathcal{Z}(r), r \in \mathcal{R} \tag{5.53}
$$

$$
\sum_{r \in \mathcal{R} \text{ with } z \in \mathcal{Z}(r)} \phi_{r,z,v}^{\text{urb}} - \sum_{r \in \mathcal{R} \text{ with } z \in \mathcal{Z}(r)} \phi_{r,z,v+1}^{\text{urb}} \geq 0 \quad \forall \ v \in \mathcal{U}_z^{\text{urb}} \setminus \{N_z^{\text{urb}}\}, z \in \mathcal{Z} \quad (5.54)
$$

$$
\phi_{r,z,v}^{\text{urb}} + \phi_{r',z,v}^{\text{urb}} - \chi_{r,r',z}^{\text{urb}} - \chi_{r',r,z}^{\text{urb}} \leq 1 \quad \forall (r,r') \in \mathcal{S}_z^{\text{urb}}, v \in \mathcal{U}_z^{\text{urb}}, z \in \mathcal{Z}
$$
\n(5.55)

$$
\chi_{r,r',z}^{\text{urb}} + \chi_{r',r,z}^{\text{urb}} \leq 1 \quad \forall (r,r') \in \mathcal{S}_z^{\text{urb}}, z \in \mathcal{Z}
$$
 (5.56)

$$
\alpha_{r',g^{r'}(z)+1} - M \cdot \chi_{r,r',z}^{\text{urb}} - \alpha_{r,g^r(z)} \leq 0 \quad \forall (r,r') \in \mathcal{S}_z^{\text{urb}}, z \in \mathcal{Z}
$$
 (5.57)

As introduced in Section 4.1, satellite capacity restrictions are implemented through assignment and sequencing of vehicle appearances. This is described for services in constraints (5.52) to (5.57). If any demand handover is decided for a service at a satellite, the assignment of a satellite unit to this service is enforced by constraints (5.52). Constraints (5.53) assure that at most one unit is assigned. Symmetries are classically broken using constraints (5.54), which

only allow a service assignment to a unit with a higher index, if the unit with the previous index has already been assigned to another service. Constraints (5.55) link the assignment and the sequencing decisions, while constraints (5.56) assure that a proper sequencing has to be defined for any pair of services visiting the same satellite. The starting times of the corresponding service segments are adjusted according to the sequencing by constraints (5.57). These mechanisms of assignment and sequencing are similarly implemented for each tour's first and last handling leg, see constraints (C.72) to (C.78) in Appendix C. Finally, the following constraints define the domains of all binary decision variables.

*Domains*

$$
\rho_k^{\text{cit}} \in \{0, 1\} \qquad \forall \ k \in \mathcal{K} \tag{5.58}
$$

$$
\rho_r^{\text{urb}} \in \{0, 1\} \qquad \forall \ r \in \mathcal{R} \tag{5.59}
$$

$$
\pi_{d,k}^{\text{cc}} \in \{0,1\} \qquad \forall \ k \in \mathcal{K}(d), d \in \mathcal{D}^{\text{c2c}} \tag{5.60}
$$

$$
\pi_{d,r,k,z}^{\text{in}} \in \{0,1\} \quad \forall (r,k,z) \in \mathcal{C}^{\text{in}}(d), d \in \mathcal{D}^{\text{e2c}} \tag{5.61}
$$

$$
\pi_{d,r,k,z}^{\text{out}} \in \{0,1\} \quad \forall \ (r,k,z) \in \mathcal{C}^{\text{out}}(d), d \in \mathcal{D}^{\text{c2e}} \tag{5.62}
$$

$$
\phi_{r,z,v}^{\text{urb}} \in \{0,1\} \qquad \forall \ v \in \mathcal{U}_z^{\text{urb}}, z \in \mathcal{Z}(r), r \in \mathcal{R}
$$
\n
$$
(5.63)
$$

$$
\phi_{k,l,z,u}^{\text{cit}} \in \{0,1\} \quad \forall \ u \in \mathcal{U}_z^{\text{cit}}, l \in \mathcal{L}^{\text{sat}}(k,z), z \in \mathcal{Z}(k), k \in \mathcal{K}
$$
\n
$$
(5.64)
$$

$$
\chi_{r,r',z}^{\text{urb}} \in \{0,1\} \quad \forall (r,r') \in \mathcal{S}_z^{\text{urb}}, z \in \mathcal{Z}
$$
\n
$$
(5.65)
$$

$$
\chi_{k,l,k',l'}^{\text{cit}} \in \{0,1\} \quad \forall \ (k,l,k',l') \in \mathcal{S}_z^{\text{cit}}, z \in \mathcal{Z}
$$
\n
$$
(5.66)
$$

# **6 Problem-Specific Approaches to Facilitating General MIP Solving**

### **6.1 Cuts and Valid Inequalities**

Cuts and valid inequalities may speed up the solving process by reducing the search space and tightening bounds. In preliminary experiments, we closely observed the feasible solutions found and tested different options of minimum requirements on selection and assignment as well as aggregation and disaggregation of constraints. The following restrictions show positive effects on the general MIP solving behavior.

*Service and Tour Selection*

$$
\rho_{r'}^{\text{urb}} - \sum_{\substack{(r,k,z) \\ \in \mathcal{C}^{\text{in}}(r')}} \sum_{d \in \mathcal{D}^{\text{e2c}}(r')} \pi_{d,r,k,z}^{\text{in}} - \sum_{\substack{(r,k,z) \\ \in \mathcal{C}^{\text{out}}(r')}} \sum_{d \in \mathcal{D}^{\text{c2e}}(r')} \pi_{d,r,k,z}^{\text{out}} \leq 0 \quad \forall r' \in \mathcal{R} \tag{6.1}
$$

$$
\rho_{k'}^{\text{cit}} - \sum_{\substack{(r,k,z) \\ \in \mathcal{C}^{\text{in}}(k')}} \sum_{d \in \mathcal{D}^{\text{e2c}}(k')} \pi_{d,r,k,z}^{\text{in}} - \sum_{\substack{(r,k,z) \\ \in \mathcal{C}^{\text{out}}(k')}} \sum_{d \in \mathcal{D}^{\text{c2e}}(k')} \pi_{d,r,k,z}^{\text{out}} - \sum_{d \in \mathcal{D}^{\text{c2c}}(k')} \pi_{d,k'}^{\text{cc}} \leq 0 \quad \forall k' \in \mathcal{K} \quad (6.2)
$$

To avoid the selection of services and tours without demand assignment, constraints (6.1) and (6.2) are added to the formulation. This strengthens the upper bound of the model by cutting solutions with unnecessarily high total fixed costs for services and tours.

*Service Interval Widths*

$$
\sum_{r \in \mathcal{R}(d)} \sum_{z \in \mathcal{Z}(r)} \left( \bar{\lambda}_{r,z}^{\text{urb}} - \lambda_{r,z}^{\text{urb}} \right) \ge h_d \quad \forall \ d \in \mathcal{D} \setminus \mathcal{D}^{\text{c2c}} \tag{6.3}
$$

$$
\sum_{k \in \mathcal{K}(d)} \left( \bar{\lambda}_k^{\text{in}} - \lambda_k^{\text{in}} \right) \ge h_d \quad \forall \ d \in \mathcal{D}^{\text{e2c}} \tag{6.4}
$$

$$
\sum_{k \in \mathcal{K}(d)} \left( \bar{\lambda}_k^{\text{out}} - \lambda_k^{\text{out}} \right) \ge h_d \quad \forall \ d \in \mathcal{D}^{\text{c2e}} \tag{6.5}
$$

Since demands cannot be rejected, every e2c and c2e demand is handed over between a service and a tour at some point. Thus, it is clear that their handover durations must be part of any service interval and constraints  $(6.3)$ ,  $(6.4)$  and  $(6.5)$  form valid inequalities for all feasible solutions. They improve the lower bound by avoiding and underestimation of the total service interval widths.

*Capacity and Linking Constraints*

$$
\pi_{d,r,k,z}^{\text{in}} - \rho_r^{\text{urb}} \le 0 \quad \forall (r,k,z) \in \mathcal{C}^{\text{in}}(d), d \in \mathcal{D}^{\text{e2c}} \tag{6.6}
$$

$$
\pi_{d,r,k,z}^{\text{in}} - \rho_k^{\text{cit}} \le 0 \quad \forall (r,k,z) \in \mathcal{C}^{\text{in}}(d), d \in \mathcal{D}^{\text{e2c}} \tag{6.7}
$$

$$
\pi_{d,r,k,z}^{\text{out}} - \rho_r^{\text{urb}} \le 0 \quad \forall (r,k,z) \in \mathcal{C}^{\text{out}}(d), d \in \mathcal{D}^{\text{c2e}} \tag{6.8}
$$

$$
\pi_{d,r,k,z}^{\text{out}} - \rho_k^{\text{cit}} \le 0 \quad \forall (r,k,z) \in \mathcal{C}^{\text{out}}(d), d \in \mathcal{D}^{\text{c2e}} \tag{6.9}
$$

$$
\sum_{(r,k,z)\in C^{in}(r')} \sum_{d\in \mathcal{D}^{e2c}(r)} vol_{d}^{\text{urb}} \cdot \pi_{d,r,k,z}^{\text{in}} \n\quad \text{with } g^{r}(z) > g' \n+ \sum_{(r,k,z)\in C^{out}(r')} \sum_{d\in \mathcal{D}^{e2e}(r)} vol_{d}^{\text{urb}} \cdot \pi_{d,r,k,z}^{\text{out}} \leq 1 \quad \forall g' \in \mathcal{G}^{\text{travel}}(r'), r' \in \mathcal{R} \n\text{with } g^{r}(z) < g' \n\sum_{\text{with } g^{r}(z) < g' \n(r,k,z) \in C^{in}(k')} \sum_{l^{k}(D(d)) > l'} vol_{d}^{\text{cit}} \cdot \pi_{d,r,k,z}^{\text{in}} \n+ \sum_{d\in \mathcal{D}^{e2c}(k') \text{ with } l^{k}(D(d)) > l'} vol_{d}^{\text{cit}} \cdot \pi_{d,k'}^{\text{cc}} \n+ \sum_{d\in \mathcal{D}^{e2c}(k') \text{ with } l^{k}(D(d)) > l'} vol_{d}^{\text{cit}} \cdot \pi_{d,r,k,z}^{\text{cut}} \leq 1 \quad \forall l' \in \mathcal{L}^{\text{travel}}(k'), k' \in \mathcal{K} \n(r,k,z) \in C^{\text{out}}(k') \text{ } d \in \mathcal{D}^{e2e}(k) \text{ with } l^{k}(O(d)) \leq l'}
$$
\n
$$
(6.11)
$$

It is well known that aggregated linking constraints like (5.5) and (5.6) lead to weak lower bounds when applying general MIP solving techniques. Our formulation being no exception, the substitution of such constraints by  $(6.6)$  to  $(6.11)$  tightens the bounds and accelerates the

 $+$ 

solving process. Note that the disaggregation of the linking constraints also requires a separation of linkage and vehicle capacity constraints. However, the positive effect on bounding exceeds a potentially negative effect caused by a significantly larger number of constraints.

## **6.2 Indicating Feasible Solutions**

Existing problem-specific knowledge can be used to indicate a first feasible solution, define a reasonable upper bound, and thus, improve the MIP solving process. In this work, we examine two options of determining a first feasible solution, where one is based on instance data and the other uses the mathematical model.

## **6.2.1 Using Demand and Tour Data**

Since demand rejection is not allowed, there must be at least one suitable tour for each demand in a feasible solution. Even if consolidation is one of the key concepts in CL, a trivial feasible solution can be constructed by assigning one individual tour to each demand. Thus, the partial solution given by an indicated assignment can be completed and an upper bound can be calculated. Considering the mathematical model, suitable demand-tour assignment variables  $\pi_{d,k}^{\text{cc}}$  exist for c2c demands. In contrast, the assignment of e2c and c2e demands to tours is done through assigning such demands to inbound and outbound connections  $(r, k, z)$ . Since an indication of the values of the assignment variables  $\pi_{d,r,k,z}^{\text{in}}$  and  $\pi_{d,r,k,z}^{\text{out}}$  requires more complex considerations of both tiers, this is not suitable for a preprocessing step. Nonetheless, the selection variables  $\rho_k^{\text{cit}}$  existing for all tours can be used to indicate which tours should be operated for the trivial assignment. This method is especially effective if short tours with unique or almost-unique assignment options exist, see tour 3 in the illustrative example in Appendix A. Thus, the trivial assignment is indicated through

- 1. sorting the tours according to increasing length (number of customers visited) and
- 2. selecting feasible tours for the trivial one-to-one assignment until all demands are covered.

The method of using a trivial first assignment takes little computational effort, while the probability of finding a feasible solution and imposing a positive effect on computation times is high. However, the complicatedness of the completion of the partial solution strongly depends on the instance's characteristics, and a solution without consolidation on the inner tier may not be favorable.

## **6.2.2 Using a Relaxation of the Model**

Using a relaxed model to obtain information on characteristics of feasible solutions is a wellknown strategy, which is also applied here. Since the satellite capacity restrictions and the waiting duration estimation constitute main computational challenges, these aspects are relaxed. Satellite capacities are discarded, i.e., neglecting constraints (5.52) to (5.57), and waiting durations are determined by using estimates based on the total handover durations as shown in Equation (4.1) only. The full relaxed model can be found in Appendix D.

Since the main components of the 2TM-CLP-SS are still covered, the assignment and selection decisions contained in a solution of the relaxed model should be suitable for finding a first feasible solution of reasonable quality. Thus, the values of the assignment variables  $\pi_{d,r,k,z}^{\text{in}},$  $\pi_{d,r,k,z}^{\text{out}},$  and  $\pi_{d,k}^{\text{cc}},$  and the selection variables  $\rho_r^{\text{urb}}$  and  $\rho_k^{\text{cit}},$  are initially indicated, while the scheduling decisions need to be made to complete this partial solution. Note that the solution space of the 2TM-CLP-SS is fully covered by the description of the relaxation and the objective function value of the relaxed model is a lower bound on the objective function value of the original model. However, more specific conclusions on the solution structure, for instance, the required number of services and tours, cannot be drawn, since the consideration of satellite capacities and the more precise estimation of the waiting durations may force significant changes or even cause infeasibility in the scheduling decisions.

#### **6.3 Fostering City Logistics Goals**

To assure demand satisfaction, the provider of a two-tier CL transportation system may rely on services and tours with a high flexibility next to the probably more restricted ones offered by external logistics service providers. On the outer-tier, feasibility-assuring services may pairwise connect each CDC with each satellite. On the inner-tier, city freighters may oscillate between a single customer location and a satellite. While these types of vehicle movements may be less critical for cargo bikes in the inner city, many short trips of trucks or trams need to be avoided to follow the key goals of CL planning.

Consequently, the solving process can be pointed to more favorable feasible solutions by taking short and highly flexible services out of consideration. The given set is checked for services with only two stops and a starting time window spanning the whole planning horizon. For such services r, the selection variables  $\rho_r^{\text{urb}}$  is forced to 0 by dedicated constraints. This method reduces the number of available connections, which may reduce the computational effort, on the one hand, but may also lead to infeasibility, on the other hand.

## **7 Computational Study**

#### **7.1 Instances**

The real-world inspired instances are generated based on Karlsruhe, a medium-size German city. The considered area with 21,000 inhabitants is close to two main highways and features a rail-based public transport system. Two CDCs, ten road-based, and four rail-based satellites



Table 2: Rules for Including Services and Tours

are chosen, e.g., parking lots of supermarkets and gas stations or tram stations with an appropriate surrounding, see Figure 14 in Appendix E. Capacities and maximum waiting durations of road-based satellites are determined in accordance with the location. For rail-based satellites, a capacity of one tram and one or several city freighters is set. The maximum waiting duration is 0 min for intermediate tram stations and 20 min for terminals. Accordingly, urban vehicles are either trams or small trucks, while city freighters are cargo bikes for all instances. Customer locations are randomly taken from the address list of the area. Distances requested from openrouteservice.org by HeiGIT and measured in travel duration of the corresponding vehicle. For all instances, the number of demands equals the number of customer locations. Demands come in five different sizes, for which a relative frequency of appearance, a handover duration and volume portions are given, see Table 15 in Appendix E.

Sets of services and tours are input to the 2TM-CLP-SS and part of each instance. The total number of existing services and tours growths exponentially in the number of considered locations and the length of the planning horizon, since the number of stops and the starting time window are individual characteristics. Thus, subsets of services and tours are selected so that temporal and spacial feasibility can best possibly be assured for all demands. Table 2 summarizes the rules for including road and rail services and tours. The composition of each set supports instance feasibility and diversity of transport options. While road services are based on a complete network between CDCs and satellites, rail services are created from given rail lines. They describe existing routes involving one or several satellite tram stations and at least one CDC tram station. For every station, a maximum number of visits per hour is given. The diversification-driven rail services implement frequent starting time options based on these restrictions and feature starting time window widths of 5 min. In contrast, the starting time window widths of road services may vary between 5 min and the full time horizon for road

Values
2h, 4h
10, 20, 30
$(1, 0, 0), (0.8, 0.2, 0), (0.65, 0.2, 0.15)$
road, rail
road: 2, 5, 10, rail: 4
$(1, 0, 0), (0, 1, 0), (0, 0, 1), (1/3, 1/3, 1/3)$
$(1, 0, 0), (0, 1, 0), (0, 0, 1), (1/3, 1/3, 1/3)$
* given by relative frequencies of the different entigers

Table 3: Overview of Varied Instance Parameters

given by relative frequencies of the different options

services, where the latter defines the maximum starting time flexibility. The bottom row of Table 2 indicates how fixed costs are determined. For tours k, the fixed costs coefficient  $c_k^{\text{cit}}$ equals the total travel duration in min. For services r, the coefficient  $c_r^{\text{urb}}$  is defined by the total travel duration multiplied by a flexibility multiplier which is 1 for regular starting time window widths and 3 for maximum starting time flexibility.

The instances can further be classified by the varied parameters shown in Table 3. Planning time horizons of 2 h and 4 h reflect more or less restricted delivery time frames in urban areas, see e.g., Muñuzuri et al. (2012) and Fontaine et al. (2021). The number of demands is varied between 10 and 30, while the demand type distributions represent the cases of (i) inbound demands only, (ii) an inbound-outbound integration and (iii) the desired maximum integration of multi-directional flows. With the majority of travel durations being less than 10 min, these numbers of demands are supposed to be shippable within the time horizons. In line with the literature, the number of road-based satellites is varied between 2 and 10, see Grangier et al. (2016), Ben Mohamed et al. (2017), Fontaine et al. (2021) and Li et al. (2023). In contrast, the 4 rail-based satellites shown in Figure 14 are taken as given, since the rail infrastructure cannot easily be adapted. A demand's time window width can either be 15 min (short), 30 min (medium), or the full planning time horizon (long). The instances cover three extreme cases with equal demand time window widths and a mixed case. Similarly, a maximum waiting duration allowed at a customer location (CWD) may be 0 min (none), 15 min (short), or the full planning time horizon (long), while the instances feature different distributions. Note that long maximum waiting durations may create an unrealistic planning flexibility that even complicates the solving process. However, a total ban of waiting may lead to an inefficiently high number of required services and tours or even the impossibility of delivering all demands.

Whenever sampling is required in the instance generation, e.g., for customer locations, demand sizes, demand types, reduced numbers of satellites, time window widths and customer waiting durations, three different samples of each parameter value are drawn and used. This results in 46,656 instances forming the basis of the computational study. Note that appropriate combinations of extreme distributions do even cover 2E-VRP and 2E-VRPTW cases. The full

Name	Description	Time limit
Standard	MIP solving	$60 \text{ min}$
<b>StartTours</b>	MIP solving with start values for tour selection based on 60 min	
	trivial feasible demand assignment	
StartAssign	MIP solving with start values for assignment variables $10 + 50$ min	
	based on relaxed model	
Reduced Flex	MIP solving with excluded maximum flexibility services	$60 \text{ min}$

Table 4: Overview of Methods Applied

data set is available online for academic use, see Lange (2024).

## **7.2 Parameters, Implementation and Structure**

Based on results of preliminary tests, the cuts and valid inequalities described in Section 6.1 are always included, and branching priority is given to assignment variables  $\pi_{d,r,k,z}^{\text{in}}$  and  $\pi_{d,r,k,z}^{\text{out}}$ . Considering the combination of multiple objectives, weights  $q_1 = q_2 = 1$  are used for the fixed costs of services and tours and the widths of activity intervals.  $q_3 = 0.01$  is chosen for the waiting durations to avoid unnecessarily spread handovers. Note that since the fixed costs are measured as travel durations, there is no further normalization required.

To gain methodological and managerial insights, four solution approaches based on the MIP model are tested and compared. As shown in Table 4, the baseline method is running the MIP solver without enhancements for 1 hour. Following the explanations in Section 6.2, two approaches using start values for variables are implemented, where the time limit is split when the relaxed model is used. Lastly, the instances are solved without services of maximum starting time window width to observe the effects of decreased planning flexibility and number of services. The mathematical formulation is implemented using Python 3.8 with the gurobipy package and solved by Gurobi 11.0. The computational experiments are run single-threaded on a machine featuring an Intel XEON SP with 2.6 GHz and 6 GB RAM.

## **7.3 Computational Results**

## **7.3.1 Results of Standard General MIP Solving**

*MIP Gap Values.* Table 5 summarizes the average MIP gap values for road-based instances solved by the Standard method. Subsets of instances are characterized by the number of demands n, the number of satellites  $Z$ , the demand type and TW widths distributions, and the planning time horizon. Different CWD distributions are not reported, since they do not have significant effects on the gaps.

Demand		$\operatorname{TW}$		Time Horizon: 2 h			Time Horizon: 4 h	
		Widths	$n=10$	$n=20\,$	$n=30\,$	$n=10$	$n=20$	$n=30$
		long	$0.0\%$	31.8%	$61.6\%$	$0.0\%$	32.0%	64.5%
Z <b>Types</b> $\overline{2}$ (0.65, 0.2, 0.15) $\overline{5}$ 10 $\overline{2}$ $\widehat{\circ}$ (0.8, 0.2, $\overline{5}$ $10\,$ $\overline{2}$ $(1,\, 0,\, 0)$ $\overline{5}$ $10\,$	medium	$0.0\%$	18.3%	$39.5\%$	$0.0\%$	22.7%	48.2%	
		short	$0.0\%$	11.9%	27.8%	$0.0\%$	19.9%	$44.4\%$ <sup>+</sup>
		$\rm mix$	$0.0\%$	21.2%	41.0%	$0.0\%$	24.8%	47.6%
		long	$0.0\%$	$29.0\%$	67.0%	$0.0\%$	29.6%	$59.4\%$
		medium	$0.0\%$	24.7%	45.2%	$0.0\%$	27.5%	54.9%
		short	$0.0\%$	17.2%	36.8%	$0.0\%$	21.9%	54.4%
		mix	$0.0\%$	$24.4\%$	47.9%	$0.0\%$	27.5%	$53.5\%$
		long	$0.0\%$	$26.2\%$	$49.1\%$	$0.0\%$	23.3%	49.4%
		medium	$0.0\%$	18.2%	35.3%	$0.0\%$	$22.1\%$	42.8%
		short	$0.0\%$	12.9%	29.8%	$0.0\%$	18.4%	39.9%
		mix	$0.0\%$	19.3%	35.4%	$0.0\%$	22.4\%	41.3%
		long	$0.0\%$	48.9%	74.5%	$0.0\%$	50.0%	72.1%
		medium	$0.0\%$	26.4%	49.8%	0.4%	$30.5\%$	59.9%
		short	$0.0\%$	18.1%	47.7%	0.6%	16.1%	58.9%
		mix	$0.0\%$	28.5%	54.5%	$0.9\%$	31.4%	59.9%
		long	8.1%	44.0%	74.7%	7.8%	44.6%	$63.3\%$
		medium	2.2%	34.9%	$50.4\%$	0.4%	35.7%	$52.0\%$
		short	$0.0\%$	26.9%	42.6%	$0.0\%$	36.6%	50.1%
		$\mbox{mix}$	2.0%	35.4%	53.4%	$0.9\%$	39.6%	58.4%
		long	$0.0\%$	35.7%	55.8%	$0.0\%$	33.1%	54.4%
		medium	$0.0\%$	28.9%	42.0%	$0.0\%$	29.2%	49.0%
		short	$0.0\%$	23.9%	36.5%	$0.0\%$	24.6%	45.2%
		$\operatorname{mix}$	$0.0\%$	27.5%	43.9%	0.1%	31.1%	50.9%
		long	$0.0\%$	45.4%	71.0%	$0.1\%$	36.1%	$63.3\%$
		medium	0.2%	25.9%	46.2%	$0.9\%$	30.3%	49.5%
		short	$0.0\%$	18.0%	$35.0\%^*$	0.2%	25.9%	47.6%
		mix	$1.5\%$	30.2%	48.5%	$1.6\%$	35.3%	56.6%
		long	$3.9\%$	41.4%	71.8%	3.4%	$40.2\%$	$62.2\%$
		medium	$0.0\%$	32.8%	$50.6\%$	$0.2\%$	34.8%	53.8%
		short	$0.0\%$	27.3%	43.1%	$0.0\%$	32.4%	52.9%
		$\mbox{mix}$	3.1%	35.9%	51.5%	2.6%	37.5%	55.0%
		long	$0.0\%$	32.1%	54.2%	$0.0\%$	28.8%	50.0%
		medium	$0.0\%$	24.2\%	41.3%	$0.0\%$	28.3%	45.3%
		short	$0.0\%$	20.9%	30.4%	$0.0\%$	26.6%	41.8%
		$\operatorname{mix}$	$0.0\%$	26.2\%	42.1%	$0.0\%$	30.0%	47.7%

Table 5: MIP Gap Values for Standard Method on Road-Based Instances

 $^+$  11% of these instances are infeasible.  $^*$  6% of these instances are infeasible.

The results show that increasing the number of demands clearly increases the complicatedness of the instances independent of the other parameters. 89% of the instances with 10 demands could be solved to optimality within 60 min, while this was possible for almost none, namely 0.05%, of the instances with 20 and 30 demands. Even if feasibility is considered in the instance generation, a few instances with 30 demands, 2 satellites and short TWs turn out infeasible due to satellite and vehicle capacities and conflicting temporal restrictions.

Considering the demand type distribution, an integrated planning of e2c, c2e and c2c flows is easiest from a MIP solving point of view followed by the e2c-only case and the e2c-c2e integration. This is caused by the required amount of inter-tier synchronization, which is highest in the e2c-c2e case, decreases when no c2e flows are considered, and decreases even more when some demands (c2c) require no synchronization.

Varying the number of satellites provides an interesting insight for practical planning. While instances with 10 satellites mostly show the smallest gaps followed by the 2-satellite case, the instances with 5 satellites feature the highest gaps for the majority of the parameter settings. This is due to the amount of available connections per demand to choose from. A smaller number of feasible connections per demand exists when the number of services is limited (2 satellites) or the set of services is geographically diverse and each satellite features little visiting connections (10 satellites). In contrast, 5-satellite instances involve a large set of visiting services for each satellites and many feasible connections for each demand.

Regarding the TW widths distribution, short TWs facilitate the MIP solving process, while long TWs complicate it independent of the other parameters. This observation is in line with the well-known results on the VRPTW (Kolen et al., 1987). Instances with mixed TW widths, which represent the most practically relevant case, show average gaps when varying this distribution.

Extending the planning time horizon from 2 h to 4 h leads to an increase in the average gap for the majority of the parameter settings. Only 18 of 108 instance sets show a decrease, out of which 14 feature long demand TWs. If planning flexibility on the demand side is high, a longer time horizon seems favorable for the MIP solving process. For 29 instance sets, the gap increases by more than 5 percentage points, where 22 sets feature 30 demands. Thus, considering different numbers of demands, the gap increase behaves proportionally to the absolute gap value. However, the highest gap differences are observed for instances with short TWs. This means that the computational advantage of tighter TWs is partially compensated when the TWs are more widely spread.

The results for the rail-based instances reported in Table 16 in Appendix F show equivalent effects when increasing the number of demands or varying the demand types and TW widths. With 4 satellites, these instances are comparable to the road-based instances with  $Z = 5$ , while the number of services is usually higher in the rail-based setting due to more frequent starting time options. Nonetheless, the rail-based instances show slightly smaller average gaps for the 2 h time horizon, which may be due to the tighter starting time windows of the services. When increasing the planning time horizon to 4 h, the increase in the average gaps is significant. This may result from a substantial increase in the number of available services and connections per demand.

Table 6: Average Percentage Differences in Objective Function Value of Instances solved to Optimality with 2 h and 4 h Time Horizon given  $n = 10$ ,  $Z = 2$  and Demand Types (0.65, 0.2, 0.15)



For reasons of validity, the following solution characteristics and the dependence on the instance parameters are only investigated for the 13,826 instances with 10 demands solved to optimality (opt10).

*Objective function values.* Table 6 displays the percentage differences in objective function values between instances with 2 h and 4 h time horizon that are solved to optimality. The instances reported here feature  $n = 10$  and  $Z = 2$ , and the demand type distribution is fixed to the fully integrated setting. For a comprehensive overview of the measures for all instances parameter settings, the reader is referred to Table 17 in Appendix F. It can be observed that the TW widths and the CWDs have a significant impact on the change in the objective function value when increasing the planning time horizon. Instances with long TW widths show a special behavior, since they feature a maximum temporal flexibility for demand satisfaction. Optimal solutions without any waiting at customer locations exist, and no changes are required when demands are more spread and waiting is restricted. Considerable objective function value increases result when TWs and CWDs are tightly bounded. For instances with short and medium TW widths, the effect of disappearing consolidation options is remarkable, while it is smaller for short TW widths, since they restrict consolidation anyway. For the case of mixed TW widths, which is assumed to be the most practically relevant, the TW widths effects seem to compensate each other so that a moderate increase of less than 10% can be observed for all CWD distributions.

*Total fixed cost.* Figure 7 shows the averaged total fixed cost of services and tours selected for road-based instances with different demand type and TW widths distributions and time horizons. It can be observed that instances with tighter TWs require a more expensive service and tour selection independent of the number of satellites involved. A high number of assignment options per demand  $(Z = 5)$  and the integration of all three demand types have a positive effect on the fixed cost. In contrast, spreading demands over a longer time horizon leads to higher total costs especially when multi-directional demands are involved. This supports the observation on the percentage difference in the overall objective function values made in Table 17. CWD policies are not explicitly shown here, since they only have one major impact. Long CWD allowances mitigate the increase in the total fixed cost when increasing the time horizon. Figure 16 in Appendix F supports these observations by plotting the averaged number of services used for instances involving all three demand types. An interesting aspect additionally revealed here is that the amount of total fixed cost seems to be driven by the costs of tours, since



Figure 7: Averaged Total Fixed Cost of Services and Tours Selected for Road-Based Instances with  $n = 10$ 

the number of services used is significantly smaller for  $Z = 2$  than for  $Z = 10$ . The behavior of the results of the rail-based instances is similar to the road-based results discussed here, see Figure 15 in the Appendix F. The only exception is that the beneficial effect of consolidating multi-directional flows is less significant. This is due to the line-based routing and frequent start options resulting in a high number of services with similar fixed cost.

*Waiting durations.* The averaged total waiting duration of all vehicles is depicted in Figure 8 for road-based instances with three demand types, different TW widths distributions and time horizons. It can be seen that vehicles tend to wait longer, if TWs are tight, the time horizon is long, and the instances feature more diverse assignment options for the demands  $(Z = 5$  and  $Z = 10$ ). Especially for instances with 2 and 5 satellites, the total waiting duration also increases with less restrictive CWD policies. This behavior is observed similarly for the rail-based instances, as shown in Figure 17 in Appendix F, and for the instances with the other two demand type distributions.

Two special types of waiting durations exist in the solutions: (i) a vehicle waiting without any handover activity and (ii) waiting not captured by the waiting duration estimates as described in Section 4.4. Waiting without handover takes place in 24.2% of the optimal solutions with an average duration of 17.11 min per waiting activity. These waiting durations appear more often, if TWs are tight, the time horizon is long, and the CWD policy is mixed. An underestimation of a waiting duration is only observed in 3.4% of the optimal solutions with an average duration of 4.42 min. A small number of satellites facilitates this behavior, since handovers involving several vehicles and demands occur more often. Tight TW widths, restrictive CWD policies and a long time horizon additionally induce underestimation.

*Resource efficiency.* The resource efficiency of the transportation system is evaluated by observing the averaged number of demands per selected service and the averaged numbers of satellites used. Figure 9 plots the former for instances with different demand type distributions



Figure 8: Averaged Total Waiting Duration for Road-Based Instances with  $n = 10$  and Demand Type Distr. (0.65, 0.2, 0.15)



Figure 9: Averaged Number of Demands per Service Used for Instances with  $n = 10$ 

and TW widths. The highest degree of consolidation is reached in solutions with five satellites and many assignment options per demand. In addition, long TW widths facilitate consolidation, while no significant effect can be reported for the length of the time horizon and the CWD policies. A similar behavior is observed for tours. Figure 18 in Appendix F illustrates that the average number of satellites used is mainly dependent on the number of existing satellites and the corresponding services and tours. The transportation system shows a reasonable satellite usage in the optimal solutions with around 4 out of 10 and around 3 out of 5 for short, medium and mixed TW widths. When TW widths are long, the averaged number of satellites used decreases, while it is almost constant for different CWD policies.

*Summary.* Table 7 summarizes the effects of the instance parameters varied. The TW widths are clearly identified as the most significant parameter, while the CWD policies do not cause major effects. Increasing the number of demands increases the complicatedness of the planning





% : increase, & : decrease, o : no significant effect, ∼ : no universal statement possible, - : not evaluate



Figure 10: Gap Difference between Facilitating Methods and Standard Method for Road-Based Instances with 2 hours Time Horizon

problem. The same holds for increasing the TW widths and the time horizon. Integrating all three demand flows seems beneficial, since it facilitates MIP solving and reduces the total fixed cost for services and tours selected. Contradicting effects are observed for instances with a high number of demand assignment options, meaning a medium number of satellites, and long TW widths. These planning problems are harder to solve, but the results feature lower total fixed cost and a higher degree of consolidation. Considering vehicles waiting in the urban area, longer TWs enable the reduction of waiting, while a longer planning time horizon necessitates it. Highly restricted maximum waiting durations at some customer locations are often partially compensated by longer waiting at satellites and other customer locations.

#### **7.3.2 Results of Enhanced General MIP Solving**

*MIP gap differences.* Figure 10 plots the differences in the MIP gaps obtained by the facilitating methods and the Standard method for instances with 20 and 30 demands and a time horizon of 2 h. Instances with 10 demands are mostly solved to optimality and rarely feature minor gap differences between -2% and 2%. Instances with 4 h time horizon show equivalent results and are therefore spared, too.

The effects of the facilitating methods are most differentiable for  $n = 20$ . Independent of the demand type distribution, the number of satellites and the TW widths, the ReducedFlex method outperforms the start value-methods especially for instances with less demand assignment options,  $Z \in \{2, 10\}$ , and limited TWs widths. Comparing the start value-strategies, StartTours achieves a higher gap decrease than StartAssign for most of the parameter settings. StartAssign even results in higher averaged remaining gaps for 26 out of 36 instance subsets. The improvement reached by a tighter upper bound is overcompensated by a decrease in the lower bound caused by the reduced computation time. For  $n = 30$ , a moderate but volatile advantageous effect of all methods on all parameter settings can be observed. The application of facilitating methods seems useful when solving larger instances, whereby the effectiveness of a certain strategy is instance parameter-specific. Further, it must be remarked that using the ReducedFlex strategy may lead to infeasibility as for instances with only e2c demands, 2 satellites and short TWs. Obviously, the services with maximum flexibility are required here to construct feasible solutions.

The gap differences of the rail-based instances are presented in Figure 19 in Appendix F. All facilitating methods show small positive effects for most of the parameter settings, but no strategy can be determined most favorable. Due to a large number of services per instance, the ReducedFlex method cannot achieve the same significant improvements as for road-based instances, but feasible solutions can always be found.

*Objective function values and lower bounds.* To illustrate the effects of the facilitating methods on the solving process in more detail, Figure 11 displays the percentage differences in the objective function values of the best solutions found and the lower bound obtained related to the results of the Standard method. For  $n = 20$ , StartTours and StartAssign do only show marginal effects, while ReducedFlex can increase the lower bound by up to 33%. However, limiting planning flexibility also causes increased objective function values especially for instances with short TWs. For  $n = 30$ , the improvements in the upper bounds obtained by StartTours and StartAssign can clearly be seen, while these methods have almost no effect on the lower bounds. The results of the ReducedFlex method are similar for both numbers of demands with more significant impacts realized on instances with 10 satellites.

*Optimality and infeasibility.* Supporting the previous observations, Table 8 summarizes the portions of instances solved to optimality or determined infeasible for each method. It becomes clear that indicating start values does not improve the solving process enough to obtain optimal solutions for larger instances. The ReducedFlex method shows a positive effect with regard to optimality through reducing the number of services. However, it involves a high risk of cutting elements required for feasible solutions.



Figure 11: Percentage Difference of UB and LB between Facilitating Methods and Standard Method for Road-Based Instances with 2 h Time Horizon





#### **7.4 Methodological Insights**

Using a general, state-of-the-art MIP solving method, the 2TM-CLP-SS can be solved to optimality in reasonable time for small instances. The results prove the capability of the mathematical formulation to capture important aspects and driving parameters of scheduling and synchronization decisions in 2TM-CL systems. Especially the determination scheme of waiting durations of vehicles based on continuous starting time variables is successfully applied. The total waiting durations are properly indicated for 96.6% of the instances solved to optimality with  $n = 10$ , and the average amount the durations are underestimated is less than 5 min.

The extensive computational study highlights that instances become challenging when (i) temporal restrictions are loose, (ii) demands are spread over a long planning time horizon, (iii) many potential demand itineraries exist, and (iv) the required amount of multi-directional flow synchronization is high. In solving larger instances, the results show that even small enhancements like indicating trivial feasible solutions based on the instance data improve the solving process. Especially interesting for further algorithmic developments are the results of the ReducedFlex method. The method removes a fraction of the available services following CL goals. Thereby, it decreases the number of assignment options per demand and reduces the temporal planning flexibility of the instance. As a result, the portion of instances solved to optimality increases, while the objective function values of the best solutions found slightly deteriorate and about 5.9% of the instances get infeasible. This supports the idea of reducing the sets of available services and tours to shrink the search space and speed up the solving process. However, the elimination rules require a more elaborate consideration of the solution structures so that instance feasibility is preserved and the increase in total costs and environmental effects is kept small.

## **7.5 Managerial Insights**

The computational results reveal important factors and mechanisms for designing, managing and operating a 2TM-CL system with satellite synchronization. Three significant implications can be summarized.

First, doability needs to be considered when deciding about vehicle fleets, satellite locations and scheduling restrictions. If the volume of commodity flows increases, vehicle and satellite capacities must suffice and a higher level of temporal planning flexibility is required. Especially the extend in which customers can restrict demand TWs is a significant parameter with a critical trade-off. Shorter TWs have a big advantageous effect in facilitating the planning process using general MIP methods, while larger TWs assure feasibility, increase planning flexibility and enable the realization of more sustainable solution with regard to resource consumption.

The amount of satellites is another important parameter with a trade-off effect. The study shows that a medium number of satellites supports consolidation, a major goal in CL, at best. For the considered area in Karlsruhe, 2TM-CL systems with 5 satellites are favorable. They feature sufficiently diverse sets of services and tours on both tiers, which provide flexibility in the demand assignment and enforce consolidation at the same time. However, the downside of a medium number of satellites and many assignment options per demand is a more complicated planning process.

A last aspect to consider is the allocation of vehicles waiting in the urban area. The results indicate that possibilities to wait are a substantial part of the temporal planning flexibility and support doability and efficiency. However, vehicles do not necessarily need the option to wait at customer locations as varying CWD policies do not significantly affect the quality of the solutions and the complicatedness of the planning process. If the maximum waiting duration at customer locations is tightly restricted, this is compensated by vehicles waiting at satellites or other customer locations. Thus, in managing and planning the transportation system, the resource consumption of vehicles can be controlled location-wise as long as enough waiting options are available in the system as a whole.

## **8 Conclusion**

In this paper, a day-before planning problem of freight transportation in urban areas, where inbound, outbound and inner-city demands must be shipped through a two-tiered network of available public and private transportation services, is considered. To reduce the use of traffic and building infrastructure, areas like supermarket parking lots and tram stations act as satellite locations featuring no storage capacities, and waiting durations of vehicles are limited and minimized. Thus, the exact synchronization of the schedules of both tiers is required and we focused on the temporal dimension of the problem.

The presented path-based modeling approach is based on binary selection and assignment decisions and continuous time variables. We proposed general concepts to synchronize multidirectional demand handovers using handover intervals and precisely estimate waiting durations through vehicle- and location-related activity intervals. The computational results support the validity of the mathematical description, which forms a promising first step in capturing the temporal requirements. It can be extended to include heterogeneous fleets, satellite opening TWs and volume-based capacities, a given assignment of demands to CDCs or satellites, and explicit considerations of demand handling activities in CDCs.

The numerical study was conducted on a vast new instance set that features road- and railbased outer-tier transportation and a variety of instance parameter settings. We discussed the performance of a general MIP solver on the model and reported improvements gained by simple facilitating methods. While optimal solutions can only be found in reasonable time for small instances, indicating a first feasible solution based on the instance data showed positive effects for larger instances. A variable fixing scheme, which forbids the selection of transportation services with few stops and high temporal flexibility, showed an interesting trade-off between beneficial solution space reduction and infeasibility. From a managerial perspective, the number of satellites and the control of the waiting durations are revealed as significant aspects. A medium number of satellites has positive effects on consolidation, while limits on waiting durations are uncritical as long as a sufficient overall temporal planning flexibility is given.

Regarding future work, we see three meaningful directions. First, the development of tailored solution approaches like matheuristics and decomposition-based methods seems promising, since complicating aspects are nicely detectable. Further, uncertainty in travel and handover times must be approached to determine reliable and resilient solutions. Thereby, renegotiation of demand TWs as considered by Hewitt (2022) might be an interesting extension. Social and environmental aspects form a last important research direction still rarely addressed. Since waiting duration restrictions can be part of the system control, fairness among people living close to satellite locations or other parking spots can be taken into account.

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# **A Illustrative 2T-CL Example**

Figure 12 extends the abstract network representation of the 2T-CL system from Section 1 by location indices and travel times for the transportation links. While an accepted waiting time of 1 time period is assumed for each customer location  $i \in \mathcal{I}$ , capacity and waiting time parameters for satellites are given in Table 9. The illustrative example features 6 demands, for which their characteristics are summarized in Table 10.



Figure 12: A 2T-CL System Example

Tables 11 summarize the sets of services and tours usable to fulfill the given demands. Four services can be selected to ship inbound demands, while six of them may carry outbound demands. Services  $r \in \{1, 2, 3, 5, 6, 7\}$  feature maximum flexibility in their starting time, while services  $r \in \{4, 8, 9\}$  have tighter starting time windows.

An optimal solution for the 2T-CL planning problem is shown in Figure 13. Time is plotted on the horizontal axes. Each location is depicted by a lane showing all the related activities and time windows. Two services, indicated by red lines, and four tours, shown by blue lines, are selected. While filled semicircles represent starting times of service stages and tour legs, blank semicircles indicate unused stops. Waiting durations, shown by purple hashed areas, occur at satellite 5 and customer location 10. Note that tour 1 waits at customer location 10, since the maximum waiting duration of satellite 5 is 2 time periods. The demand handovers taking place at satellite 5 demonstrate the proper implementation of the satellite capacity restrictions. This schedule is realized with total fixed costs of 46 and a total waiting duration of 2.5.

$\boldsymbol{z}$	$N_z^{\rm urb}$	$N_z^{\text{cit}}$	$w_z^{\text{sat}}$
3	2	2	2
		2	
h			к

Table 9: Overview of Satellite Characteristics

Table 10: Overview of Demands and their Characteristics

$\mathfrak{a}$	Type	O(d)	D(d)	$h_d$	$vol^\mathrm{cit}_d$	$vol_d^{\text{urb}}$	$\tilde{a}_d$	$\left[b_d^{\text{pick}}, \bar{b}_d^{\text{pick}}\right]$	$\left[b_{d}^{\textrm{deli}}, \bar{b}_{d}^{\textrm{deli}}\right]$
	e2c		9	$0.5\,$	1/2	1/4	4		[10, 14]
$\overline{2}$	c2e	10	Х	$0.5\,$	1/4	1/8	$\qquad \qquad -$	[2, 6]	
3	e2c	Х	11	$0.5\,$	1/4	1/8	5	$\overline{\phantom{a}}$	[16, 18]
4	e2c	Х	6	1	2/3	1/3	$\theta$	-	[5, 7]
5	c2e		Х	0.5	1/3	1/6		[2, 6]	
6	c2c	8	10	0.5	1/2	-		[6, 9]	[8, 11]

Table 11: Overview of Services and Tours and their Characteristics

(a) Services  $r \in \mathcal{R}$ 

(b) Tours  $k \in \mathcal{K}$ 







Figure 13: Optimal Schedule for the Illustrative Example

# **B Notation**

Note that variables are always denoted by Greek letters, while indices, sets and parameters are represented by Latin letters.

## **Sets**



 $\mathcal{D}^{e2c} = \{d \in \mathcal{D} \mid O(d) = X, D(d) \in \mathcal{I}\}\$ e2c (inbound) demands  $\mathcal{D}^{\text{e2c}}(k) = \{d \in \mathcal{D}^{\text{e2c}}\}\$  $D(d) \in \mathcal{I}(k)$  $\mathcal{D}^{\mathrm{c2e}}(k) = \{$  $O(d) \in \mathcal{I}(k)$  $\mathcal{D}^{\mathrm{c2c}}(k) = \{$  $(O(d) \in \mathcal{I}(k)) \wedge$  $(D(d) \in \mathcal{I}(k))$  $\mathcal{D}(k) = \mathcal{D}^{e2c}(k) \cup \mathcal{D}^{c2e}(k) \cup \mathcal{D}^{c2c}(k)$  demands transportable by tour k  $\mathcal{D}^{\text{e2c}}(r) = \{d \in \mathcal{D}^{\text{e2c}} \mid O(r) = X\}$  $\mathcal{D}^{\mathrm{c2e}}(r) = \{d \in \mathcal{D}^{\mathrm{c2e}} \mid D(r) = X\}$  $\mathcal{D}(r) = \mathcal{D}^{\text{e2c}}(r) \cup \mathcal{D}^{\text{c2e}}(r)$  demands transportable by service r  $\mathcal{E} = \{e \mid e = 1, \ldots, E\}$  city distribution centers (CDC)  $\mathcal{E}(r) \subseteq \mathcal{E}$  CDCs service r is visiting  $\mathcal{G}(r)=\{g\mid g=0,\ldots,m_{r}^{\text{seg}}\}$  $\mathcal{G}^{\text{travel}}(r) \subset \mathcal{G}(r)$  $\mathcal{G}^{\text{hand}}(r) \subset \mathcal{G}(r)$  $\mathcal{I}(k) \subset \mathcal{I}$  customer locations tour k is visiting  $\mathcal{K} = \{k \mid k = 1, \ldots, K\}$  tours (by city freighters)  $\mathcal{K}(d) \subseteq \mathcal{K}$  tours fitting to satisfy demand d  $\mathcal{L}(k) = \{l \mid l = 0, \ldots, m_k^{\text{leg}}\}$  $\mathcal{L}^{\text{travel}}(k) \subset \mathcal{L}(k)$  $\mathcal{L}^{\text{pd}}(k) \subset \mathcal{L}(k)$  $\mathcal{L}^{\text{sat}}(k,z) \subset \{0,m_k^{\text{leg}}\}$ k  $\mathcal{R} = \{r \mid r = 1, \ldots, R\}$  services (by urban vehicle)  $\mathcal{R}(d) \subset \mathcal{R}$  services fitting to satisfy demand d  $\mathcal{S}_z^\text{cit}=\big\{(k,l,k',l'$  $l'\in\dot{\mathcal{L}}^\mathrm{sat}(k'$  $k' \in \mathcal{K} \setminus \{k\}$  with  $z \in \mathcal{Z}(k'),$  $l \in \mathcal{L}^{\text{sat}}(k, z), k \in \mathcal{K}$  with  $z \in \mathcal{Z}(k)$  $\mathcal{S}_z^{\text{urb}} = \big\{ (r,r'$  $r' \in \mathcal{R} \setminus \{r\}$  with  $z \in \mathcal{Z}(r')$  $r \in \mathcal{R}$  with  $z \in \mathcal{Z}(r)$  $\mathcal{U}_z^{\text{cit}} = \{u \mid u = 1, \ldots, N_z^{\text{cit}}\}$  $\mathcal{U}_z^{\text{urb}} = \{v \mid v = 1, \ldots, N_z^{\text{urb}}\}$  $\mathcal{Z} = \{z \mid z = 1, \ldots, Z\}$  satellites  $\mathcal{Z}(k) \subseteq \mathcal{Z}$  satellites tour k is visiting  $\mathcal{Z}(r) \subseteq \mathcal{Z}$  satellites service r is visiting

 $e2c$  demands deliverable by tour  $k$  $c2e$  demands pickable by tour k  $c2c$  demands transportable by tour k

e2c demands transportable by service  $r$ c2e demands transportable by service r segment indices of service  $r$ indices of all travel segments of service  $r$ indices of all handling segments of service  $r$  $\mathcal{I} = \{i \mid i = 1, \ldots, I\}$  customer locations, alternative index: j leg indices of tour  $k$ indices of all travel legs of tour  $k$ indices of all pickup or delivery legs of tour  $k$  $\log$  indices of tour k visiting satellite z ) | pairs of tour legs potentially competing for a unit at satellite z

> ) | pairs of services potentially competing for a unit at ), satellite z

city freighter units of satellite  $z$ urban vehicle units of satellite  $z$ 

#### **Parameters**



 $[b_{d}^{\rm{pick}}]$  $\bar{b}_d^{\text{pick}}, \bar{b}_d^{\text{pick}}$ d time window for the starting time of pickup of a demand  $d \in \mathcal{D}^{c2e} \cup$  $\mathcal{D}^{\text{c2c}}, b_d^{\text{pick}} \geq 0, \overline{b}_d^{\text{pick}} \geq b_d^{\text{pick}}$ d  $[b_d^{\text{deli}}, \bar{b}_d^{\text{deli}}]$ time window for the starting time of delivery of demand  $d \in \mathcal{D}^{e2c} \cup$  $\mathcal{D}^{\text{c2c}}, \, b_d^{\text{deli}} \geq 0, \bar{b}_d^{\text{deli}} \geq b_d^{\text{deli}}$  $c_r^{\rm{urb}}$ fixed cost of operating service  $r$  $c_k^{\rm{cit}}$ fixed cost of operating tour  $k$  $D(d) \in \{X\} \cup \mathcal{I}$  destination of demand d  $D(r) \in \mathcal{Z} \cup \mathcal{E}$  destination of service r  $D(k) \in \mathcal{Z}$  destination of tour k  $g^{r}(z) \in \{0, 1, \ldots, m_{r}^{\text{seg}}\}$ segment of service r representing a stop at satellite  $z$  $h_d > 0$  handover time of demand d at satellite or customer location  $l^k(i) \in \{2,\ldots,m_k^{\text{leg}}\}$  $\log$  of tour k representing pickup or delivery at customer location i  $m_k^{\operatorname{leg}}$ index of the last leg of tour  $k$  $m_k^{\operatorname{leg}}$ dummy leg representing the end of tour  $k$  $m_r^{\rm seg}$ index of the last segment of service  $r$  $m_r^{\text{seg}}$ dummy segment representing the end of service  $r$  $N_z^{\text{cit}}\in\mathbb{Z}_+$ number of city freighter parking units at satellite  $z$  $\tilde{N_z^{\text{urb}}} \in \mathbb{Z}_+$  $N_z^{\text{urb}} \in \mathbb{Z}_+$  number of urban vehicle parking units at satellite  $z$ <br>  $O(d) \in \{X\} \cup \mathcal{I}$  origin of demand d origin of demand  $d$  $O(r) \in \mathcal{Z} \cup \mathcal{E}$  origin of service r  $O(k) \in \mathcal{Z}$  origin of tour k  $q_1 > 0$  weight of the fixed costs in the objective function  $q_2 > 0$  weight of the activity interval widths in the objective function  $q_3 > 0$  weight of the waiting times in the objective function  $tt_{k,l}^{\mathrm{cit}}>0$ travel time of travel leg  $l \in \mathcal{L}^{\text{travel}}(k)$  of tour k  $tt_{r,a}^{\text{urb}}$ travel time of segment  $q \in \mathcal{G}^{\text{travel}}(r)$  of service r  $vol_{d}^{\mathrm{cit}}$ city freighter volume portion required by demand  $d$  $vol_d^{\text{urb}} \in (0,1]$ urban vehicle volume portion required by demand  $d$  $w_z^{\rm sat}$ maximum waiting time of any vehicle at satellite  $z$  $w_i^{\text{cust}}$  $w_i^{\text{cust}} \geq 0$  maximum waiting time of a city freighter at customer location i<br>  $X$  external zone external zone

## **Variables**





# **C Full Mathematical Model**

$$
q_1 \cdot \left[ \sum_{r \in \mathcal{R}} c_r^{\text{urb}} \cdot \rho_r^{\text{urb}} + \sum_{k \in \mathcal{K}} c_k^{\text{cit}} \cdot \rho_k^{\text{cit}} \right]
$$
  
+ 
$$
q_2 \cdot \left[ \sum_{r \in \mathcal{R}} \sum_{z \in \mathcal{Z}(r)} (\bar{\lambda}_{r,z}^{\text{urb}} - \lambda_{r,z}^{\text{urb}}) + \sum_{k \in \mathcal{K}} (\bar{\lambda}_k^{\text{in}} - \lambda_k^{\text{in}} + \bar{\lambda}_k^{\text{out}} - \lambda_k^{\text{out}}) \right]
$$
  
+ 
$$
q_3 \cdot \left[ \sum_{r \in \mathcal{R}} \sum_{z \in \mathcal{Z}(r)} \omega_{r,z}^{\text{urb}} + \sum_{k \in \mathcal{K}} (\omega_k^{\text{in}} + \omega_k^{\text{out}}) + \sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{I}(k)} \omega_{k,i}^{\text{cust}} \right] \rightarrow \min!
$$
(C.1)

## **subject to**

*Demand Assignment*

$$
\sum_{(r,k,z)\in \mathcal{C}^{\text{in}}(d)} \pi^{\text{in}}_{d,r,k,z} = 1 \quad \forall \ d \in \mathcal{D}^{\text{e2c}} \tag{C.2}
$$

$$
\sum_{(r,k,z)\in \mathcal{C}^{\text{out}}(d)} \pi_{d,r,k,z}^{\text{out}} = 1 \quad \forall \ d \in \mathcal{D}^{\text{c2e}} \tag{C.3}
$$

$$
\sum_{k \in \mathcal{K}(d)} \pi_{d,k}^{\text{cc}} = 1 \quad \forall \ d \in \mathcal{D}^{\text{c2c}} \tag{C.4}
$$

*Assignment-Selection Linkage and Vehicle Capacities*

$$
\sum_{\substack{(r,k,z)\in\mathcal{C}^{\text{in}}(r')\\ \text{with } g^{r}(z)>g'\\ \text{with } g^{r}(z)>g'\\ \text{with } g^{r}(z)>g'\\ \text{with } g^{r}(z)
$$

*Linking Selection and Scheduling*

$$
\alpha_{r,g} - M \cdot \rho_r^{\text{urb}} \le 0 \quad \forall \ g \in \mathcal{G}(r), r \in \mathcal{R}
$$
\n
$$
(C.7)
$$

$$
\beta_{k,l} - M \cdot \rho_k^{\text{cit}} \le 0 \quad \forall \ l \in \mathcal{L}(k), k \in \mathcal{K}
$$
\n(C.8)

$$
\lambda_{r,z}^{\text{urb}} - M \cdot \rho_r^{\text{urb}} \le 0 \quad \forall \ z \in \mathcal{Z}(r), r \in \mathcal{R}
$$
 (C.9)

$$
\bar{\lambda}_{r,z}^{\text{urb}} - M \cdot \rho_r^{\text{urb}} \le 0 \quad \forall \ z \in \mathcal{Z}(r), r \in \mathcal{R}
$$
\n(C.10)

$$
\lambda_k^{\text{in}} - M \cdot \rho_k^{\text{cit}} \le 0 \quad \forall \ k \in \mathcal{K} \tag{C.11}
$$

$$
\bar{\lambda}_k^{\text{in}} - M \cdot \rho_k^{\text{cit}} \le 0 \quad \forall \ k \in \mathcal{K}
$$
\n(C.12)

$$
\lambda_k^{\text{out}} - M \cdot \rho_k^{\text{cit}} \le 0 \quad \forall \ k \in \mathcal{K}
$$
\n(C.13)

$$
\bar{\lambda}_k^{\text{out}} - M \cdot \rho_k^{\text{cit}} \le 0 \quad \forall \ k \in \mathcal{K}
$$
\n(C.14)

*Interval Widths for Handovers, Pickups and Deliveries*

$$
\gamma_{r,k,z}^{\text{sat}} + \sum_{d \in \mathcal{D}^{\text{e2c}}(k)} h_d \cdot \pi_{d,r,k,z}^{\text{in}} + \sum_{d \in \mathcal{D}^{\text{c2e}}(k)} h_d \cdot \pi_{d,r,k,z}^{\text{out}} = \bar{\gamma}_{r,k,z}^{\text{sat}} \quad \forall (r,k,z) \in \mathcal{C}
$$
\n(C.15)

$$
\gamma_{d,k}^{\text{pick}} + h_d \cdot \sum_{(r',k',z') \in \mathcal{C}^{\text{out}}(d,k)} \pi_{d,r',k',z'}^{\text{out}} = \bar{\gamma}_{d,k}^{\text{pick}} \quad \forall \ k \in \mathcal{K}(d), d \in \mathcal{D}^{\text{c2e}} \tag{C.16}
$$

$$
\gamma_{d,k}^{\text{deli}} + h_d \cdot \sum_{(r',k',z') \in \mathcal{C}^{\text{in}}(d,k)} \pi_{d,r',k',z'}^{\text{in}} = \bar{\gamma}_{d,k}^{\text{deli}} \quad \forall \ k \in \mathcal{K}(d), d \in \mathcal{D}^{\text{e2c}} \tag{C.17}
$$

$$
\gamma_{d,k}^{\text{pick}} + h_d \cdot \pi_{d,k}^{\text{cc}} = \bar{\gamma}_{d,k}^{\text{pick}} \quad \forall \ k \in \mathcal{K}(d), d \in \mathcal{D}^{\text{c2c}} \tag{C.18}
$$

$$
\gamma_{d,k}^{\text{deli}} + h_d \cdot \pi_{d,k}^{\text{cc}} = \bar{\gamma}_{d,k}^{\text{deli}} \quad \forall \ k \in \mathcal{K}(d), d \in \mathcal{D}^{\text{c2c}} \tag{C.19}
$$

*Customer Time Windows*

$$
b_d^{\text{pick}} \le \gamma_{d,k}^{\text{pick}} + M \cdot \left(1 - \pi_{d,r,k,z}^{\text{out}}\right) \quad \forall \ (r,k,z) \in \mathcal{C}^{\text{out}}(d), d \in \mathcal{D}^{\text{c2e}} \tag{C.20}
$$

$$
\gamma_{d,k}^{\text{pick}} \le \bar{b}_d^{\text{pick}} + M \cdot \left(1 - \pi_{d,r,k,z}^{\text{out}}\right) \quad \forall \ (r,k,z) \in \mathcal{C}^{\text{out}}(d), d \in \mathcal{D}^{\text{c2e}} \tag{C.21}
$$

$$
b_d^{\text{deli}} \le \gamma_{d,k}^{\text{deli}} + M \cdot \left(1 - \pi_{d,r,k,z}^{\text{in}}\right) \quad \forall \ (r,k,z) \in \mathcal{C}^{\text{in}}(d), d \in \mathcal{D}^{\text{e2c}} \tag{C.22}
$$

$$
\gamma_{d,k}^{\text{deli}} \le \bar{b}_d^{\text{deli}} + M \cdot \left(1 - \pi_{d,r,k,z}^{\text{in}}\right) \quad \forall \ (r,k,z) \in \mathcal{C}^{\text{in}}(d), d \in \mathcal{D}^{\text{e2c}} \tag{C.23}
$$
\n
$$
L^{\text{pick}} \le L^{\text{pick}} \cdot M \cdot (1 - \pi_{d,r,k,z}^{\text{cc}}) \quad \forall \ k \in \mathcal{K}(d), d \in \mathcal{D}^{\text{c2c}} \tag{C.24}
$$

$$
b_d^{\text{pick}} \le \gamma_{d,k}^{\text{pick}} + M \cdot \left(1 - \pi_{d,k}^{\text{cc}}\right) \qquad \forall \ k \in \mathcal{K}(d), d \in \mathcal{D}^{\text{c2c}} \tag{C.24}
$$

$$
\gamma_{d,k}^{\text{pick}} \le \bar{b}_d^{\text{pick}} + M \cdot (1 - \pi_{d,k}^{\text{cc}}) \qquad \forall \ k \in \mathcal{K}(d), d \in \mathcal{D}^{\text{c2c}} \tag{C.25}
$$

$$
b_d^{\text{deli}} \le \gamma_{d,k}^{\text{deli}} + M \cdot (1 - \pi_{d,k}^{\text{cc}}) \qquad \forall \ k \in \mathcal{K}(d), d \in \mathcal{D}^{\text{c2c}} \tag{C.26}
$$

$$
\gamma_{d,k}^{\text{deli}} \le \bar{b}_d^{\text{deli}} + M \cdot (1 - \pi_{d,k}^{\text{cc}}) \qquad \forall \ k \in \mathcal{K}(d), d \in \mathcal{D}^{\text{c2c}} \tag{C.27}
$$

*Demand Availability Times*

$$
\tilde{a}_d \le \alpha_{r,0} + M \cdot \left(1 - \pi^{\text{in}}_{d,r',k,z}\right) \quad \forall \ (r,k,z) \in \mathcal{C}^{\text{in}}(d), d \in \mathcal{D}^{\text{e2c}} \tag{C.28}
$$

*Services: Scheduling and Waiting Times*

$$
\alpha_{r,0} \ge 0 \qquad \forall \ r \in \mathcal{R} \tag{C.29}
$$

$$
\alpha_{r,1} + M \cdot (1 - \rho_r^{\text{urb}}) \ge a_r \qquad \forall \ r \in \mathcal{R} \qquad (C.30)
$$

$$
\alpha_{r,1} \le \bar{a}_r \qquad \forall \ r \in \mathcal{R} \tag{C.31}
$$

$$
\alpha_{r,g} + tt_{r,g}^{\text{urb}} \cdot \rho_r^{\text{urb}} = \alpha_{r,g+1} \qquad \forall \ g \in \mathcal{G}^{\text{travel}}(r), r \in \mathcal{R} \tag{C.32}
$$
\n
$$
\alpha_{r,g} \leq \lambda^{\text{urb}} \qquad \forall \ s \in \mathcal{Z}(r) \ r \in \mathcal{R} \tag{C.33}
$$

$$
\alpha_{r,g^r(z)} \leq \lambda_{r,z}^{\text{urb}} \qquad \forall z \in \mathcal{Z}(r), r \in \mathcal{R} \tag{C.33}
$$
  

$$
\overline{\lambda}_{r,z}^{\text{urb}} \leq \alpha_{r,g^r(z)+1} \quad \forall z \in \mathcal{Z}(r), r \in \mathcal{R} \tag{C.34}
$$

$$
\lambda_{r,z}^{\text{urb}} \le \gamma_{r,k,z}^{\text{sat}} \qquad \forall (r,k,z) \in \mathcal{C}
$$
\n(C.35)

$$
\bar{\gamma}_{r,k,z}^{\text{sat}} \le \bar{\lambda}_{r,z}^{\text{sub}} \qquad \forall (r,k,z) \in \mathcal{C}
$$
\n(C.36)

$$
\lambda_{r,z}^{\text{urb}} \le \bar{\lambda}_{r,z}^{\text{urb}} \qquad \forall z \in \mathcal{Z}(r), r \in \mathcal{R} \tag{C.37}
$$

$$
\alpha_{r,g'} = \alpha_{r,g'+1} \qquad \forall \ g' \in \mathcal{G}^{\text{hand}}(r) : \exists \ e \in \mathcal{E}
$$

with 
$$
g^r(e) = g', r \in \mathcal{R}
$$
 (C.38)

$$
\alpha_{r,g^r(z)+1} - \alpha_{r,g^r(z)} - \left(\bar{\lambda}_{r,z}^{\text{urb}} - \lambda_{r,z}^{\text{urb}}\right) = \hat{\omega}_{r,z}^{\text{urb}} \qquad \forall z \in \mathcal{Z}(r), r \in \mathcal{R}
$$
\n
$$
\alpha_{r,g^r(z)+1} - \alpha_{r,g^r(z)}
$$
\n(C.39)

$$
-\sum_{\substack{(r',k,z')\in \mathcal{C}^{\text{in}} \text{ with } d\in \mathcal{D}^{\text{e2c}}(k) \\ r'=r \text{ and } z'=z}} \sum_{\substack{(r',k,z')\in \mathcal{C}^{\text{in}} \text{ with } d\in \mathcal{D}^{\text{e2c}}(k) \\ (r',k,z')\in \mathcal{C}^{\text{out}} \text{ with } d\in \mathcal{D}^{\text{e2e}}(k)}} h_d \cdot \pi_{d,r',k,z'}^{\text{in}} = \tilde{\omega}_{r,z}^{\text{urb}} \qquad \forall z \in \mathcal{Z}(r), r \in \mathcal{R}
$$
\n(C.40)

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$$
\hat{\omega}_{r,z}^{\text{urb}} \le \omega_{r,z}^{\text{urb}} \qquad \forall \ z \in \mathcal{Z}(r), r \in \mathcal{R} \tag{C.41}
$$

$$
\tilde{\omega}_{r,z}^{\text{urb}} \le \omega_{r,z}^{\text{urb}} \qquad \forall z \in \mathcal{Z}(r), r \in \mathcal{R} \tag{C.42}
$$

$$
\omega_{r,z}^{\text{urb}} \le \omega_z^{\text{sat}} \qquad \forall \ z \in \mathcal{Z}(r), r \in \mathcal{R} \tag{C.43}
$$

*Tours: Scheduling and Waiting Times*

 $(\beta_{k,1} - \beta_{k,0}) -$ 

$$
\beta_{k,0} \ge 0 \qquad \qquad \forall \ k \in \mathcal{K} \tag{C.44}
$$

$$
\beta_{k,0} \le \lambda_k^{\text{in}} \qquad \forall \ k \in \mathcal{K} \tag{C.45}
$$

$$
\bar{\lambda}_k^{\text{in}} \le \beta_{k,1} \qquad \forall \ k \in \mathcal{K} \tag{C.46}
$$

$$
\lambda_k^{\text{in}} - M \cdot \left(1 - \pi_{d,r,k,z}^{\text{in}}\right) \le \gamma_{r,k,z}^{\text{sat}} \qquad \forall (r,k,z) \in \mathcal{C}^{\text{in}}(d), d \in \mathcal{D}^{\text{e2c}} \qquad (C.47)
$$
\n
$$
\bar{z}^{\text{sat}} - M \cdot \left(1 - \pi^{\text{in}}\right) < \bar{\lambda}^{\text{in}} \qquad \forall (r,k,z) \in \mathcal{C}^{\text{in}}(d), d \in \mathcal{D}^{\text{e2c}} \qquad (C.48)
$$

$$
\bar{\gamma}_{r,k,z}^{\text{sat}} - M \cdot \left(1 - \pi_{d,r,k,z}^{\text{in}}\right) \leq \bar{\lambda}_k^{\text{in}} \qquad \forall (r,k,z) \in \mathcal{C}^{\text{in}}(d), d \in \mathcal{D}^{\text{e2c}} \qquad \text{(C.48)}
$$

$$
\lambda_k^{\text{in}} \leq \bar{\lambda}_k^{\text{in}} \qquad \forall k \in \mathcal{K} \qquad \text{(C.49)}
$$

$$
(\beta_{k,1} - \beta_{k,0}) - (\bar{\lambda}_k^{\text{in}} - \lambda_k^{\text{in}}) = \hat{\omega}_k^{\text{in}} \qquad \forall \ k \in \mathcal{K}
$$
 (C.50)

$$
\hat{\omega}_k^{\text{in}} \le \omega_k^{\text{in}} \qquad \forall \ k \in \mathcal{K} \tag{C.51}
$$

$$
\sum_{(r,k,z)\in\mathcal{C}^{\text{in}}(k')} \sum_{d\in\mathcal{D}^{\text{e2c}}(k)} h_d \cdot \pi_{d,r,k,z}^{\text{in}} = \tilde{\omega}_k^{\text{in}} \qquad \forall \ k'\in\mathcal{K} \tag{C.52}
$$

$$
\tilde{\omega}_k^{\text{in}} \le \omega_k^{\text{in}} \qquad \forall \ k \in \mathcal{K} \tag{C.53}
$$

$$
\omega_k^{\text{in}} \le w_{O(k)}^{\text{sat}} \qquad \forall \ k \in \mathcal{K} \tag{C.54}
$$

$$
\beta_{k,m_k^{\text{leg}}} \le \lambda_k^{\text{out}} \qquad \forall \ k \in \mathcal{K} \tag{C.55}
$$

$$
\bar{\lambda}_k^{\text{out}} \le \beta_{k,m_k^{\text{leg}}+1} \qquad \forall \ k \in \mathcal{K} \tag{C.56}
$$

$$
\lambda_k^{\text{out}} - M \cdot \left(1 - \pi_{d,r,k,z}^{\text{out}}\right) \le \gamma_{r,k,z}^{\text{sat}} \qquad \forall (r,k,z) \in \mathcal{C}^{\text{out}}(d), d \in \mathcal{D}^{\text{c2e}} \qquad \text{(C.57)}
$$
  

$$
\bar{\gamma}_{r,k,z}^{\text{sat}} - M \cdot \left(1 - \pi_{d,r,k,z}^{\text{out}}\right) \le \bar{\lambda}_k^{\text{out}} \qquad \forall (r,k,z) \in \mathcal{C}^{\text{out}}(d), d \in \mathcal{D}^{\text{c2e}} \qquad \text{(C.58)}
$$

$$
\lambda_k^{\text{out}} \le \bar{\lambda}_k^{\text{out}} \qquad \forall \ k \in \mathcal{K} \tag{C.59}
$$

$$
(\beta_{k,m_k^{\text{leg}}+1} - \beta_{k,m_k^{\text{leg}}}) - (\bar{\lambda}_k^{\text{out}} - \lambda_k^{\text{out}}) = \hat{\omega}_k^{\text{out}} \qquad \forall \ k \in \mathcal{K}
$$
 (C.60)

$$
\hat{\omega}_k^{\text{out}} \le \omega_k^{\text{out}} \qquad \forall \ k \in \mathcal{K} \tag{C.61}
$$

$$
\left(\beta_{k,m_k^{\text{leg}}+1} - \beta_{k,m_k^{\text{leg}}}\right) - \sum_{k=1}^{\infty} \sum_{k=1}^{\infty} h_k \cdot \pi_{d,r,k',z}^{\text{out}} = \tilde{\omega}_k^{\text{out}} \qquad \forall \ k \in \mathcal{K}
$$
\n(C.62)

$$
\begin{array}{c}\n d \in \mathcal{D}^{\text{c2e}}(k) \ (r, k', z) \in \mathcal{C}^{\text{out}} \\
 \text{with } k' = k\n \end{array}
$$

$$
\tilde{\omega}_k^{\text{out}} \le \omega_k^{\text{out}} \qquad \forall \ k \in \mathcal{K} \tag{C.63}
$$

$$
\omega_k^{\text{out}} \le w_{D(k)}^{\text{sat}} \qquad \forall \ k \in \mathcal{K} \tag{C.64}
$$

$$
\beta_{k,l} + t t_{k,l}^{\text{cit}} \cdot \rho_k^{\text{cit}} = \beta_{k,l+1} \qquad \forall l \in \mathcal{L}^{\text{travel}}(k), k \in \mathcal{K} \tag{C.65}
$$

$$
\beta_{k,l^k(O(d))} \le \gamma_{d,k}^{\text{pick}} \qquad \forall \ k \in \mathcal{K}(d), d \in \mathcal{D}^{\text{c2e}} \cup \mathcal{D}^{\text{c2c}} \qquad (C.66)
$$

$$
\bar{\gamma}_{d,k}^{\text{pick}} \le \beta_{k,l^k(O(d))+1} \ \forall \ k \in \mathcal{K}(d), d \in \mathcal{D}^{\text{c2e}} \cup \mathcal{D}^{\text{c2c}} \tag{C.67}
$$

$$
\beta_{k,l^k(D(d))} \le \gamma_{d,k}^{\text{deli}} \qquad \forall \ k \in \mathcal{K}(d), d \in \mathcal{D}^{\text{e2c}} \cup \mathcal{D}^{\text{c2c}} \qquad \text{(C.68)}
$$

$$
\bar{\gamma}_{d,k}^{\text{deli}} \leq \beta_{k,l^k(D(d))+1} \ \forall \ k \in \mathcal{K}(d), d \in \mathcal{D}^{\text{e2c}} \cup \mathcal{D}^{\text{c2c}} \qquad \text{(C.69)}
$$
\n
$$
-\sum_{d \in \mathcal{D}^{\text{e2c}}(k)} \sum_{(r',k',z')} h_d \cdot \pi_{d,r',k',z'}^{\text{in}} h_d \cdot \pi_{d,r',k',z'}^{\text{in}} \n-\sum_{d \in \mathcal{D}^{\text{c2e}}(k)} \sum_{(r',k',z')} h_d \cdot \pi_{d,r',k',z'}^{\text{out}} \n-\sum_{d \in \mathcal{D}^{\text{c2e}}(k)} \sum_{(r',k',z')} h_d \cdot \pi_{d,r',k',z'}^{\text{out}} \n-\sum_{d \in \mathcal{D}^{\text{c2e}}(k)} h_d \cdot \pi_{d,k}^{\text{c}} = \omega_{k,i}^{\text{cust}} \qquad \forall \ i \in \mathcal{I}(k), k \in \mathcal{K} \qquad \text{(C.70)}
$$
\n
$$
\alpha_{d \in \mathcal{D}^{\text{c2c}}(k) \text{ with}} \qquad \text{(O(d)=i)\lor(D(d)=i)} \qquad \text{cust} \qquad \text{cust} \qquad \text{(C.71)}
$$

$$
\omega_{k,i}^{\text{cuts}} \le w_i^{\text{cuts}} \qquad \forall \ i \in \mathcal{I}(k), k \in \mathcal{K} \tag{C.71}
$$

*Satellite Capacities*

$$
\sum_{d \in \mathcal{D}^{\text{e2c}}(k)} \sum_{\substack{(r',k',z') \in \mathcal{C}^{\text{in}}(k) \\ \text{with } z' = O(k)}} \pi^{\text{in}}_{d,r',k',z'}
$$
\n
$$
- \left| \mathcal{D}^{\text{e2c}}(k) \right| \cdot \sum_{u \in \mathcal{U}^{\text{cit}}_{O(k)}} \phi^{\text{cit}}_{k,0,O(k),u} \leq 0 \quad \forall k \in \mathcal{K}
$$
\n
$$
\sum_{d \in \mathcal{D}^{\text{c2e}}(k)} \sum_{\substack{(r',k',z') \in \mathcal{C}^{\text{out}}(k) \\ \text{with } z' = D(k)}} \pi^{\text{out}}_{d,r',k',z'}
$$
\n
$$
(C.72)
$$

$$
- | \mathcal{D}^{\text{c2e}}(k) | \cdot \sum_{u \in \mathcal{U}_{D(k)}^{\text{cit}}} \phi_{k,m_k^{\text{leg}},D(k),u}^{\text{cit}} \le 0 \quad \forall \ k \in \mathcal{K}
$$
 (C.73)

$$
\sum_{u \in \mathcal{U}_z^{\text{cit}}} \phi_{k,l,z,u}^{\text{cit}} \le 1 \quad \forall \ l \in \mathcal{L}^{\text{sat}}(k,z), z \in \mathcal{Z}(k), k \in \mathcal{K}
$$

$$
(\mathrm{C.74})
$$

$$
\sum_{\substack{k \in \mathcal{K} \\ w \text{with} \\ z \in \mathcal{Z}(k)}} \sum_{l \in \mathcal{L}^{\text{sat}}(k,z)} \phi_{k,l,z,u}^{\text{cit}} - \sum_{\substack{k \in \mathcal{K} \\ w \text{with} \\ z \in \mathcal{Z}(k)}} \sum_{l \in \mathcal{L}^{\text{sat}}(k,z)} \phi_{k,l,z,u+1}^{\text{cit}} \ge 0 \quad \forall u \in \mathcal{U}_z^{\text{cit}} \setminus \{N_z^{\text{cit}}\}, z \in \mathcal{Z}
$$
 (C.75)

$$
\phi_{k,l,z,u}^{\text{cit}} + \phi_{k',l',z,u}^{\text{cit}} - \chi_{k,l,k',l'}^{\text{cit}} - \chi_{k',l',k,l}^{\text{cit}} \leq 1 \quad \forall (k,l,k',l') \in \mathcal{S}_z^{\text{cit}}, u \in \mathcal{U}_z^{\text{cit}}, z \in \mathcal{Z}
$$
\n(C.76)

$$
\chi_{k,l,k',l'}^{\text{cit}} + \chi_{k',l',k,l}^{\text{cit}} \leq 1 \quad \forall \ (k,l,k',l') \in \bigcup_{z \in \mathcal{Z}} \mathcal{S}_z^{\text{cit}} \tag{C.77}
$$

$$
\beta_{k',l'+1} - M \cdot \chi_{k,l,k',l'}^{\text{cit}} - \beta_{k,l} \leq 0 \quad \forall (k,l,k',l') \in \bigcup_{z \in \mathcal{Z}} \mathcal{S}_z^{\text{cit}} \tag{C.78}
$$

$$
\sum_{\substack{(r,k,z)\in\mathcal{C}^{\text{inc}}(r',z')\\ \mathcal{C}^{\text{inc}}(r',z')}} \sum_{d\in\mathcal{D}^{\text{enc}}(k)} \pi^{\text{in}}_{d,r,k,z} + \sum_{\substack{(r,k,z)\in\mathcal{C}^{\text{inc}}(r',z')\\ \mathcal{C}^{\text{out}}(r',z')}} \sum_{d\in\mathcal{D}^{\text{c2e}}(k)} \pi^{\text{out}}_{d,r,k,z} - \mid \mathcal{D}(r') \mid \sum_{v\in\mathcal{U}^{\text{urb}}_{z'}} \phi^{\text{urb}}_{r',z',v} \leq 0 \quad \forall z' \in \mathcal{Z}(r'), r' \in \mathcal{R}
$$
\n(C.79)

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$$
\sum_{v \in \mathcal{U}_z^{\text{urb}}} \phi_{r,z,v}^{\text{urb}} \le 1 \quad \forall \ z \in \mathcal{Z}(r), r \in \mathcal{R}
$$
 (C.80)

$$
\sum_{r \in \mathcal{R} \text{ with } z \in \mathcal{Z}(r)} \phi_{r,z,v}^{\text{urb}} - \sum_{r \in \mathcal{R} \text{ with } z \in \mathcal{Z}(r)} \phi_{r,z,v+1}^{\text{urb}} \geq 0 \quad \forall \ v \in \mathcal{U}_z^{\text{urb}} \setminus \{N_z^{\text{urb}}\}, z \in \mathcal{Z} \qquad \text{(C.81)}
$$

$$
\phi_{r,z,v}^{\text{urb}} + \phi_{r',z,v}^{\text{urb}} - \chi_{r,r',z}^{\text{urb}} - \chi_{r',r,z}^{\text{urb}} \leq 1 \quad \forall (r,r') \in \mathcal{S}_z^{\text{urb}}, v \in \mathcal{U}_z^{\text{urb}}, z \in \mathcal{Z}
$$
\n(C.82)

$$
\chi_{r,r',z}^{\text{urb}} + \chi_{r',r,z}^{\text{urb}} \leq 1 \quad \forall (r,r') \in \mathcal{S}_z^{\text{urb}}, z \in \mathcal{Z}
$$
 (C.83)

$$
\alpha_{r',g^{r'}(z)+1} - M \cdot \chi_{r,r',z}^{\text{urb}} - \alpha_{r,g^{r}(z)} \leq 0 \quad \forall (r,r') \in \mathcal{S}_z^{\text{urb}}, z \in \mathcal{Z}
$$
 (C.84)

*Domains*

$$
\rho_k^{\text{cit}} \in \{0, 1\} \qquad \forall \ k \in \mathcal{K} \tag{C.85}
$$

$$
\rho_r^{\text{urb}} \in \{0, 1\} \qquad \forall \ r \in \mathcal{R} \tag{C.86}
$$

$$
\pi_{d,k}^{\text{cc}} \in \{0,1\} \qquad \forall \ k \in \mathcal{K}(d), d \in \mathcal{D}^{\text{c2c}} \tag{C.87}
$$

$$
\pi_{d,r,k,z}^{\text{in}} \in \{0,1\} \quad \forall \ d \in \mathcal{D}^{\text{e2c}}(k), (r,k,z) \in \mathcal{C}^{\text{in}} \tag{C.88}
$$

$$
\pi_{d,r,k,z}^{\text{out}} \in \{0,1\} \quad \forall \ d \in \mathcal{D}^{\text{c2e}}(k), (r,k,z) \in \mathcal{C}^{\text{out}} \tag{C.89}
$$

$$
\phi_{r,z,v}^{\text{urb}} \in \{0, 1\} \qquad \forall \ v \in \mathcal{U}_z^{\text{urb}}, z \in \mathcal{Z}(r), r \in \mathcal{R}
$$
\n(C.90)

$$
\phi_{k,l,z,u}^{\text{cit}} \in \{0,1\} \quad \forall \ u \in \mathcal{U}_z^{\text{cit}}, l \in \mathcal{L}^{\text{sat}}(k,z), z \in \mathcal{Z}(k), k \in \mathcal{K}
$$
\n(C.91)

$$
\chi_{r,r',z}^{\text{urb}} \in \{0,1\} \quad \forall (r,r') \in \mathcal{S}_z^{\text{urb}}, z \in \mathcal{Z}
$$
\n(C.92)

$$
\chi_{k,l,k',l'}^{\text{cit}} \in \{0,1\} \quad \forall \ (k,l,k',l') \in \mathcal{S}_z^{\text{cit}}, z \in \mathcal{Z}
$$
\n(C.93)

## **D Relaxed Mathematical Model**

$$
q_1 \cdot \left[ \sum_{r \in \mathcal{R}} c_r^{\text{urb}} \cdot \rho_r^{\text{urb}} + \sum_{k \in \mathcal{K}} c_k^{\text{cit}} \cdot \rho_k^{\text{cit}} \right] + q_3 \cdot \left[ \sum_{r \in \mathcal{R}} \sum_{z \in \mathcal{Z}(r)} \omega_{r,z}^{\text{urb}} + \sum_{k \in \mathcal{K}} (\omega_k^{\text{in}} + \omega_k^{\text{out}}) + \sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{I}(k)} \omega_{k,i}^{\text{cust}} \right] \rightarrow \min! \tag{D.1}
$$

#### **subject to**

*Demand Assignment:* (C.2) to (C.4)

*Assignment-Selection Linkage and Vehicle Capacities* (C.5) and (C.6)

*Linking Selection and Scheduling:* (C.7) and (C.8)

*Interval Widths for Handovers, Pickups and Deliveries:* (C.15) to (C.19)

*Customer Time Windows:* (C.20) to (C.27)

*Demand Availability Times:* (C.28)

*Services: Scheduling and Waiting Times*

(C.29) to (C.32)

$$
\alpha_{r,g^r(z)} \le \gamma_{r,k,z}^{\text{sat}} \qquad \forall (r,k,z) \in \mathcal{C}
$$
\n(D.2)

$$
\bar{\gamma}_{r,k,z}^{\text{sat}} \le \alpha_{r,g^r(z)+1} \quad \forall \ (r,k,z) \in \mathcal{C}
$$
\n(D.3)

$$
\alpha_{r,g} \le \alpha_{r,g+1} \qquad \forall \ g \in \mathcal{G}(r), r \in \mathcal{R} \tag{D.4}
$$

(C.38), (C.40) and (C.42)

*Tours: Scheduling and Waiting Times*

(C.44)

$$
\beta_{k,0} \le \beta_{k,1} \qquad \forall \ k \in \mathcal{K} \tag{D.5}
$$

$$
\beta_{k,0} - M \cdot \left(1 - \pi_{d,r,k,z}^{\text{in}}\right) \le \gamma_{r,k,z}^{\text{sat}} \quad \forall \ (r,k,z) \in \mathcal{C}^{\text{in}}(d), d \in \mathcal{D}^{\text{e2c}} \tag{D.6}
$$

$$
\bar{\gamma}_{r,k,z}^{\text{sat}} - M \cdot \left(1 - \pi_{d,r,k,z}^{\text{in}}\right) \leq \beta_{k,1} \qquad \forall \ (r,k,z) \in \mathcal{C}^{\text{in}}(d), d \in \mathcal{D}^{\text{e2c}} \tag{D.7}
$$

(C.53) and (C.54)

$$
\beta_{k,m_k^{\text{leg}}} \le \beta_{k,m_k^{\text{leg}}+1} \quad \forall \ k \in \mathcal{K}
$$
\n(D.8)

$$
\beta_{k,m_k^{\text{leg}}} - M \cdot \left(1 - \pi_{d,r,k,z}^{\text{out}}\right) \le \gamma_{r,k,z}^{\text{sat}} \qquad \forall \ (r,k,z) \in \mathcal{C}^{\text{out}}(d), d \in \mathcal{D}^{\text{c2e}} \tag{D.9}
$$

$$
\bar{\gamma}_{r,k,z}^{\text{sat}} - M \cdot \left(1 - \pi_{d,r,k,z}^{\text{out}}\right) \leq \beta_{k,m_k^{\text{leg}}+1} \quad \forall \ (r,k,z) \in \mathcal{C}^{\text{out}}(d), d \in \mathcal{D}^{\text{c2e}} \tag{D.10}
$$

(C.63) to (C.71)

*Domains:* (C.85) to (C.89)

# **E Details on the Instances**



Demand Size	ХS	S	M		X <sub>L</sub>
Frequency of Appearance	(0.1)	0.2	0.4	02	0 <sup>1</sup>
Handover Duration (min)	$\mathfrak{h}$	$\mathfrak{h}$			10
Volume in Cargo Bike	0.02	0.03	0.05	0.1	0.125
Volume in Truck	0.001	Ი ᲘᲘ25	0.004	0.008	0.01
Volume in Tram		0.00125	0.002	(1.004)	0.005

Table 15: Demand Sizes and Their Characteristics

## **F Extended Results**

This section contains different results and evaluations that complement and support the conclusions drawn in this work. Instances are differentiated by the number of demands  $n$ , the number of satellites Z, the demand type, the TW widths and the CWD distributions, and the planning time horizon. Instance parameters which are not used for classification do not show significant effects on the considered measures.

At first, extended results obtained by applying a general MIP solver as the Standard Method with a computation time limit of 60 minutes are reported. Table 16 summarizes the averaged MIP gap values for the rail-based instances. Table 17 shows the averaged percentage differences in the objective function values for instances solved to optimality with  $n = 10$  when increasing the planning time horizon from 2 h to 4 h. Figure 15 plots the averaged total fixed cost of services and tours selected for rail-based instances with  $n = 10$ . For all instances with 10 demands and all three demands types involved, Figure 16 shows the averaged number of services used, while Figure 18 plots the number of satellites used. Figure 17 summarizes the averaged total waiting durations for the rail-based instances with  $n = 10$  and the fully integrated demand type setting.

Demand	TW	Time Horizon: 2 h			Time Horizon: 4 h			
<b>Types</b>	Widths	$n=10$	$n=20$	$n=30$	$n=10$	$n=20$	$n=30$	
	long	$0.0\%$	31.9%	51.6%	$0.7\%$	35.1%	58.6%	
(0.65, 0.2, 0.15)	medium	$0.0\%$	17.4%	38.0%	$0.0\%$	25.8%	46.1\%	
	short	$0.0\%$	11.7%	30.9%	$0.0\%$	24.4\%	44.6%	
	mix	$0.0\%$	25.3%	38.0%	$0.0\%$	36.7%	47.0%	
	long	$4.0\%$	36.9%	61.2%	11.7%	41.0%	61.4%	
	medium	$0.0\%$	28.3%	44.8%	$1.4\%$	48.2%	51.6%	
(0.8, 0.2, 0)	short	$0.0\%$	22.9%	38.1%	$0.0\%$	48.9%	50.1%	
	mix	$0.8\%$	30.6%	46.9%	$2.5\%$	47.9%	57.5%	
	long	$2.9\%$	36.4%	59.0%	7.9%	38.7%	60.2%	
	medium	$0.0\%$	33.2%	46.0%	$0.2\%$	44.9%	55.7%	
(1, 0, 0)	short	$0.0\%$	27.2%	40.1%	$0.5\%$	46.7%	55.7%	
	mix	0.4%	34.6%	48.9%	2.5%	45.0%	57.9%	

Table 16: Averaged MIP Gap Values for Standard Method on Rail-Based Instances  $(Z = 4)$ 

	Dem. Types			(0.65, 0.2, 0.15)				(0.8, 0.2, 0)				(1, 0, 0)	
	<b>CWDs</b>	long	short	$\mathbf{n}\mathbf{o}$	mix	long	short	no	mix	long	short	no	mix
Ζ	TWs												
$\overline{2}$	long	$-0.96$	$-0.96$	$-0.96$	$-0.96$	$-0.89$	$-0.89$	$-0.89$	$-0.89$	1.96	2.26	2.26	2.08
	med	9.15	18.48	25.11	16.47	9.67	18.37	23.13	15.55	9.92	13.73	20.29	13.85
	short	$-0.56$	11.04	15.00	7.27	$-0.04$	7.80	11.17	4.91	1.44	5.76	10.08	4.05
	mix	0.88	4.96	8.19	3.83	2.08	4.49	6.53	2.64	2.98	4.28	6.25	4.94
$\overline{4}$	long	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	med	3.22	15.67	23.00	11.52	4.22	10.29	15.33	5.41	5.89	13.44	19.22	7.60
	short	$-2.78$	1.89	10.22	3.04	$-0.22$	5.67	7.44	1.59	0.13	6.13	10.56	3.46
	mix	$-1.04$	2.44	8.81	1.56	0.76	3.41	4.20	1.29	2.05	3.21	1.08	2.37
5	long	0.22	0.22	0.22	0.22	$\mathbf{L}^*$	$\overline{\phantom{a}}^*$	$\overline{\phantom{a}}^*$	$-2.00$	$-2.78$	$-2.33$	$-2.56$	$-2.45$
	med	5.59	11.26	12.89	9.84	8.15	10.50	11.15	9.63	4.81	6.96	7.74	4.86
	short	0.48	6.96	4.19	3.40	0.78	3.22	3.70	2.11	1.44	2.63	2.48	1.81
	mix	0.02	1.88	1.90	1.06	1.87	3.29	3.83	2.28	2.49	2.44	3.29	2.56
10	long	$-3.00$	$-3.00$	$-3.00$	$-3.00$	$-0.67$	$-0.67$	$-0.75$	$-0.67$	$-0.22$	$-0.22$	$-0.22$	$-0.22$
	med.	3.78	11.11	14.89	8.11	4.00	6.89	5.89	4.04	2.56	3.00	4.89	2.74
	short	0.56	8.11	1.00	2.26	$1.56\,$	2.33	1.56	1.56	0.11	2.11	0.56	0.56
	mix	$-0.41$	0.78	1.26	0.11	0.00	0.96	0.85	0.64	2.96	3.00	3.19	2.79

Table 17: Averaged Percentage Differences in Objective Function Value (in %) of Instances with 2 h and 4 h Time Horizon for Instances solved to Optimality with  $n = 10$ 

<sup>∗</sup> Not evaluated, since not solved to optimality.



Figure 15: Averaged Total Fixed Cost of Services and Tours Selected for Rail-Based Instances with  $n=10\,$ 



Figure 16: Averaged Number of Services Used for Instances with  $n = 10$  and Demand Type Distr. (0.65, 0.2, 0.15)



Figure 17: Averaged Total Waiting Duration for Rail-Based Instances with  $n = 10$  and Demand Type Distr. (0.65, 0.2, 0.15)

![](_page_57_Figure_1.jpeg)

Figure 18: Averaged Number of Satellites for Instances with  $n = 10$  and Demand Type Distr. (0.65, 0.2, 0.15)

The following figures depict the comparative evaluation of the facilitation methods and the Standard method on the rail-based instances with 2 h time horizon. The methods yield similar results for instances with a time horizon of 4 h. Figure 19 shows the differences in the MIP gap values obtained by the facilitating methods compared to the Standard method within 1 hour of computation time and Figure 20 plots the percentage differences in the lower bounds and the upper bounds.

![](_page_57_Figure_4.jpeg)

Figure 19: Gap Difference between Facilitating Methods and Standard Method for Rail-Based Instances with 2 h Time Horizon

![](_page_58_Figure_1.jpeg)

Figure 20: Percentage Difference of UB and LB between Facilitating Methods and Standard Method for Rail-Based Instances  $(Z = 4)$  with 2 h Time Horizon