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The Two-Echelon Commodity-Split Multi-Compartment Capacitated Arc Routing Problem

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Abstract. The shift from traditional waste treatment methods to recycling has introduced complexities in the collection and transportation of waste, and in the design of waste management networks. To maximize routing efficiency and minimize logistical costs, it is beneficial to introduce intermediate dumping sites with multi-waste stream skips between the collection area and the far away treatment plants. To that end, we introduce the Two-Echelon Commodity-Split Multi-Compartment Capacitated Arc Routing Problem. In the first-echelon, multi-compartment vehicles collect different waste streams and then unload their compartments in skips at one or more dumping sites. In the second-echelon, skips are transported by tractors to their relative treatment plants and brought back. The aim of the problem is to minimize the first- and second-echelon routing costs and the cost of locating skips at dumping sites. We introduce a mathematical formulation for the problem and a two-phase matheuristic solution approach. The first phase, the vehicle mix selection phase, selects a subset of cost-attractive vehicle assignments, which is given as input to the routing and skip assignment phase. In the second phase, a novel 2-echelon multi-commodity location-routing tour splitting algorithm is presented. The proposed solution is tested on 60 real-life instances from five Danish regions. Our comprehensive computational experiments and analyses show how the proposed matheuristic can yield high-quality solutions efficiently. Finally, several managerial insights are provided regarding the number and location of dumping sites, skip usage with different degrees of sorting, as well as the interaction between the routing cost of each echelon.

Keywords: Arc routing, waste management, multi-compartment, matheuristic, two-echelon routing.

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1. Introduction

Over the past two decades, environmental policies relating to waste management have shifted from more polluting waste treatment solutions, such as landfills and incinerators, towards circular treatment options, such as recycling waste into various streams to repurpose them as new materials (European Comission, 2008). Recycling waste into different streams complicates waste management systems. It requires separating waste into streams at a specific point in the system, collecting the streams either individually or collectively, and transporting each stream to its respective treatment plant.

Treatment plants are usually built on a regional or provincial level, while waste collection takes place at a municipal level, resulting in the treatment plants being far away from the waste collection area. In a non-recycling setting, it is typical to collect the single waste stream using collection vehicles that dump the waste at a central depot, where larger trucks would transport the waste to the landfill/incinerator. This is no longer viable in a recycling setting, due to the nature of the different waste streams (e.g., organic waste is wet). Proper storage of each stream at the depot is therefore required, which comes in the form of large open- or closed-faced rectangular containers (called skips), which are then assigned to the different waste streams. Logistically, skip transportation is different than the transportation of general waste to landfills or incinerators, and is usually done by trucks called tractors that can move one or two skips at a time.

With the existence of a multitude of treatment plants, the situation could arise where the central depot is located too far from certain plants, leading to long travel distances for the skip transportation tractors from the depot to each plant. To address operational inefficiency, the central depot can be replaced by several intermediate dumping sites that serve as consolidation hubs and house skips for different streams. The locations of the dumping sites are chosen to be advantageously spread-out, resulting in a minimal travel distance to the plants, while still being close enough to the waste collection area. This streamlines the unloading operations at treatment plants, optimizes the routing of waste collection and skip transport, and significantly reduces operational costs.

Given a curbside recyclable collection service based on source separation of the waste and co-collection with multi-compartment vehicles - which has been proven to be the best collection policy for multi-stream waste (Zbib and Laporte, 2020) - once the capacity of a compartment is reached, each collection vehicle would then travel to one or more of the intermediate dumping sites to empty their compartments in a skip assigned to the same waste streams, before returning to the vehicle depot. Once all collection operations are done, the skips are transported by tractors to their respective treatment plants, and delivered back to their dumping site.

In this paper, we study the above-described setting, which we coin as the Two-Echelon Commodity-Split Multi-Compartment Capacitated Arc Routing Problem (2E-CSMC-CARP). The objective of the 2E-CSMC-CARP is to minimize the total operational cost, given by the first-echelon waste collection costs, the second-echelon skip transportation costs, and the cost of locating skips at the intermediate dumping sites. We propose a mathematical model for the 2E-CSMC-CARP, and, given the complexity of the problem, propose a two-phase matheuristic approach that decomposes the 2E-CSMC-CARP into smaller sub-problems that are handled sequentially. We test our solution approach on real-life waste collection instances from five Danish regions drawn from the literature (Kiilerich and Wøhlk, 2018), and compare it to solving the mathematical model using a commercial solver, before presenting some insightful managerial insights.

The remainder of this paper is organized as follows. Section 2 provides an overview of relevant literature on waste collection routing problems, Section 3 describes the problem setting, Section 4 presents the mathematical formulation, and Section 5 describes the matheuristic approach to solve it. We present the results of the computational experiments in Section 6, followed by our conclusions in Section 7.

2. Literature review

The Capacitated Arc Routing Problem (CARP) has several real-world applications, such as waste collection. Although the CARP has been extensively studied in the literature (Corberán et al., 2021), its waste collection variants have been scarcely studied (Kiilerich and Wøhlk, 2018). The 2E-CSMC-CARP is an extension of the CARP that integrates two important aspects of a recyclable waste management network: the management of multiple waste streams by multi-compartment vehicles, and the utilization of intermediate facilities. In what follows, we review the literature on multi-compartment CARPs and multi-echelon waste management networks.

2.1. Multi-compartment arc routing

The multi-compartment capacitated arc routing problem (MC-CARP) was first introduced by Muyldermans and Pang (2010) and it is mainly studied within the context of waste collection. In the MC-CARP, a required edge can have a demand for different commodities. Since these commodities must be kept separate during transportation, multi-compartment vehicles with limited capacity for each commodity are used. The objective of the MC-CARP is to find a set of minimum cost routes starting and ending at the depot while ensuring that all demands are collected without exceeding the compartment capacities of vehicles. Eydi and Javazi (2011) extend the problem to include the minimization of the number of vehicles as a second objective. The study of the MC-CARP in the context of waste management was pioneered by Kiilerich and Wøhlk (2018) where several real-world curbside waste collection variants of the CARP are introduced. They present two policies for the collection of waste by a heterogeneous fleet of multi-compartment vehicles. The first policy is represented by the No-Split Multi-Compartment CARP (NSMC-CARP), where all the bins of a household have to be collected by the same vehicle (similarly to Muyldermans and Pang (2010)). The second policy allows for different bins at a household to be collected by different vehicles, represented by the Commodity-Split Multi-Compartment CARP (CSMC-CARP).

Given the complexity of the MC-CARP, various algorithms have been developed to solve it. For the NSMC-CARP, Muyldermans and Pang (2010) presented a local search algorithm, Eydi and Javazi (2011) a multi-objective genetic algorithm, and Zbib and Wøhlk (2019) a multi-move chain descent heuristic to solve the large-scale real-life Danish instances proposed by Kiilerich and Wøhlk (2018). The work of Zbib and Laporte (2020) is the only solution approach in the literature for the CSMC-CARP. The authors developed a data-driven matheuristic that decomposes the problem into incomplete solution representations and heuristically solve one or more decisions at a time using a multi-commodity tour splitting algorithm for the routing phase.

To the best of our knowledge, all existing studies on the MC-CARP focus solely on single-echelon systems. However, waste management systems are multi-echelon in nature, including several stages such as collection, storage, and treatment. Introducing a multi-echelon framework for the MC-CARP adds to the complexity of the problem and necessitates the development of intricate algorithms to address its unique challenges.

2.2. Two-echelon waste management networks

The transition towards the circular economy has accelerated the utilization of waste as a valuable resource (Chagas et al., 2023). Therefore, modern waste management systems necessitate multi-echelon routing networks to efficiently collect, treat, and distribute materials for recycling and repurposing. The optimization of the network structure and its underlying operations are important aspects of two-echelon waste management networks that are shared with two-echelon routing problems, which have been extensively studied (see the reviews by Cuda et al. (2015) and Sluijk et al. (2023)). Decisions regarding mutli-commodity facility location (Boccia et al., 2018), synchronization mechanisms (Escobar-Vargas and Crainic, 2024), load assignments (Araújo et al., 2023), and routing strategies (Darvish et al., 2019) that are studied in the two-echelon location routing literature are also central to enhancing the efficiency and minimizing the costs of two-echelon waste management networks. Nonetheless, waste management networks also have application-specific challenges such as coordinating collection and disposal points, utilizing intermediate facilities for dumping or sorting sites, and managing different types of waste streams.

Rodrigues and Soeiro Ferreira (2015) and Hemmelmayr et al. (2013) address the importance of intermediate facilities in optimizing vehicle routing for waste collection. Other works highlight the need for efficient coordination between the echelons of the network to reduce the number of collection vehicles required and the distance travelled (Ghiani et al., 2021; Markov et al., 2016; Rahmanifar et al., 2024). Even though curbside waste collection from households is often modelled as an arc routing problem, research on two-echelon waste collection problems in a node routing setting has received more attention than its arc routing counterpart (Hemmelmayr et al., 2013; Benjamin and Beasley, 2013; Wei et al., 2019; Liu and Liao, 2021; Ghiani et al., 2021; Lavigne et al., 2023).

De Rosa et al. (2002) introduced the arc routing and scheduling problem with transshipment and discussed its application to waste collection. Waste is collected from edges requiring service by specially equipped vehicles, then, shredded or compacted, it is transferred to an intermediate facility to finally be transported to a final destination. They present a lower bound procedure for the problem which is obtained by relaxing an integer linear formulation. The problem is solved using a Tabu Search heuristic. Mourão and Almeida (2000) and Mourão and Amado (2005) study a two-echelon waste collection problem, but they also only consider one recycling center. The vehicles leave the depot to execute waste collection and bring their load to the dumping site before returning to the depot.

Notably, the decision of where to locate the intermediate facilities is not addressed in these studies. In the context of the arc routing problem with intermediate facilities, Willemse and Joubert (2016) developed four constructive heuristics to minimize either the total cost or the fleet size. Later, Willemse and Joubert (2019) improved the solution algorithm for the mixed capacitated arc routing problems under time restrictions with intermediate facilities by adapting five commonly usedlocal search operators for arc routinh. The closest study to ours is that of Wei et al. (2024), who studied a multi-level CARP for waste collection with two levels of intermediate facilities. Two types of vehicles are considered, manually operated vehicles and vehicles with compressors. The goal is to minimize the total travel cost by finding a set of routes for the two types of vehicles. An extended adaptive large neighbourhood search metaheuristic is proposed to solve the problem. In contrast, our 2E-CSMC-CARP extends beyond the scope of Wei et al. (2024) to encompass the management of multiple waste streams, co-collection by multi-compartment vehicles, multi-stream skip assignment to intermediate facilities, and distinct treatment plants for each waste stream.

3. Problem description

The Two-Echelon Commodity-Split Multi-Compartment Capacitated Arc Routing Problem (2E-CSMC-CARP) addresses a complex waste recycling management system. Due to the large distances between the collection points and treatment plants and the need to collect multiple waste streams, transporting waste directly from collection points to treatment plants is inefficient and time consuming. Therefore, a two-echelon structure is preferable in this setting. In the first echelon, multiple collection vehicles traverse different edges to collect different types of waste and unload them at intermediate dumping sites. In the second echelon, larger vehicles transport each waste stream directly from these dumping sites to their respective treatment plants.

The intermediate dumping sites are equipped with skips where collection vehicles can unload waste. These sites serve as consolidation points, from where larger vehicles can transport the waste to treatment plants in a more efficient manner. Multi-compartment vehicles with a limited capacity in each compartment are available at the depot in the first echelon. These vehicles traverse several edges with positive waste demand for the different waste streams and fill up the allocated compartment(s) for each type of waste. Once done with the collection phase, they unload the compartments into the dedicated skips for each type of waste at the intermediate dumping sites. Another set of capacitated trucks (tractors) departs from these sites to bring the skips to the treatment plants and and then returns them.

The problem aims to optimize simultaneously the decisions pertaining to both echelons. The key decisions are the design of the first-echelon waste collection routes, compartment allocation for different waste streams in each vehicle, assignment of the vehicles to intermediate dumping sites, and the number of skips for each waste stream to have in each dumping site. In the second-echelon, the focus is on determining the optimal routes for transporting the located skips from the dumping sites to the treatment plants.

The objective of the 2E-CSMC-CARP is to determine a set of least-cost routes for both echelons, as well as the number of skips for each waste stream to locate at each dumping site. In the first-echelon, waste streams are assigned to the compartments of each vehicle, and all collection routes have to start and end at the depot, such that demand of all required edges for all waste streams are collected exactly once by one of the vehicles' compartments, without violating the compressed capacities of each compartment in each vehicle. Moreover, before returning to the depot, each route needs to visit at least one dumping site containing skips corresponding to each waste stream it has collected. In the second-echelon, all skips that are located at dumping sites have to be picked up by exactly one transportation truck route and transported to exactly one treatment plant.

4. Mathematical formulation

In this section, we introduce the mathematical formulation for the 2E-CSMC-CARP. This formulation captures the six interdependent decisions of the problem: 1) selecting which first-echelon collection vehicles to use, 2) assigning waste streams to the compartments of the selected collection vehicles, 3) routing the collection vehicles, 4) selecting the intermediate dumping sites to visit by each collection route, 5) locating skips at the dumping sites, and 6) routing the second-echelon skip transportation trucks.

Our formulation employs a mixed graph of edges and arcs to efficiently integrate the two-echelon network encompassing diverse node types. For the first-echelon collection part of the graph in our model, edges are used for the demand and traversal edges, connected by linking nodes, and the traversal direction is inconsequential. This aligns with the primary CARP models where servicing requirements are tied to edges, and the routing decisions are direction-agnostic (Golden and Wong, 1981). Edges are also implemented to connect dumping sites and treatment plants. However, for the remainder of the first echelon, a directed representation is useful to ensure the routes that perform the collection and unloading are structured in a specified order, thereby, capturing the inherent directionality of these routing decisions. Routes in the first echelon must go through the collection and unloading cycle by visiting the different types of nodes in a specific order (depot $\rightarrow$ demand nodes $\rightarrow$ dumping sites $\rightarrow$ depot) and directly returning to a previous section is not allowed. The usage of arcs in these segments of the network facilitates the accurate modelling of route sequencing and directionality without requiring auxiliary flow variables and constraints, which is pivotal for efficiently modelling the coordination between the two echelons.

The 2E-CSMC-CARP is defined on a mixed graph $G = (N, E, A)$, where N is the set of nodes, E is the set of edges, A is the set of arcs, and $F(> 1)$ the set of waste streams. Let $v_0 \in N$ be the depot, $N^l \subseteq N$ the set of linking nodes, $N^D \subseteq N$ the set of intermediate dumping sites, and N_f^P the set of treatment plants assigned to each waste stream $f \in F$. The set of edges is composed by the pairs $E = \{(i,j) : i,j \in N^l\} \cup \{(i,j) : i,j \in N^D\} \cup \{(i,j) : i \in N^D, j \in N_f^P\} \cup \{(i,j) : i,j \in N_f^P\}$, while the set of arcs is defined by the connections between the different kinds of node types in the first echelon, with $A = \{(i,j) : i = v_0, j \in N^l\} \cup \{(i,j) : i \in N^l, j \in N^D\} \cup \{(i,j) : i \in N^D, j = v_0\}$. We denote $E^{'} \subseteq E$ as the set of edges in the first echelon. Figure 1 depicts a simplified version of the two-echelon network structure and the topology of the graph.

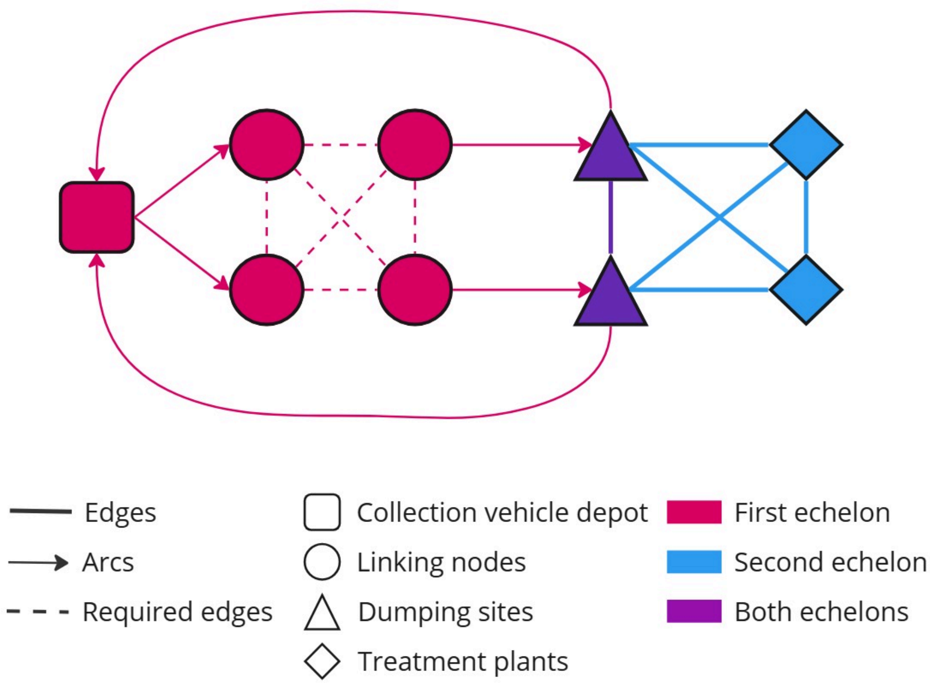

Figure 1: Illustration of the Two-Echelon Network

With every edge $(i, j) \in E$ and arc $(i, j) \in A$, there is associated a non-negative traversal cost c_{ij}. Moreover, each edge $(i, j) \in E'$ has an associated demand of waste stream $f \in F$, given by q^f_{ij}. We denote by $E^r \subseteq E'$ the set of required edges, such that every $(i, j) \in E^r$ has at least one positive demand for one waste stream, $\sum_{f \in F} q^f_{ij} > 0$. We also denote by $E^r_f \subseteq E^r$ the set of all required edges with a positive demand for waste stream $f \in F$. Assigned to each waste stream $f \in F$ is at least one node $N^P_f \subseteq N$ as a treatment plant that can process only that type of waste stream.

The vehicles in the first-echelon undertake two activities: collecting waste streams from required edges, and unloading the collected streams at the intermediate dumping sites N^D. In this echelon, a multi-compartment unlimited heterogeneous fleet of waste collection vehicles K is located at the depot, where M^k, $(|M^k| \leq |F|)$ is the set of compartments of vehicle $k \in K$. Each compartment $m \in M^k$ has an uncompressed capacity Q^{km}. With each waste stream $f \in F$ and vehicle $k \in K$ is associated a compression factor γ^{fk}, such that if the waste stream is collected by any of the compartments of the vehicle, the total demand of the waste stream in the vehicle is compressed by a factor γ^{fk}. The parameter $Q^{fkm} = \gamma^{fk} Q^{km}$ is referred to as the compressed capacity of $f \in F$ if assigned to $m \in M^k$.

A typical first-echelon route consists of the vehicle starting at the depot, traversing a subset of the required edges to collect waste, and when the capacity of one or more of its compartments is reached, visiting one or more dumping sites in order to unload the waste, before returning to the depot. The fleet of vehicles is responsible for the collection of the demand for all waste streams at all required edges.

The waste arriving at a dumping site is unloaded into skips. Each skip can contain a single waste stream $f \in F$ and has a capacity of S. All skips have the same capacity irrespective of the waste stream they contain. The vehicles in the second-echelon, which are a homogeneous fleet of skip transport trucks, are responsible for the transport of filled skips from dumping sites to the treatment plants. Each vehicle has a capacity for transporting h skips at a time. A typical second-echelon route consists of starting at a given dumping site, travelling between dumping sites and picking up h skips from a maximum of h different sites, visiting a maximum of h treatment plants to unload the content of each skip for subsequent treatment, before returning each empty skip to the dumping site it was picked up from, and returning to the original dumping site. We explicitly enumerate all the possible second-echelon routes, which define the set R of second-echelon routes. This is because the number of dumping sites, treatment plants, and the capacity of trucks is small enough to allow us to enumerate all possible routes and formulate the second-echelon using a set partitioning formulation.

A route $r \in R$ is defined as $r = (d_1^{f_1}, \ldots, d_n^{f_n}, p_1, \ldots, p_n) : d_1, \ldots, d_n \in N^D, p_1, \ldots, p_n \in N_f^P$ $1 \leq n \leq h$, where $d_n^{f_n}$ represents the n^{th} dumping site visited in the route where a skip containing waste stream $f_n \in F$ is picked, and p_n represents the n^{th} plant visited to unload that skip. Route r can only pick a skip containing f if there is a plant $p \in N_f^P$ in the route. We denote by η_{jr}^f the number of skips for waste stream $f \in F$ that have be collected from dumping site $j \in N^D$ by route $r \in R$. We assign to each route $r \in R$ a non-negative traversal cost $c_r^{'}$ consisting of the cost of all the traversed edges in the route, and considering that the route ends at the starting dumping site d_1.

It should be noted that there is no need to consider time synchronization between the first-echelon and second-echelon routes since typically in waste management systems, the first-echelon collection is executed during the day, and the skips transport is conducted at the end of the day or overnight, once all first-echelon vehicles are done with waste collection.

Given that the number of skips for each waste stream at each dumping site depends on the total load dumped at the dumping site by the first-echelon collection vehicles, the number of skips to locate at each dumping site for each type of waste is, therefore, a decision variable and is not pre-determined. To that end, we associate with each route $r \in R$ a fixed cost h_r which consists of the cost of locating the skips picked up in route r at their respective dumping sites.

Table 1 summarizes the associated decision variables with each echelon.

Table 1: 2E-CSMC-CARP model decision variables

Echelon	Variable	Description
First-echelon	x_{ij}^{k}	Binary variable equal to one if vehicle $k \in K$ traverses $(i, j) \in E^{'} \cup A$ in the direction from i to j, and zero otherwise.
	y_{ij}^{fk}	Binary variable equal to one if waste stream $f \in F$ on edge $(i, j) \in E_f^r$ is collected by vehicle $k \in K$, and zero otherwise.
	z^{fkm}	Binary variable equal to one if waste stream $f \in F$ is assigned to compartment $m \in M^k$ of vehicle $k \in K$, and zero otherwise.
	s_j^{fk}	Quantity of waste stream $f \in F$ unloaded at dumping site $j \in N^D$ by vehicle $k \in K$.
	o_{ij}^{k}	Non-negative continuous auxiliary flow variable that takes positive values if vehicle k traverses edge $(i, j) \in E^{'} \cup A$ in the direction from i to j.
Second-echelon	u_r	Number of skip trucks traversing route $r \in R$.

The mathematical formulation of the 2E-CSMC-CARP is as follows:

$$\min \sum_{k \in K} \sum_{(i,j) \in E^{'} \cup A} c_{ij} x_{ij}^{k} + \sum_{r \in R} (c_r^{\prime} + h_r) u_r \tag{1}$$

subject to:

$$\sum_{k \in K} y_{ij}^{fk} = 1 \quad \forall f \in F, (i, j) \in E_f^r \tag{2}$$

$$\sum_{f \in F} z^{fkm} \leq 1 \quad \forall k \in K, m \in M^k \tag{3}$$

$$\sum_{(i,j) \in E^r} q_{ij}^{f} y_{ij}^{fk} \leq \sum_{m \in M_k} z^{fkm} Q^{fkm} \quad \forall f \in F, k \in K \tag{4}$$

$$|M_k|(x_{ij}^{k} + x_{ji}^{k}) \geq \sum_{f \in F} y_{ij}^{fk} \quad \forall k \in K, (i, j) \in E^r \tag{5}$$

$$\sum_{\substack{i \in N: \\ (i,j) \in E^{'} \cup A}} x_{ij}^{k} - \sum_{\substack{i \in N: \\ (j,i) \in E^{'} \cup A}} x_{ji}^{k} = 0 \quad \forall k \in K, j \in N \tag{6}$$

$$\sum_{(i,j)\in E^r} q_{ij}^f y_{ij}^{fk} \leq \sum_{j\in N^D} s_j^{fk} \quad \forall f \in F, k \in K \tag{7}$$

$$\sum_{f\in F} s_j^{fk} \leq \sum_{f\in F} \sum_{(i,j)\in E^r} q_{ij}^f \sum_{\substack{i\in N: \\ (i,j)\in E^{'}\cup A}} x_{ij}^k \quad \forall k \in K, j \in N^D \tag{8}$$

$$\sum_{r\in R} \eta_{jr}^f u_r \geq \frac{\sum_{k\in K} s_j^{fk}}{S} \quad \forall f \in F, j \in N^D \tag{9}$$

$$\sum_{\substack{j\in N: \\ (i,j)\in E^{'}\cup A}} \left(o_{ij}^k - o_{ji}^k\right) \leq \sum_{\substack{j\in N: \\ (i,j)\in E^{'}\cup A}} x_{ij}^k \quad \forall k \in K, i \in N : i \neq 0 \tag{10}$$

$$o_{ij}^k \leq |N|^2 x_{ij}^k \quad \forall k \in K, (i,j) \in E^{'} \cup A \tag{11}$$

$$x_{ij}^k \in \{0,1\} \quad \forall (i,j) \in E^{'} \cup A \tag{12}$$

$$y_{ij}^{fk}, z^{fkm} \in \{0,1\} \quad \forall f \in F, k \in K, (i,j) \in E^r, m \in M_k \tag{13}$$

$$s_j^{fk} \geq 0 \quad \forall f \in F, k \in K, j \in N^D \tag{14}$$

$$o_{ij}^k \geq 0 \quad \forall (i,j) \in E^{'} \cup A \tag{15}$$

$$u_r \in \mathbb{Z}^+ \quad \forall r \in R \tag{16}$$

The objective function (1) minimizes the total cost consisting of the total routing cost of the first- and second-echelon and the total cost of assigning skips to dumping sites. Constraints (2) ensure that all waste streams are collected from every required edge. Constraints (3) ensure that each compartment of each vehicle is assigned to hold at most one waste stream. Constraints (4) make sure that the total demand for a waste stream collected by a vehicle from all its serviced edges does not exceed the compressed capacity of all compartments assigned to that waste stream. Constraints (5) guarantee that a given waste stream can be collected from an edge by a vehicle only if that vehicle is traversing that edge in any direction. Constraints (6) are the flow conservation constraints for every vehicle on every node in the graph. Constraints (7) ensure that all the collected waste for each stream by each vehicle is unloaded at the dumping sites. Constraints (8) state that a vehicle can only unload waste at dumping sites that it visits. The quantity of unloaded waste s_j^{fk} can be greater than zero only if vehicle k visits dumping site j at any point of the route. The network's demand $\sum_{(i,j)\in E^r} q_{ij}^f$ is used as an upper bound. Constraints (9) ensure that the number of skips of a waste stream that has to be transported by the second-echelon routes r from each dumping site corresponds to the number of skips needed to hold the quantity of that waste stream unloaded by first-echelon vehicles at that site, given by $\frac{\sum_{k\in K} s_j^{fk}}{S}$. Constraints (10) and (11) are sub-tour elimination constraints, which together with the flow conservation constraints (6) and the use of arcs A, ensure that a first-echelon route is composed by a closed cycle of nodes visited in the correct order (depot $\rightarrow$ required edges $\rightarrow$ dumping sites $\rightarrow$ depot). Note that we adopted the subtour elimination constraints of Golden and Wong (1981) since it allows us to include the dumping site arcs, contrary to other formulations formed by only using demand edges (Letchford and Salazar-González, 2015). Finally, constraints (12)–(16) define the domains of the variables.

5. Solution algorithm

The 2E-CSMC-CARP model is governed by six decisions: 1. selecting the first-echelon vehicle mix, 2. assigning waste streams to the compartments of the selected vehicles, 3. creating the first echelon routes for the selected vehicles, 4. assigning dumping sites to the selected vehicles, 5. allocating skips to dumping sites, and 6. selecting the second-echelon routes to transport skips to treatment plants. The first three decisions relate to the first-echelon, the fourth and fifth address the interaction between the two echelons, while the last one is related to the second-echelon.

Since the 2E-CSMC-CARP combines multi-commodity waste collection in the first-echelon, corresponding to the CSMC-CARP (Zbib and Laporte, 2020), and the two-echelon road-train problem (Araújo et al., 2023), it is inherently more complex than these two NP-hard problems and presents a larger solution space. This renders any solution approach that tackles all the decisions at the same time combinatorially prohibitive. To address these intricate challenges, we propose a two-phase matheuristic approach that decomposes the 2E-CSMC-CARP into smaller sub-problems that are handled sequentially. These two phases are the first-echelon vehicle mix phase and the routing and skips assignment phase.

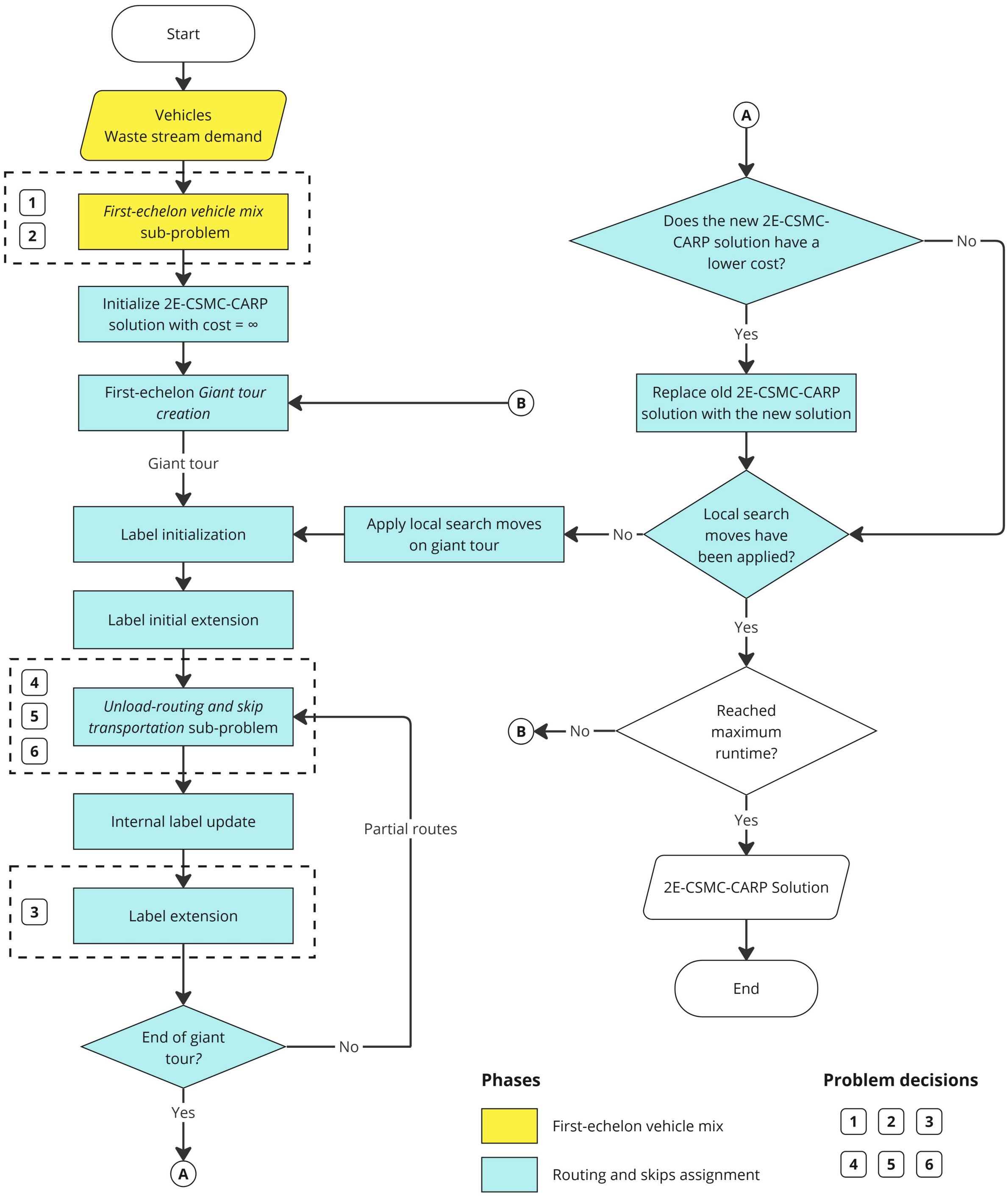

Figure 2: Flowchart of the 2E-CSMC-CARP algorithm

Figure 2 presents an overview of the solution method, indicating where in the solution method the six decisions are being addressed. The first phase, i.e., the first-echelon vehicle mix phase, deals with decisions 1 and 2, while the second phase, the routing and skips assignment phase, deals with decisions 3–6. The routing and skips assignment phase further decomposes the sub-problem given by decisions 3–6 into two smaller sub-problems which are iteratively and sequentially solved.

5.1. First-echelon vehicle mix phase

Given a first-echelon waste collection vehicle $k \in K$ in the heterogeneous vehicle fleet, there are $|F|^{|M^k|}$ possible ways to assign the different waste streams to the compartments of the vehicle. We define S^k as the set of all possible compartment assignments for $k \in K$, with $|S^k| = |F|^{|M^k|}$. For example, with $|F| = 6$ and $|M^k| = 4$, there are $|S^k| = 1,296$ possible assignments for that vehicle. This renders very large the decision space of choosing the mix of vehicles and compartment assignments, and considering all possible vehicles and assignments in the routing and skips assignment phase can be computationally prohibitive. Given this, the rationale behind the first-echelon vehicle mix phase is to select a diverse subset of cost-attractive vehicle assignments $\bar{S} \subseteq \cup_{k \in K} S^k$, which is then given as input to the routing and skips assignment phase.

Let θ_s^k be a dummy cost associated with each assignment $s \in S^k, k \in K$. We define parameter a_s^{fm}, which for every assignment $s \in S^k$ is set to one if waste stream $f \in F$ is assigned to compartment $m \in M^k$, and zero otherwise. The total compressed capacity of waste stream $f \in F$ in assignment $s \in S^k, k \in K$ is denoted by Q_s^f, where $Q_s^f = \sum_{m \in M^k} a_s^{fm} Q^{fkm}$. Finally, let δ_s^k be a non-negative integer variables corresponding to the number of selected vehicles $k \in K$ with assignment $s \in S^k$.

To obtain the subset $\bar{S}$, we iteratively solve the sub-problem defined by (17)–(19) and update the dummy costs in each iteration. The objective function (17) minimizes the total dummy cost of vehicles and assignments. Constraints (18) ensure that the total compressed capacity of all compartments collecting waste stream $f \in F$ is sufficient to cover the total demand of all required edges $e \in E_f^r$. Constraints (19) define the domain of the variables.

$$\text{minimize} \sum_{k \in K} \sum_{s \in S^k} \theta_s^k \delta_s^k \tag{17}$$

$$\text{subject to} \qquad \sum_{k \in K} \sum_{s \in S^k} Q_s^f \delta_s^k \geq \sum_{e \in E_f^r} q_e^f \quad f \in F \tag{18}$$

$$\delta_s^k \in \mathbb{Z}^+ \quad s \in S^k, k \in K \tag{19}$$

In the first-echelon vehicle mix phase, the sub-problem (17)–(19) is solved iteratively until a time limit T_{vm} is reached. To initialize, all assignment costs $\theta_s^k = 1$. After each iteration, assignments with $\delta_s^k > 0$ are added to $\bar{S}$, and this assignment is penalized in subsequent iterations to encourage diversification. For each $\delta_s^k > 0, s \in \bar{S}$, we update the costs θ_s^k in two ways to ensure diversification as follows:

$$\theta_s^k = \begin{cases} \theta_s^k + 1 & \text{if } s \notin \bar{S}, k \in K \\ \mathcal{M} & s \in \bar{S}. \end{cases}$$

This cost update gives a very large cost $\mathcal{M}$ to an already chosen assignment. Concurrently, for each remaining unselected assignment for the same vehicle, the cost θ_s^k is incremented by one. This modification reduces the desirability of these unselected assignments in favor of vehicles that have been selected less frequently.

To further diversify the subset $\bar{S}$, we iteratively remove the most dominant waste stream from the set F and resolve the subproblem for a number of iterations. This process aims to have an assignment that combines less dominant streams. To do so, we calculate the average compression factor $\bar{\gamma^f} = \frac{\sum_{k \in K} \gamma^{fk}}{|K|}, f \in F$ over all vehicle types, which are then used to calculate the occupation rate $\bar{q^f}, f \in F$. The occupation rate approximates the average proportion of the compressed waste stream f relative to the total compressed demand of all waste streams. The rate $\bar{q}^f$ for each $f \in F$ is calculated as $\bar{q^f} = \frac{\sum_{e \in E_f^r} q_e^f \bar{\gamma}^f}{\sum_{f \in F} \sum_{e \in E_f^r} q_e^f \bar{\gamma^f}}$. This rate helps in ranking waste streams from the most dominant to the least dominant in terms of total demand. We then run the subproblem until time limit T_{vm} is reached while removing the next most dominant waste stream $f \in F$, where f is the one with the maximum $\bar{q^f}$.

5.2. Routing and skips assignment phase

The output of the first-echelon vehicle mix phase is a subset $\bar{S} \subseteq \cup_{k \in K} S^k$ of attractive vehicle-assignment pairs that is then given as input to the routing and skips assignment phase. Even under the subset $\bar{S}$, the 2E-CSMC-CARP still yields a very large decision space, requiring a heuristic algorithm to solve it. To solve the 2E-CSMC-CARP, we have developed a matheuristic approach named the 2-echelon multi-commodity location-routing tour splitting algorithm (2EMCLR). We start by describing the rationale behind the 2EMCLR.

We denote by $\phi \in \Phi, \phi \in F, |\phi| = 1, ..., |F|$ a combination of waste streams, with Φ being the set of all possible combinations, such that $|\Phi| = \sum_{i=1}^{|F|} \binom{|F|}{i} = 2^{|F|} - 1$. Let $g(F)$ be a feasible solution to the 2E-CSMC-CARP. Similarly, let $g(\phi)$ be a feasible solution to the 2E-CSMC-CARP where only waste streams $f \in \phi$ are collected. $g(\phi)$ is a partial solution of $g(F)$. In fact, the solution $g(F)$ can be obtained from the concatenation of a pair of partial solutions with smaller cardinality $g(\phi_i) \oplus g(\phi_j) : \phi_i, \phi_j \in \Phi, \phi_i \cup \phi_j = F, \phi_i \cap \phi_j = \emptyset$. In the same spirit, we can obtain any partial solution $g(\phi), |\phi| > 1$ by concatenating a pair of partial solutions $g(\phi_i) \oplus g(\phi_j) : \phi_i, \phi_j \in \Phi, \phi_i \cup \phi_j = \phi, \phi_i \cap \phi_j = \emptyset$. To obtain the solution $g(F)$, we can recursively concatenate pairs of partial solutions $g(\phi_i), g(\phi_j)$ starting from solutions with cardinality $|\phi| = 1$ to $|\phi| = |F|$. For any $\phi \in \Phi$, there exists $\sum_{m=1}^{|\phi| - \lceil \frac{|\phi|}{2} \rceil} \binom{|\phi| - m}{m}$ concatenations of solution pairs $g(\phi_i) \oplus g(\phi_j)$ to obtain $g(\phi)$. Therefore, a least-cost solution $g(\phi)$ over all possible concatenations corresponds to $g(\phi) = \underset{\phi_i, \phi_j \in \Phi, \phi_i \cup \phi_j = \phi, \phi_i \cap \phi_j = \emptyset}{\arg\min} \{g(\phi_i) \oplus g(\phi_j)\}$. Given this, the rationale behind the 2EMCLR is to solve the 2E-CSMC-CARP on each $\phi \in \Phi$, and then obtain the least-cost solution $g(F)$ by recursively concatenating all possible concatenations from $|\phi| = 1$ to $|\phi| = |F|$.

By solving the problem for a specific set of waste streams $f \in \phi$, the first-echelon collection problem reduces from being a CSMC-CARP to a No-Split Multi-Compartment CARP (NSMC-CARP), which is easier to solve than its commodity-split counter part as it essentially boils down to being a CARP with multiple capacities that need to be respected simultaneously. However, solving the 2E-NSMC-CARP is still challenging. To counter that, we employ a tour splitting based approach in a two-echelon setting.

In a typical tour splitting algorithm for the CARP, the required edges single giant tour. This tour is then transformed into an auxiliary directed acyclic graph, where each edge is represented as a node. Each arc in this graph corresponds to a capacity-feasible CARP route that begins and ends at the depot, servicing the subsequence of edges in the order they appear between the start and end nodes of the arc (Prins et al., 2009). Extending this method to the NSMC-CARP involves incorporating arcs that comply with the capacity constraints of all compartments in the vehicle. The splitting algorithm assigns a cost label to each node and iteratively extends each label to subsequent nodes in the auxiliary graph, comparing the cost of extending the label with that of the current label cost at the node.

To adapt the tour splitting algorithm to the 2E-NSMC-CARP, we redefine cost labels to correspond to the cost of a 2E-NSMC-CARP solution, which includes decisions 3 to 6. Consequently, the arcs in the auxiliary graph are redefined as paths that start at the depot and end at the end node given by the arc in the auxiliary graph (which corresponds to the third decision). To include decisions 4 to 6 in the tour splitting algorithm, we superpose at each label extension a subgraph formed by the end nodes of all the paths in the current node being extended in the auxiliary graph, the dumping sites, the end depot, and the second-echelon routes. By defining the subgraph locally at each label extension, we can efficiently solve the subproblem related to decisions 4 to 6 using a mathematical solver. This subproblem involves determining for each path in the current label: which dumping sites the vehicle should visit to unload waste before returning to the depot, how many skips to place at each dumping site, and which second-echelon routes to use to transport the skips to treatment plants and back. Therefore, at each label of the algorithm, we locally know what is the least-cost solution for the 2E-NSMC-CARP that includes the first-echelon routes given by the label.

To tie the tour splitting algorithm with the concatenation of solutions to obtain $g(F)$ for the 2E-CSMC-CARP, once we have extended the labels of all subsequent nodes to a current node for all combinations $\phi \in \Phi$, and therefore the $|\Phi|$ labels at that node are final, we execute an internal label update procedure where we recursively concatenate the 2E-NSMC-CARP labels of different combinations $g(\phi) = \underset{\phi_i, \phi_j \in \Phi, \phi_i \cup \phi_j = \phi, \phi_i \cap \phi_j = \emptyset}{\arg\min} \{g(\phi_i) \oplus g(\phi_j)\}$ as described previously until we obtain the label $g(F)$ for that node. Therefore, the 2EMCLR iteratively deals with the four decisions in the routing and skips assignment phase by dividing them into three iteratively sequential procedures solved for each node in the auxiliary graph.

5.2.1. Two-echelon multi-commodity location-routing tour splitting algorithm

We now detail the components of the 2EMCLR as presented in Figure 2 and Algorithm 1, mainly the giant tour creation, the labels initialization, the labels extension, and the internal labels update procedures. The 2EMCLR is run iteratively by generating a number of giant tours and splitting them, until the time limit T_{max} given to the algorithm is reached.

The first step of the 2EMCLR is the giant tour creation procedure. It consists of a randomized nearest neighbor algorithm that accounts for the trade-off that exists between the distance of first-echelon required edges and the packing of the multi-compartments of the first-echelon collection vehicles. The procedure starts by calculating the ratio $\bar{q}_e = \frac{\underset{f \in F: q_e^f > 0}{\text{average}} \left\{ q_e^f \bar{\gamma}^f \right\}}{\max_{f \in F: q_e^f > 0} \left\{ q_e^f \bar{\gamma}^f \right\}}, e \in E^r$. This ratio indicates the skewness of the compressed demands of the waste streams on the edge. Edges are added to the giant tour w in an iterative way by keeping track of the average $\bar{q}_e$ for all links already added to the giant tour, and adding an edge at random from the nearest edges to the last added link whose $\bar{q}_e \leq average\, \{\bar{q}_e\}$. Once an edge is chosen, since each edge has two possible orientations, we choose the orientation that minimizes the distance to the previous edge when adding it to w. The procedure terminates when all edges $e \in E^r$ are added to w.

Algorithm 1 2EMCLR algorithm

Require: $\Phi, \bar{S}, T_{max}$

Initialize the best 2E-CSMC-CARP solution to ∞
while T_{max} is not reached **do**
    Generate a giant tour $w = (1, ..., e)$
    Pre-process the partial labels $\mathcal{D}_i, \mathcal{W}_i^f, i = 0, ..., e, f \in F$
    Initialize labels $\omega_0^\phi = 0, \omega_i^\phi = \infty, \omega_0^\phi \in \Omega_0, \omega_i^\phi \in \Omega_i, i = 1, ..., e, \phi \in \Phi$
    for $i = 0$ to e **do**
        for $j = i + 1$ to e **do**
            Internal label update for $\Omega_i, i \neq 0$
            for $\phi \in \Phi$ **do**
                Label extension of ω_i^ϕ from i to j
            end for
        end for
    end for
end while

Once a giant tour w is generated as an ordering of the edges $(1, ..., |E^r|)$, we associate the auxiliary graph $G_w = (N_w, A_w)$ with w, where N_w is a set of ordered nodes, $|N_w| = |E^r| + 1$ with the first node being a dummy sink node, and the remaining nodes associated respectively with the edges in w. Moreover, each arc $(i, j)^\phi \in A_w, i \in j, \phi \in \Phi$ corresponds to a capacity-feasible path that starts at the depot node $v_0 \in N$, collects the waste streams in ϕ from the edges given by the ordering $(i + 1, ..., j)$, and ends at j. A cost π_{ij} is associated with each $(i, j)^\phi$, which corresponds to the cost of the path. We note that parameters related to edges in the graph G, which were previously indexed by ij, will now be indexed by the corresponding index i in the auxiliary graph G_w.

We associate a set Ω_i of cost labels of cardinality $|\Phi|$ with each node $i \in N_w$. The label ω_i^ϕ corresponds to the least-cost of the 2E-NSMC-CARP solution $g(\phi)$ for the ordering $(1, ..., i), i \leq |w|$ in the giant tour. We store inside each label the solution cost as well as the predecessor nodes in the auxiliary graph to be able to build the final solution recursively at the outset of the 2EMCLR. All labels $\omega_0^\phi \in \Omega_0$ are initialized to zero, and labels $\omega_i^\phi \in \Omega_i, i > 0$ to ∞. In order to speed up the label extension procedure of the 2EMCLR, we pre-calculate partial cost labels $\mathcal{D}_i$, and load labels $\mathcal{W}_i^f, f \in F$ for each node $i \in N_w$. This reduces the time of each label extension in the auxiliary graph from $\mathcal{O}(E^r)$ to $\mathcal{O}(1)$. The cost π_{ij} for $(i, j)^\phi \in A_w$ can then be calculated as $\pi_{ij} = d_{v_0, i+1} + \mathcal{D}_j - \mathcal{D}_{i+1} + c_{i+1}, \exists s \in \bar{S} : \mathcal{W}_j^f - \mathcal{W}_{i+1}^f \leq Q_s^f, \forall f \in \phi$, with $\mathcal{D}_i = c_1 + \sum_{l=2}^{i} (d_{l-1,l} + c_l)$, where $d_{l-1,l}$ is the shortest path distance between the end node of edge $l - 1 \in N_w$ and the start node of edge $l \in N_w$, $\mathcal{W}_i^f = \sum_{l=1}^{i} q_l^f, f \in F$.

The next steps of the algorithm is to iterate through the nodes $i \in N_w$ in order, and extend the labels $\omega_i^\phi \in \Omega_i$ for each $\phi \in \Phi$ to each node $j \in N_w, j > i$. At any step of the algorithm, for each $\omega_i^\phi \in \Omega_i$, we know what first-echelon paths are included in the label thus far, and what nodes $v \in N^l$ each path ends in. Let A_{ij}^ϕ be the set of all paths stored in the current label ω_i^ϕ plus the path given by $(i,j)^\phi$, $N_{ij}^\phi \subseteq N$ the set of end nodes of each path $e \in A_{ij}^\phi$, and W_e^f the total load of the path $e \in A_{ij}^\phi$ for waste stream $f \in F$. We define at each label extension the subgraph $G_\sigma = (N_\sigma, E_\sigma, A_\sigma, R) \subset G$ such that $N_\sigma = N_{ij}^\phi \cup N^D \cup v_0$, $E_\sigma = \{(i,j) : i,j \in N^D, i \neq j\}$, and $A_\sigma = \{(i,j) : i \in N_{ij}^\phi, j \in N^D\} \cup \{(i,j) : i \in N^D, j = v_0\}$. In order to obtain a solution for the 2E-NSMC-CARP extending label ω_i^ϕ to node j, we define the subproblem corresponding to decisions 4–6 on G_σ, and solve it to optimality using a mathematical solver. We denote the subproblem the unload-routing and skip transportation problem (URSTP) and present it as given by (20)–(29) below.

Let x_{ml}^e be a binary variable equal to 1 if the vehicle servicing the path $e \in A_{ij}^\phi$ traverses $(m,l) \in E_\sigma \cup A_\sigma$, and zero otherwise, s_l^{ef} a continuous variable corresponding to the load of waste stream $f \in F$ unloaded by the vehicle servicing the path $e \in A_{ij}^\phi$ at dumping site $l \in N^D$, and u_r a non-negative integer variable corresponding to the number of skip trucks traversing route $r \in R$. The objective function (20) minimizes the total cost of the two-echelon problem collecting the edges in the paths A_{ij}^ϕ (i.e., the 2E-NSMC-CARP solution $g(\phi)$ on A_{ij}^ϕ). It consists of three components: the first-echelon routing cost π_e of the paths (which is a parameter), the cost of forming a closed route from each first-echelon path by unloading at one or more dumping sites and returning to the depot, and the second-echelon cost of locating skips at the dumping sites and transporting them to their respective treatment plants and back. Constraints (21)–(23) are connectivity constraints for each path to close it into a cycle that visits dumping sites and returns to the depot node, and constraints (24)–(26) are the equivalent of the dumping site and second-echelon route constraints (7)–(9) in the full model.

To evaluate the extension of the label $\omega_i^\phi \in \Omega_i$ to j with the existing label $\omega_j \in \Omega_j$, we compare the value of the objective function (20) with the cost of label ω_j, and, if the value of (20) is smaller than ω_j, we update ω_j to correspond to the extension of ω_i^ϕ to j.

$$\min \sum_{e \in A_{ij}^\phi} \pi_e + \sum_{e \in A_{ij}^\phi} \sum_{(m,l) \in E_\sigma \cup A_\sigma} c_{ml} x_{ml}^e + \sum_{r \in R} (c_r' + h_r) u_r \tag{20}$$

subject to

$$\sum_{(v,l) \in A_\sigma} x_{vl}^e = 1 \quad \forall e \in A_{ij}^\phi \tag{21}$$

$$\sum_{(m,l)\in E_\sigma\cup A_\sigma} x^e_{ml} - \sum_{(l,m)\in E_\sigma\cup A_\sigma} x^e_{lm} = 0 \quad \forall m \in N_\sigma \setminus N^\phi_{ij}, e \in A^\phi_{ij} \tag{22}$$

$$\sum_{(m,v_0)\in A_\sigma} x^e_{mv_0} = 1 \quad \forall e \in A^\phi_{ij} \tag{23}$$

$$\sum_{l\in N^D} s^{ef}_l \geq W^f_e \quad \forall f \in F, e \in A^\phi_{ij} \tag{24}$$

$$\sum_{f\in F} s^{ef}_l \leq \sum_{e\in A^\phi_{ij}} W^f_e \sum_{(m,l)\in E_\sigma\cup A_\sigma} x^e_{ml} \quad \forall l \in N^D, e \in A^\phi_{ij} \tag{25}$$

$$\sum_{r\in R} \eta^f_{lr} u_r \geq \frac{\sum_{e\in A^\phi_{ij}} s^{ef}_l}{S} \quad \forall f \in F, l \in N^D \tag{26}$$

$$x^e_{ml} \in \{0,1\} \quad \forall (m,l) \in E_\sigma \cup A_\sigma, e \in A^\phi_{ij} \tag{27}$$

$$s^{ef}_l \geq 0 \quad \forall f \in F, l \in N^D, e \in A^\phi_{ij} \tag{28}$$

$$u_r \in \mathbb{Z}^+ \quad \forall r \in R \tag{29}$$

Once all the labels of the preceding nodes are extended to the labels of the current node $i \in N_w$, the internal labels update procedure is triggered for i. As mentioned earlier, it consists of recursively concatenating the 2E-NSMC-CARP labels $\omega^\phi_i \in \Omega_i$ of different combinations $g(\phi) = \underset{\phi_i,\phi_j\in\Phi,\phi_i\cup\phi_j=\phi,\phi_i\cap\phi_j=\emptyset}{\arg\min} \{g(\phi_i) \oplus g(\phi_j)\}$ until we obtain the label $g(F)$ for that node. More specifically, the procedure checks for each label $\omega^\phi_i, |\phi| \geq 2$ if there exists two labels $\omega^{\phi_j}_i$ and $\omega^{\phi_h}_i$ such that $\omega^\phi_i > \min_{\phi_j,\phi_h\in\Phi,\phi_j\cup\phi_h=\phi,\phi_j\cap\phi_h=\emptyset} \left\{\omega^{\phi_j}_i + \omega^{\phi_h}_i\right\}$, and if there is, we set $\omega^\phi_i = \omega^{\phi_j}_i + \omega^{\phi_h}_i$. Once the 2EMCLR runs on the giant tour w, the label $\omega^F_{|N_w|}$ is returned as it corresponds to the value of the objective function of the current solution of the 2E-CSMC-CARP on w.

6. Computational experiments

In this section, we present and discuss the results of our computational experiments. The solution method presented in Section 5 is implemented in Python and Gurobi 10.0 is used as the MIP solver. All implementations are executed on an Intel E5-2683 v4 Broadwell CPU at 2.1GHz with one core and 125G of memory. In Section 6.1, we introduce the instances used for the experiments, followed by the parameter tuning procedure in Section 6.2. Computational results are presented in Section 6.3, and we draw several managerial insights in Section 6.4.

6.1. Instance generation

The first-echelon data is adapted from a subset of 60 of the benchmark CSMC-CARP instances of Kiilerich and Wøhlk (2018) from real-life waste data in Denmark (which can be found on `http://www.optimization.dk`). The subset consists of 20 graphs spread around five regions: Norddjurs (N), Syddjurs (S), the two counties of Skanderborg and Odder (K), Odense (O), and Frederiksberg (F). Each graph is paired with a degree of sorting and a set of vehicle types to form an instance. The degree of sorting represents the number of waste streams considered and are denoted as B, D, and E (three, four, and six waste streams, respectively). For each region and degree of sorting, Kiilerich and Wøhlk (2018) consider the bi-weekly waste stream demand aggregated at the street level for each required edge. The smallest graph contains 26 nodes and 19 required edges, and the largest one contains 5,102 nodes and 5,518 required edges. The cost of traveling an edge corresponds to its length in meters. The vehicle files contain four to six vehicle types with one to four compartments.

To this dataset, we add the data related to the second echelon as follows. Four waste companies operate in the five regions: Renodjurs in (N) and (S), Renosyd in (K), Odense Renovation in (O), and ARC in (F). Each waste company owns six to ten recycling centres. We chose two to four of these centres as dumping sites for each region, prioritizing their proximity to the centroid of the required edges. Any number of skips with a capacity of 10,000 litres can be placed at every dumping site at a cost of 1,000 per skip.

Treatment plants are randomly located within a 25 km radius from the centroid of the set of required edge locations. Each plant is dedicated to process a single waste stream. The assignment of waste streams to treatment plants is performed based on the proximity to the centroid of the required edge locations and the total demand of each waste stream, where waste streams with higher demand are assigned to the closest plants.

6.2. Parameter tuning

The algorithm presented in Section 5 relies on three parameters to limit the CPU run time, T_{max}, T_{vm}, and T_{2E}. To determine the maximum allowable run time, T_{max} for each instance, we use a linear scaling function defined as $\alpha = |E^r||F|$. With the desired run time set to five minutes for the smallest instance and a maximum of 180 minutes for the largest instance based on α, the maximum run time in minutes is obtained by $T_{max} = 0.0053\alpha + 4.7$.

The parameter T_{vm}, which limits the run time available for the first echelon vehicle mix phase, is limited to the 10% of the total run time T_{max}. This proportion proved to provide sufficient number of compartment assignments before iterations failed to yield significant improvements.

For the parameter T_{2E}, which controls the run time limit to solve each URSTP subproblem, we tested values ranging from one to five minutes. Our tests indicated that, while most instances of the URSTP subproblem could be solved to optimality in one or two seconds, some of them (2%) required several minutes. A higher time limit for T_{2E} yields better results for the URSTP, but setting T_{2E} too high risks not completing even one full iteration of the 2EMCLR if multiple time-intensive URSTP instances occur. Based on the preliminary experiments, we selected $T_{2E} = 3$ minutes, which is the highest value that consistently allows the algorithm to complete multiple splitting iterations and achieve feasible solutions across all instances.

6.3. Computational results

In this section, we evaluate the performance of the proposed algorithm on the 60 instances from the literature. First, we compare the performance of the algorithm with the solution obtained by the MIP solver, considering two available dumping sites. The results are presented in Table 2 where each instance is identified by the *Graph* name and the number of required edges ($|E_r|$). Table 2 shows the upper bound (UB) and the lower bound (LB) from the MIP solver, along with the average (*Avg.*) and best solutions (*Best*) over and among three experimental runs of the algorithm. Since both the MIP solver and the algorithm reach the time limit for all instances, the time is not reported. The only exceptions are the smallest three instances F13_B, F13_D, and F13_E, where the solver reaches optimality in less than one minute. The values in **boldface** in Table 2 are the best solution obtained for each instance.

The results show that for small instances (with 19 required edges) where the MIP solver reaches optimality, the algorithm offers an average optimality gap of 3% ($GAP_{Avg} = \dfrac{Avg - LB}{LB}$). However, the solver is unable to reach optimality in graphs with more than 72 edges. For these graphs, the solver and the algorithm exhibit an average optimality gap of 1% and 6%, respectively. For graphs with the number of required edges ranging between 170 and 702 edges, the algorithm outperforms the MIP solver on all instances, yielding an average improvement of 12%, where $Improvement_{Avg} = \dfrac{Avg - UB}{UB}$, with an average optimality gap of 7%. Moreover, except for N12_B, the solver is not able to find any feasible solutions for large instances with more than 702 required edges within the time limit, whereas the algorithm yields solutions with an average gap of 17% from the relaxed solution obtained as LB. We also observe that increasing the number of waste streams makes the problem more difficult to solve. Given the same number of required edges, the algorithm yields an average improvement compared to the MIP solver of 16%, 13%, and 10% for 3, 4, and 6 waste streams, respectively.

Table 2: Comparison of the results from the MIP solver and the proposed algorithm for DS =2

		MIP Solver		Algorithm				MIP Solver		Algorithm	
Graph	$\|E_r\|$	UB	LB	Avg.	Best	Graph	$\|E_r\|$	UB	LB	Avg.	Best
F13_B	19	**22,991**	22,991	23,698	23,617	N12_B	702	760,349	560,803	654,091	**614,137**
F13_D	19	**37,571**	37,571	38,424	38,093	N12_D	702	-	688,872	785,127	**755,786**
F13_E	19	**62,265**	62,265	63,835	63,511	N12_E	702	-	812,384	932,934	**894,974**
F12_B	72	**56,800**	55,989	60,814	60,535	F1_B	783	-	1,485,741	1,754,002	**1,658,232**
F12_D	72	**86,751**	85,830	89,174	88,698	F1_D	783	-	1,916,484	2,138,029	**2,017,490**
F12_E	72	**118,954**	118,023	124,569	121,286	F1_E	783	-	2,095,086	2,427,973	**2,352,131**
O13_B	170	155,518	131,783	139,299	**138,794**	K12_B	803	-	990,011	1,031,982	**1,017,974**
O13_D	170	118,914	107,331	111,482	**109,720**	K12_D	803	-	1,361,279	1,616,212	**1,551,915**
O13_E	170	191,970	171,261	175,396	**172,006**	K12_E	803	-	1,588,690	1,997,277	**1,960,246**
F11_B	174	172,844	149,309	156,729	**155,528**	S11_B	961	-	907,938	1,026,996	**954,685**
F11_D	174	240,710	208,273	214,959	**211,478**	S11_D	961	-	804,499	981,262	**892,543**
F11_E	174	283,444	248,054	266,077	**263,466**	S11_E	961	-	1,101,163	1,265,524	**1,182,086**
S13_B	176	224,074	192,265	197,564	**194,018**	N11_B	1606	-	1,316,316	1,623,211	**1,520,963**
S13_D	176	164,111	143,311	149,033	**146,960**	N11_D	1606	-	1,637,724	1,996,105	**1,793,447**
S13_E	176	285,716	250,075	267,334	**264,879**	N11_E	1606	-	2,231,077	2,494,039	**2,376,189**
K13_B	283	312,378	274,298	284,379	**279,532**	O11_B	2132	-	1,610,961	1,877,055	**1,863,222**
K13_D	283	401,978	355,376	381,139	**380,125**	O11_D	2132	-	1,988,164	2,391,948	**2,376,214**
K13_E	283	418,043	360,700	381,107	**377,668**	O11_E	2132	-	2,415,962	2,792,505	**2,670,635**
N13_B	366	333,520	272,829	285,478	**282,359**	S10_B	2221	-	2,224,538	2,571,401	**2,351,903**
N13_D	366	378,215	331,334	340,101	**333,288**	S10_D	2221	-	2,560,455	3,127,024	**3,077,249**
N13_E	366	645,723	555,425	596,146	**584,406**	S10_E	2221	-	3,118,142	3,851,453	**3,761,171**
F10_B	377	470,739	387,119	406,960	**404,867**	K11_B	2281	-	1,946,599	2,144,849	**2,080,420**
F10_D	377	645,922	539,760	604,244	**584,515**	K11_D	2281	-	2,163,971	2,680,288	**2,598,210**
F10_E	377	784,106	677,984	768,352	**760,283**	K11_E	2281	-	2,758,869	3,257,149	**2,936,348**
S12_B	407	566,730	397,717	421,275	**417,977**	N10_B	2802	-	2,349,714	2,618,568	**2,498,977**
S12_D	407	565,254	390,764	442,892	**420,757**	N10_D	2802	-	2,805,447	3,280,157	**3,122,581**
S12_E	407	681,541	485,793	533,671	**530,701**	N10_E	2802	-	4,177,348	4,707,075	**4,670,936**
O12_B	535	617,784	434,620	491,268	**462,133**	K10_B	3744	-	3,424,390	4,250,187	**3,973,482**
O12_D	535	626,914	443,874	513,137	**504,507**	K10_D	5518	-	6,202,439	7,780,155	**7,408,505**
O12_E	535	-	424,275	494,034	**483,081**	K10_E	5518	-	4,848,469	5,789,879	**5,158,025**

6.4. Managerial insights

In this section, we derive several managerial insights on the skip usage and the cost of each echelon. Table 3 reports the breakdown of the average results obtained by the algorithm, sectioning the total cost in three categories: skip allocation cost, first echelon transportation cost, and second echelon transportation cost. The values in **boldface** are the highest cost among the cost elements within an instance.

Table 3 shows that, in most instances, the first echelon transportation cost is the highest. This is due to the use of vehicles with less capacity in the first echelon, making transportation less efficient. However, some instances have a higher cost in the second echelon transportation, specifically instances F12, F11, and F10, showing an interaction with network's topology and size. This underscores the effectiveness of the proposed algorithm in identifying cost-saving opportunities in different stages of the network and enhancing the efficiency of waste management logistics.

Table 3: Average cost elements for the proposed algorithm for DS =2

Graph	Skip Cost	$Cost_{firstEchelon}$	$Cost_{secondEchelon}$	Graph	Skip Cost	$Cost_{firstEchelon}$	$Cost_{secondEchelon}$
F13_B	4,333	4,880	**14,237**	N12_B	8,667	**527,815**	117,944
F13_D	5,333	3,964	**29,421**	N12_D	8,333	**635,628**	141,443
F13_E	7,000	5,175	**51,056**	N12_E	9,667	**679,290**	244,312
F12_B	8,000	19,579	**33,236**	F1_B	102,333	**1,087,818**	564,073
F12_D	9,667	23,588	**56,254**	F1_D	102,667	**1,250,468**	785,230
F12_E	11,333	28,226	**85,261**	F1_E	103,333	**1,441,676**	882,885
O13_B	4,333	**98,273**	36,543	K12_B	5,667	**989,597**	37,066
O13_D	5,000	**59,892**	46,070	K12_D	6,667	**1,552,270**	57,300
O13_E	7,333	**98,556**	69,704	K12_E	8,333	**1,898,713**	90,248
F11_B	19,333	58,758	**78,305**	S11_B	8,333	**903,450**	114,921
F11_D	19,000	67,722	**127,905**	S11_D	8,000	**856,281**	116,654
F11_E	20,667	85,592	**159,935**	S11_E	9,667	**1,094,400**	161,461
S13_B	3,000	**160,044**	34,130	N11_B	14,333	**1,367,369**	241,512
S13_D	4,000	**92,708**	51,898	N11_D	15,000	**1,656,035**	324,739
S13_E	6,333	**168,431**	92,572	N11_E	16,333	**2,041,282**	436,488
K13_B	4,333	**255,099**	24,649	O11_B	33,667	**1,488,823**	354,631
K13_D	4,333	**340,814**	35,700	O11_D	34,000	**1,985,539**	371,850
K13_E	6,333	**319,843**	55,266	O11_E	35,333	**2,372,833**	384,055
N13_B	5,667	**208,970**	71,261	S10_B	21,333	**2,281,165**	268,907
N13_D	6,333	**236,517**	97,252	S10_D	22,333	**2,773,573**	331,124
N13_E	8,333	**383,638**	204,279	S10_E	23,667	**3,366,646**	461,223
F10_B	42,333	**195,776**	168,698	K11_B	22,333	**1,966,082**	156,754
F10_D	43,667	253,232	**307,680**	K11_D	23,667	**2,452,498**	204,769
F10_E	45,667	334,469	**388,552**	K11_E	24,333	**2,949,858**	282,662
S12_B	4,333	**365,472**	51,220	N10_B	20,000	**2,321,789**	276,190
S12_D	5,000	**366,894**	70,722	N10_D	20,000	**2,917,443**	342,353
S12_E	7,333	**414,374**	112,159	N10_E	21,000	**4,175,326**	510,680
O12_B	8,000	**398,585**	84,627	K10_B	37,667	**3,890,886**	321,981
O12_D	9,333	**400,206**	103,336	K10_D	37,000	**7,370,842**	371,984
O12_E	10,000	**387,790**	95,913	K10_E	38,667	**5,341,953**	409,907

By comparing different degrees of sorting (in three, four, and six waste streams), we find that increasing the number of skips increases all types of costs, but not uniformly. Going from three to four waste streams increases the skip allocation cost, first echelon cost, and second echelon cost by 3%, 36%, and 30%, respectively. Whereas increasing the number of waste streams from four to six causes an increment of 8%, 9%, and 30% of the respective costs. This shows the importance of considering the variable impact that the number of waste streams can have on two-echelon networks, specially when each echelon uses vehicles with varying capacities and multi-compartment capabilities.

Table 4 reports the minimum number of skips needed, the total number of skips used, the average skip utilization rate, calculated as $\frac{\text{Skip Capacity} \times \text{Total number of skips}}{\text{Total Demand}}$, and the cost ratio, calculated as $\frac{Cost_{firstEchelon}}{Cost_{secondEchelon}}$. The minimum number of skips shows that the demand volume is independent from the number of edges, since some regions are more densely populated. The densely populated region F (Frederiksberg) requires a high amount of skips for medium sized graphs, whereas regions K and S require relatively few skips for similar sized graphs. The algorithm shows capability to find solutions with a number of skips used in par with the minimum required, especially in low-demand instances. The results also show that the utilization rate per skip has a positive correlation with the minimum number of skips and the number of required edges, and a negative one with the number of waste streams. On average, every minimum required skip increases the skip utilization by 1.4% and every 100 required edges increases it by 2%, whereas the addition of a waste fraction decreases the utilization rate by 11.4%. On the other hand, the cost ratio is highly influenced by the region's topology. Regions F, O, S, K, and N have an average cost ratio of 0.8, 3.7, 5.9, 15.0, and 4.9, respectively.

Further analysis (detailed results in Appendix Table A.6) shows that increasing the size of the graph by adding more dumping sites results in a larger and more difficult instance, but has a positive impact on the efficiency of the network when running the algorithm under the same run time limits. In general, increasing the number of dumping sites from two to three offers a cost improvement of 3%, and going from three to four dumping sites improves the solution by 5%. The cost efficiency gained by adding a dumping site is related to the number of waste streams, yielding an improvement when adding a third dumping site of 2.1%, 3.4% and 4.5%, for 3, 4, and 6 waste streams, respectively. When the fourth dumping site is added, the variation of cost efficiency improvement among number of waste streams decreases, but the variation among regions increases, yielding an improvement of 4.9% for the highest (N) and 1.4% for the lowest (S).

This improvement typically occurs by positioning a dumping site closer to the required edges or treatment plants, reducing the distance travelled by the vehicles of each echelon. For example, in region N, the fourth dumping site is located on average 3 km closer to the treatment plants than the third dumping site, whereas in region S the fourth dumping site is 13 km farther.

Table 4: Statistics for instances with DS =2

Graph	$\|E_r\|$	$\|F\|$	Skip Min N.	Skip N. Used	Skip Utilization rate	Cost ratio	Graph	$\|E_r\|$	$\|F\|$	Skip Min N.	Skip N. Used	Skip Utilization rate	Cost ratio
F13_B	19	3	4	4	38%	0.3	N12_B	702	3	6	8	74%	4.5
F13_D	19	4	5	5	29%	0.1	N12_D	702	4	7	8	71%	4.5
F13_E	19	6	7	7	19%	0.1	N12_E	702	6	9	9	56%	2.8
F12_B	72	3	8	8	70%	0.6	F1_B	783	3	96	102	96%	1.9
F12_D	72	4	9	9	57%	0.4	F1_D	783	4	97	102	93%	1.6
F12_E	72	6	11	11	38%	0.3	F1_E	783	6	98	103	89%	1.6
O13_B	170	3	4	4	53%	2.7	K12_B	803	3	5	5	70%	26.7
O13_D	170	4	5	5	43%	1.3	K12_D	803	4	6	6	61%	27.1
O13_E	170	6	7	7	29%	1.4	K12_E	803	6	8	8	40%	21.0
F11_B	174	3	18	19	86%	0.8	S11_B	961	3	7	8	82%	7.9
F11_D	174	4	18	19	78%	0.5	S11_D	961	4	8	8	80%	7.3
F11_E	174	6	19	20	64%	0.5	S11_E	961	6	9	9	65%	6.8
S13_B	176	3	3	3	41%	4.7	N11_B	1606	3	12	14	82%	5.7
S13_D	176	4	4	4	31%	1.8	N11_D	1606	4	13	15	74%	5.1
S13_E	176	6	6	6	21%	1.8	N11_E	1606	6	15	16	61%	4.7
K13_B	283	3	4	4	35%	10.4	O11_B	2132	3	32	33	97%	4.2
K13_D	283	4	4	4	39%	9.6	O11_D	2132	4	33	34	93%	5.3
K13_E	283	6	6	6	26%	5.8	O11_E	2132	6	34	35	85%	6.2
N13_B	366	3	4	5	62%	2.9	S10_B	2221	3	19	21	85%	8.5
N13_D	366	4	5	6	50%	2.4	S10_D	2221	4	20	22	81%	8.4
N13_E	366	6	7	8	33%	1.9	S10_E	2221	6	21	23	70%	7.3
F10_B	377	3	41	42	93%	1.2	K11_B	2281	3	21	22	87%	12.5
F10_D	377	4	41	43	83%	0.8	K11_D	2281	4	22	23	84%	12.0
F10_E	377	6	42	45	71%	0.9	K11_E	2281	6	23	24	74%	10.4
S12_B	407	3	4	4	63%	7.1	N10_B	2802	3	20	20	93%	8.4
S12_D	407	4	5	5	51%	5.2	N10_D	2802	4	20	20	93%	8.5
S12_E	407	6	7	7	34%	3.7	N10_E	2802	6	21	21	80%	8.2
O12_B	535	3	8	8	70%	4.7	K10_B	3744	3	35	37	95%	12.1
O12_D	535	4	8	9	65%	3.9	K10_D	5518	4	35	37	93%	19.8
O12_E	535	6	9	10	54%	4.0	K10_E	5518	6	35	38	87%	13.0

This explains the differences in cost improvement when adding dumping sites to the network, and brings light to the importance of considering the topology of the region when planning two-echelon waste collection services.

Table 5 reports the statistics related to the number of skips, the skip utilization rate, and the second echelon cost for the instances solved to optimality by the solver. For these instances, as shown in Table A.6 in Appendix A, using three dumping sites leads to the same results as using two, since the additional dumping site is not utilized due its inconvenient location. On the other hand, the fourth dumping site improves the solution by an average of 5%. When the fourth dumping site is activated, the number of skips increases on average by 2, and the skip utilization rate decreases by 3.4%. Despite this, the cost of transportation in the second echelon is reduced on average by 15%, sufficiently to provide an overall benefit. This confirms that the use of intermediate dumping sites can improve the efficiency of the waste collection network, if they are placed in convenient locations.

Table 5: Statistics of instances solved to optimality by the MIP solver

Graph	DS=2			DS=3			DS=4		
	Number skips	Skip utilization rate	2^{nd} Echelon cost	Number skips	Skip utilization rate	2^{nd} Echelon cost	Number skips	Skip utilization rate	2^{nd} Echelon cost
F13_B	4	37%	17,710	4	37%	17,710	6	34%	15,394
F13_D	5	30%	35,569	5	30%	35,569	7	27%	30,159
F13_E	7	20%	57,764	7	20%	57,764	9	16%	48,575

7. Conclusions

This paper studies the Two-Echelon Commodity-Split Multi-Compartment Capacitated Arc Routing Problem (2E-CSMC-CARP), an important problem in the design of efficient recyclable waste management systems. The problem is inspired by real logistical challenges faced in the design of networks within the context of the co-collection of multiple waste streams from households by multi-compartment vehicles. In the first echelon, multi-stream collection is done by multi-compartment collection vehicles, which then travel to dumping sites in order to unload the load of their compartments in multi-stream skips. In the second echelon, the skips are transported by tractors to their respective treatment plants and back. By introducing stream intermediate dumping sites as consolidation hubs within the waste management network, the operational efficiency and logistical routing costs of the network are optimized.

We have introduced a MIP formulation for the 2E-CSMC-CARP, and a solution approach in the form of a two-phase matheuristic. Due to the high combinatorial complexity of the problem, our solution approach decomposes the six decisions of the problem into smaller sub-problems that are solved sequentially. The first phase of the algorithm, the first-echelon vehicle mix phase, returns a subset of cost-attractive vehicle assignments among all possible vehicles, which is given as input to the second phase, the routing and skips assignment phase. The core of the routing and skips assignment phase consists of a novel 2-echelon multi-commodity location-routing tour splitting algorithm (2EMCLR). The core of the algorithm consists in solving a Two-Echelon No-Split Multi-Compartment CARP (2E-NSMC-CARP) problem for each possible subset of waste streams, and then concatenating the solution of the 2E-NSMC-CARP problems in order to obtain the full 2E-CSMC-CARP solution. Each 2E-NSMC-CARP solution is obtained by a location-routing tour splitting procedure that combines the extension of the labels for the first-echelon routes with an optimal selection at each extension of the dumping sites to unload in, the number of skips to locate, and the second-echelon routes to take. The former follows a classical tour splitting label extension procedure, while the latter is done by solving with a MIP solver a new sub-problem, the Unload-Routing and Skip Transportation problem.

The effectiveness of our algorithm to solve the 2E-CSMC-CARP was tested on 60 real-life waste collection instances from five regions in Denmark. The graphs included up to 5518 required edges and six waste streams. Instances were complemented with the addition of one treatment plant per waste stream and two to four dumping sites.

The algorithm demonstrates remarkable scalability, handling large-scale waste collection instances that are typically challenging for standard solvers. Computational experiments considering strict CPU run-time limits have consistently shown that our algorithm outperforms a commercial MIP solver by providing near-optimal solutions and achieving lower operational costs and better route optimization. This cost-effectiveness is more evident as the number of required edges increases. In a practical context, our computational results showed the benefit of implementing intermediate dumping sites to reduce the transportation cost of multiple waste streams. We have shown that networks with more dumping sites are more cost-efficient, as long as the location of each added dumping site is appropriate to the topology of the graph.

Further research could explore the integration of variable demand and planning of dumping site activation to enhance the decision-making process and adaptability of the algorithm under dynamic urban environments. Finally, our solution strategy can be adapted to solve several multi-echelon routing problems.

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Appendix A. Detailed results

Table A.6: Computational results for each instance

Graph	Vehicle	T_{max}	DS=2 UB (Solver)	DS=2 LB (Solver)	DS=2 Avg. (Algorithm)	DS=3 UB (Solver)	DS=3 LB (Solver)	DS=3 Avg. (Algorithm)	DS=4 UB (Solver)	DS=4 LB (Solver)	DS=4 Avg. (Algorithm)
F13_B	M3-2	5.0	**22,991**	22,991	23,698	**22,991**	22,991	23,972	**22,645**	22,645	23,140
F13_D	M3-1	5.1	**37,571**	37,571	38,424	**37,571**	37,571	38,504	**35,173**	35,173	36,282
F13_E	M3-1	5.3	**62,265**	62,265	63,835	**62,265**	62,265	64,346	**58,499**	58,499	60,317
F12_B	M4-2	5.8	**56,800**	55,989	60,814	**56,800**	55,989	60,682	**55,317**	54,319	56,538
F12_D	M4-1	6.2	**86,751**	85,830	89,174	**87,910**	85,830	88,377	**85,876**	84,208	85,886
F12_E	M4-1	7.0	**118,954**	118,023	124,569	**120,012**	118,023	123,308	**109,423**	108,279	118,572
O13_B	M3-2	7.5	155,518	131,783	**139,299**	141,986	127,645	**134,928**	142,615	125,359	**130,037**
O13_D	M3-1	8.4	118,914	107,331	**111,482**	117,843	105,760	**110,843**	117,552	99,764	**104,543**
O13_E	M3-1	10.2	191,970	171,261	**175,396**	185,863	163,413	**175,497**	185,208	162,342	**178,673**
F11_B	M4-2	7.4	172,844	149,309	**156,729**	176,288	145,429	**156,375**	160,427	145,429	**152,678**
F11_D	M4-1	8.3	240,710	208,273	**214,959**	234,216	192,891	**205,140**	220,727	192,891	**201,460**
F11_E	M4-1	10.1	283,444	248,054	**266,077**	278,133	238,246	**253,082**	249,146	238,246	**247,501**
S13_B	M3-2	7.5	224,074	192,265	**197,564**	229,367	188,293	**195,846**	216,642	188,293	**198,900**
S13_D	M3-1	8.4	164,111	143,311	**149,033**	157,580	137,837	**142,838**	155,048	137,837	**142,245**
S13_E	M3-1	10.3	285,716	250,075	**267,334**	286,172	242,180	**273,196**	270,853	242,180	**264,738**
K13_B	M3-2	9.2	312,378	274,298	**284,379**	322,046	272,285	**281,799**	298,880	270,934	**281,413**
K13_D	M3-1	10.7	401,978	355,376	**381,139**	389,689	353,648	**371,153**	387,886	350,279	**363,743**
K13_E	M3-1	13.7	418,043	360,700	**381,107**	413,409	358,764	**381,339**	-	352,681	**374,638**
N13_B	M2-1	10.7	333,520	272,829	**285,478**	305,991	267,414	**292,996**	299,057	267,414	**282,695**
N13_D	M2-1	12.7	378,215	331,334	**340,101**	384,631	325,073	**337,078**	644,103	325,073	**334,576**
N13_E	M2-1	16.7	645,723	555,425	**596,146**	616,876	535,989	**577,438**	-	535,989	**548,143**
F10_B	M6-2	10.5	470,739	387,119	**406,960**	442,890	371,207	**391,662**	-	371,207	**386,154**
F10_D	M6-1	12.5	645,922	539,760	**604,244**	607,107	500,429	**521,527**	-	500,429	**516,998**
F10_E	M6-1	16.3	784,106	677,984	**768,352**	794,532	650,489	**712,137**	-	650,489	**716,099**
S12_B	M3-2	11.2	566,730	397,717	**421,275**	788,860	394,430	**415,506**	-	394,430	**403,541**
S12_D	M3-1	13.3	565,254	390,764	**442,892**	-	385,495	**438,742**	-	385,495	**422,370**
S12_E	M3-1	17.6	681,541	485,793	**533,671**	-	481,864	**511,344**	-	481,864	**506,116**
O12_B	M4-2	13.2	617,784	434,620	**491,268**	864,264	432,132	**479,650**	-	426,083	**453,573**
O12_D	M4-1	16.0	626,914	443,874	**513,137**	-	436,663	**472,581**	-	432,799	**487,231**
O12_E	M4-1	21.7	-	424,275	**494,034**	-	417,476	**484,606**	-	410,746	**495,910**

Table A.7: Computational results for each instance (Continued)

Graph	Vehicle	T_{max}	DS=2 UB (Solver)	DS=2 LB (Solver)	DS=2 Avg. (Algorithm)	DS=3 UB (Solver)	DS=3 LB (Solver)	DS=3 Avg. (Algorithm)	DS=4 UB (Solver)	DS=4 LB (Solver)	DS=4 Avg. (Algorithm)
N12_B	M4-2	17.1	760,348	560,802	**654,091**	1,093,662	546,831	**640,492**	**-**	546,831	**594,331**
N12_D	M4-1	21.3	**-**	688,872	**785,127**	**-**	676,078	**751,528**	**-**	676,078	**732,886**
N12_E	M4-1	29.6	**-**	812,384	**932,934**	**-**	786,639	**838,013**	**-**	786,639	**810,832**
F1_B	M8-2	15.9	**-**	1,485,741	**1,754,002**	**-**	1,447,871	**1,557,775**	**-**	1,447,871	**1,567,549**
F1_D	M8-1	19.6	**-**	1,916,483	**2,138,029**	**-**	1,876,036	**2,068,522**	**-**	1,876,036	**2,009,055**
F1_E	M8-1	27.0	**-**	2,095,085	**2,427,973**	**-**	2,059,725	**2,207,300**	**-**	2,059,725	**2,164,686**
K12_B	M2-1	17.5	-	990,010	**1,031,982**	-	989,295	**1,026,122**	-	986,643	**1,049,426**
K12_D	M2-1	21.7	-	1,361,278	**1,616,212**	-	1,355,489	**1,521,576**	-	1,352,641	**1,573,631**
K12_E	M2-1	30.2	-	1,588,689	**1,997,277**	-	1,586,736	**1,846,271**	-	1,579,163	**1,732,812**
S11_B	M4-2	20.0	-	907,937	**1,026,996**	-	898,367	**1,135,425**	-	898,367	**1,121,463**
S11_D	M4-1	25.1	-	804,499	**981,262**	-	796,876	**969,625**	-	796,876	**885,684**
S11_E	M4-1	35.3	-	1,101,162	**1,265,524**	-	1,091,456	**1,251,940**	-	1,091,456	**1,202,674**
N11_B	M4-2	30.2	-	1,316,316	**1,623,211**	-	1,309,997	**1,598,676**	-	1,309,997	**1,571,134**
N11_D	M4-1	38.7	-	1,637,723	**1,996,105**	-	1,617,138	**1,954,069**	-	1,617,138	**1,799,461**
N11_E	M4-1	55.8	-	2,231,077	**2,494,039**	-	2,213,094	**2,638,996**	-	2,213,094	**2,472,535**
O11_B	M6-2	38.6	-	1,610,961	**1,877,055**	-	1,576,171	**1,842,843**	-	1,561,109	**1,732,560**
O11_D	M6-1	49.9	-	1,988,164	**2,391,948**	-	1,982,565	**2,192,566**	-	1,945,185	**2,166,209**
O11_E	M6-1	72.5	-	2,415,961	**2,792,505**	-	2,383,562	**2,608,716**	-	2,362,681	**2,607,859**
S10_B	M5-2	41.0	-	2,224,538	**2,571,401**	-	2,218,377	**2,385,174**	-	2,218,377	**2,405,046**
S10_D	M5-1	53.1	-	2,560,455	**3,127,024**	-	2,535,877	**2,862,815**	-	2,535,877	**2,918,249**
S10_E	M5-1	77.2	-	3,118,142	**3,851,453**	-	3,100,415	**3,711,482**	-	3,100,415	**3,648,484**
K11_B	M5-2	40.0	-	1,946,599	**2,144,849**	-	1,932,476	**2,254,085**	-	1,924,803	**2,171,940**
K11_D	M5-1	51.8	-	2,163,971	**2,680,288**	-	2,154,537	**2,679,842**	-	2,133,715	**2,592,557**
K11_E	M5-1	75.3	-	2,758,869	**3,257,149**	-	2,754,455	**3,024,624**	-	2,734,581	**3,001,934**
N10_B	M5-2	49.3	-	2,349,714	**2,618,568**	-	2,333,531	**2,640,148**	-	2,333,531	**2,555,635**
N10_D	M5-1	64.1	-	2,805,446	**3,280,157**	-	2,772,304	**3,079,094**	-	2,772,304	**3,004,002**
N10_E	M5-1	93.8	-	4,177,348	**4,707,075**	-	4,168,913	**4,397,240**	-	4,168,913	**4,374,782**
K10_B	M6-2	64.2	-	3,424,390	**4,250,187**	-	3,402,604	**3,742,244**	-	3,376,052	**3,667,021**
K10_D	M6-1	121.7	-	6,202,439	**7,780,155**	-	6,179,812	**7,293,925**	-	6,155,064	**7,043,307**
K10_E	M6-1	180.0	-	4,848,468	**5,789,879**	-	4,835,560	**5,507,878**	-	4,812,608	**5,613,908**

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