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CIRRELT-2023-33

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October 2023

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Multi-attribute Two-echelon Location Routing: Formulation and Dynamic Discretization Discovery Approach[ł]

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Abstract. We study the two-echelon location-routing system under tight synchronization constraints, in addition to several other interacting attributes. Prompted, in particular, by city-logistics applications, the system we address concerns a two-echelon distribution layout composed of a set of platform facilities and a set of intermediate satellite facilities to deliver freight from supply zones outside the city to customers within. The problem setting includes time-dependent multicommodity demand, time windows, lack of storage capacity at intermediate facilities, and synchronization at these facilities of the fleets operating on different echelons. The problem requires the selection of facilities at both levels, the allocation of suppliers to platforms and of customers to satellites, and the routing and scheduling of vehicles at each echelon, in order to deliver the freight from platforms to customers, through the satellites. The lack of storage capacity of the shared facilities, the satellites, requires tight scheduling of the vehicle routes and demand itineraries, i.e., departure times from the platforms and satellites, and the synchronization of vehicle routes at satellites for efficient transshipment operations. We introduce the problem setting, present a mixed-integer programming formulation, and a dynamic discretization discovery-based exact solution method for the problem. We perform thorough analyses to assess the impact of the problem attributes and requirements on the system behavior and algorithm performance.

Keywords: Transportation, time-dependent two-echelon location-routing, synchronization, dynamic discretization discovery, city logistic

Acknowledgements. Partial funding for this project was provided by the Natural Sciences and Engineering Council of Canada (NSERC) through its Discovery Grants Program. We also thank the Centre Sur l'Intelligence² en Gestion de Systèmes complexes (CRI²GS), ESG UQAM, for partial funding through the Excellence Student Fellowship program. While working on this paper, the second author was Adjunct Professor, Department of Computer Science and Operations Research, Université de Montréal, Montréal, Canada. The authors are grateful to Ms. Diane Larin for her invaluable linguistic and editorial assistance.

[ł] Revised version of the CIRRELT-2022-37

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Dépôt légal – Bibliothèque et Archives nationales du Québec
Bibliothèque et Archives Canada, 2023

1 Introduction

Locating/selecting facilities and building vehicle routes are two of the most critical problems arising in planning and managing transportation and logistics systems. The *Location Routing Problem* (*LRP*) class combines the two problems (under suitably defined harmonizing cost and demand values) into a single formulation providing a more accurate and refined representation of the impact of facility selections on the functioning and performance of the resulting system. One refers to the *Two-Echelon Location Routing Problem*, *2E-LRP*, when intertwined facility-selection and vehicle-routing decisions are to be taken for systems involving two sets of facilities and vehicles must be routed between the first and the second level of facilities, and between facilities on the second level and customers making up the “third” level. The interest and vast application areas of LRP and 2E-LRP are emphasized by a very large body of literature and the numerous surveys synthesizing the field (e.g., Prodhon and Prins, 2014; Schneider and Drexl, 2017; Mara et al., 2021).

However, the literature addressing richer problem settings characterized by several interacting attributes is still very limited (Sluijk et al., 2022). Existing 2E-LRP models and methods are unable to keep pace with the rapidly evolving planning challenges that require more comprehensive and rich problem representations. This gap is particularly evident in applications such as Physical Internet (Crainic et al., 2022) and City Logistics (Crainic et al., 2021b), which involve multiple origin-to-destination commodities, time-dependent demand, scheduled and synchronized activities, and other complex considerations. The complexity arising from the need for specialized modelling and heuristics to handle the interactions among these diverse attributes restricts the applicability of existing 2E-LRP models and methods, especially when considering time-dependent aspects (e.g., Bala et al., 2017). Our objective is to contribute toward filling these gaps in the knowledge, through contributions to the modelling of such systems and decisions, as well as to the body of solution methods for these formulations.

We address a 2E-LRP with multiple interacting attributes, including time-dependent multicommodity origin-to-destination (OD) demand, time windows, limited storage capacity at intermediate facilities, and synchronization at the intermediary facilities of the fleets operating on different echelons. This new *Two-Echelon Multi-Attribute Location-Routing Problem with fleet Synchronization* at intermediate facilities (*2E-MALRPS*) thus requires 1) the selection of facilities on both levels, 2) the routing, scheduling, and synchronization of vehicles at second-echelon (intermediate) facilities , and 3) the allocation of OD demands to the selected facilities and their delivery using sequences of synchronized routes. The goal of the 2E-MALRPS is to minimize the total cost of the system, composed of the facility-selection cost at both levels and the transportation (fleet-utilization) cost, while satisfying the demand and the capacities of the system elements.

The time dependencies of demand and scheduled operations require time to be explicitly represented in the formulation. Time-space networks constitute a widely-known modelling technique to efficiently capture and handle temporal information (e.g., Ford and Fulkerson, 1962; Crainic and Hewitt, 2021). Most contributions in the literature use a classic time-space representation in which the duration of the plan to be built is discretized, according to a given granularity, into a number of consecutive periods, and the nodes in the physical network representing the system studied are duplicated at the points in time defining the periods. While such a modelling provides the means to adequately represent the level of detail of time-dependent activities and planning, the size of the corresponding formulations grows very rapidly with the refinement of the time discretization, making exact solution methods impractical. We address this challenge through the model-building and algorithm-design aspects. On the modelling front, we propose a hybrid formulation, where the nodes standing for facilities, which can be active at any moment while the system works, are duplicated at all periods, while customer nodes appear only at relevant periods (when they can be reached within their time windows), while a continuous time representation is used to capture the timing of vehicles arriving and departing to/from intermediary facilities and customers.

On the algorithmic front, we propose an exact solution method based on the *Dynamic Discretization Discovery* (*DDD*) strategy introduced by Boland et al. (2017) for the *Scheduled Service Network Design* (*SSND*) problem. In that context, the DDD consists in iteratively solving a mixed-integer model formulated on a sparser version of the time-space network, also known as the *reduced time-space network*, and refining this network (i.e., refining the granularity of the time representation), until an optimal solution is found. The refinement procedure is the core of the DDD, in which all the nodes and arcs within the reduced network undergoing refinement must be reviewed and adjusted. Refinement generates an iterative network growth. It is hoped, and achieved (e.g., Boland et al., 2017), that growth will be significantly smaller than a full time-space network while adequately representing the time attributes of the problem in hand.

The iterative refinement process inherent in the DDD enables the dynamic discretization of relevant time moments in the network, resulting in a systematic reduction of computational complexity. This characteristic offers promising opportunities for addressing complex time-dependent problems by simplifying the representation of time (Vu et al., 2019). However, effectively tackling the two-echelon definition and the combinatorial nature of the 2E-MALRPS, which integrates location, routing, and time aspects, requires extending and adapting the original DDD methodology. These extensions are crucial for guiding the refinement process, controlling network growth, and preserving attribute interaction. To achieve these goals, we define specific properties and procedures for computing bounds, handling solution degeneracy, refining granularity, and controlling iterative growth of the time-space network.

The proposed DDD solution method thus enables an efficient time-space representation of the system, while overcoming the scalability limitations of the explicit representation of time. The results of the computational study illustrate the behaviour and very good performance of the proposed modelling approach and solution method, and emphasize the importance of explicitly accounting for the time attributes of the problem elements, and of the associated fleet synchronization requirements within time-sensitive distribution systems.

The paper is organized as follows. The problem definition is given in Section 2, and an overview of related literature in Section 3. Section 4 is dedicated to the hybrid modelling of time we use and the 2E-MALRPS formulation. Section 5 describes the DDD solution method we propose. Computational results are presented and analyzed in Section 6. We summarize our work in Section 7.

2 Problem Setting

The two-echelon system is composed of sets of suppliers (demand origins), platforms (primary facilities), satellites (intermediate facilities), customers (demand destinations), and two garages. *Platforms* are large-sized infrastructures where one performs the storage, sorting, and consolidation of the inbound freight provided by supply points through various modes of transportation. *Satellites*, on the other hand, are medium- to small-sized facilities, located within the city limits and providing reduced or null storage capacity, where first and second-echelon vehicles meet and freight is transshipped and consolidated for the second part of transportation to customers. Freight transportation is performed by two fleets of homogeneous and limited-capacity vehicles, each operating within a specific echelon and able to transport the products making up the OD demand. Vehicles are assumed to be available at the garage of their corresponding echelon, where they start and end their routes.

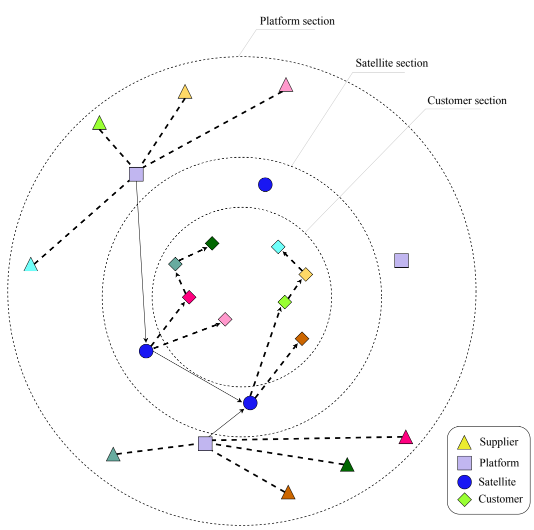


Figure 1: Two-echelon distribution system topology

The *multi-commodity origin-to-destination (OD) demand* is defined between suppliers and customers, each individual demand being characterized by origin, destination, volume, availability time, which is then adjusted for each platform, and due time window at destination. As depicted in Figure 1, each OD demand has to be assigned to an open platform and an open satellite. OD demands are consolidated at platforms, and are then moved by first-echelon vehicles to their selected satellites for the second part of their journeys. Once delivered at satellites, demand flows are transshipped and consolidated into second-echelon vehicles, which perform the deliveries to the final destinations. First and second echelon vehicle routes visit sequences of satellites and customers, respectively. To simplify the presentation of the system, garage nodes are not displayed in any of the illustrations of the paper.

The problem requires the selection of facilities at both levels, the allocation of suppliers to platforms and of customers to satellites, as well as the routing and scheduling of vehicles at each echelon to deliver the freight from platforms to customers, through satellite facilities. Vehicle routes and the OD-demand itineraries, determining the sites (facilities and customers) visited and the visit schedules, i.e., arrival, waiting, and departure times, must be determined within the time restrictions imposed by each OD demand (at platforms and customer), as well as by the need to synchronize vehicles at satellites due to the time dependency of demand and the lack of satellite storage capacity. Transportation costs are assumed to be equal to the travel times of the corresponding inter-site movements, while, to simplify the presentation but without loss of generality, waiting times at the various sites do not yield additional costs. The main objective of the resulting 2E-MALRPS is to minimize the total cost of the system, composed of the cost of selecting/opening facilities at both levels and the transportation costs, while satisfying the demand and the capacities of the system elements.

3 Literature Review

This section aims to situate the 2E-MALRPS within the relevant literature on the multi-attribute 2E-LRP and LRP, pointing out the gaps in knowledge with respect to time dependencies, time windows, origin-destination demand, and fleet synchronization. Given the available space, the review of the advances of each specific attribute is out of the scope of this paper. Therefore, our focus is on multi-attribute 2E-LRP and LRP, which encompass applications characterized by the interplay of two or more of these attributes within the same problem setting. A brief discussion on time-space formulations is also provided, focusing on dynamic discretization schemes used as solution frameworks. For an overview of the different problem variants in single- and two-echelon distribution systems and location routing problems that are out of the scope of this work, we refer the interested reader to recent surveys by Albareda-Sambola and Rodríguez-Pereira (2019); Crainic et al. (2021a); Mara et al. (2021), and Sluijk et al. (2022). We also refer readers to well-known surveys by Lopes et al. (2013) and Prodhon and Prins (2014), which provide a broad compilation of the early works and surveys for single-echelon LRPs with less attribute considerations.

The LRP has been the object of numerous studies since Maranzana (1964), research spanning a wide range of problem settings. Most LRPs focus on a single key attribute to represent the time or demand requirements of the problem under consideration. One notices that, time windows are used in most cases when temporal dependencies are addressed (Farham et al., 2018), while issues related to non-substitutable demand and fleet synchronization have scarcely been addressed from an optimization point of view (Boccia et al., 2018). Contributions to rich, multi-attribute LRPs, and the influence that the simultaneous consideration of several interacting attributes may have on the decision-making, are still very limited (Mara et al., 2021).

The literature on multi-attribute 2E-LRP, a more challenging problem setting due to the inter-echelon relationships, is much more recent and rather sparse. Research in the field has largely centered on the study of time-sensitive applications, focusing on customer time windows (e.g., Wang et al., 2018) and multi-period settings only (e.g., Darvish et al., 2019). Rich, multi-attribute 2E-LRPs incorporating multiple interacting sources of time-dependency, such as time-dependent non-substitutable demand and fleet synchronization, have been scarcely investigated. To the best of our knowledge, two contributions only have addressed problem settings with at least two attributes, which are relevant for this research: Bala et al. (2017) address a 2E-LRP with synchronized production schedules and time windows, while Mirhedayatian et al. (2019) consider a pick-up and delivery with fleet synchronization setting.

Location-routing and routing problems are related. Although surveying the Two-Echelon Vehicle Routing Problem (2E-VRP) literature (see, e.g., Crainic et al., 2021a; Sluijk et al., 2022) is out of the scope of this paper, it is worth noticing that more 2E-VRP variants with time-dependency and synchronization constraints were explored compared to the 2E-LRP. Yet, similarly to the 2E-VRP, the study of rich, multi-attribute 2E-VRP settings is still largely lacking.

From a modelling perspective, 2E-LRP contributions tend to share a compact-type structure, also known as flow formulations, to model the problem setting, with temporal and demand decisions being recorded through additional variables or indexes linked with the vehicle routing/flow variables (Contardo et al., 2012; Mara et al., 2021). The complexity of the 2E-LRP makes exact solution methods for these formulations impractical in most cases, in particular as the problem size or the number of attributes being considered grows, even when combining formulation-strengthening valid inequalities and column-generation mechanisms (Farham et al., 2018) (Albareda-Sambola and Rodríguez-Pereira, 2019). Meta-heuristics are thus generally proposed (e.g., Mirhedayatian et al., 2019; Abbassi et al., 2020).

It is noteworthy that the scientific literature on problems with tight or complex time considerations has widely adopted the use of time-space network representations (Crainic and Hewitt, 2021). Also noteworthy is the growing research effort addressing the iterative refining of the discretization of time-space networks, in order to efficiently deal with the trade-offs between the precision of the model, brought by refined time discretizations, and the significant computational challenges of addressing the corresponding large formulations. Introduced by Boland et al. (2017) for the Service Network Design Problem (SNDP), research on the *Dynamic Discretization Discovery* approach focuses mostly on the design of reduced time-space networks through a sparse discretization of time, which are iteratively refined to derive lower and upper bounds to the problem without the use of a highly detailed time-space representation. Very few efforts target topics outside the SNDP. Thus, Vu et al. (2019) propose a DDD-inspired solution method for the time-dependent travelling salesman problem with time windows, while Lagos et al. (2022) address the impact of time discretization on a continuous-time inventory-routing problem.

The 2E-LRP requires particular developments with respect to the modelling and algorithmic challenges that arise when addressing multiple interacting time-sensitive attributes, including the interest of alternative exact solution frameworks. This research work aims towards filling these gaps in the literature by proposing a hybrid time-space formulation and bv adapting and enhancing the guidelines of the DDD methodology for the 2E-MALRPS.

4 2E-MALRPS Modelling

Section 4.1 is dedicated to the formal problem definition, while Section 4.2 introduces the time-space representation. Finally, Section 4.3 presents the 2E-MALRPS formulation.

4.1 Problem definition and notation

Let $\mathcal{G}^{\mathrm{ph}} = (\mathcal{V}^{\mathrm{ph}}, \mathcal{A}^{\mathrm{ph}})$ be the weighted directed graph representing the physical network on which the problem is defined. The set of vertices $\mathcal{V}^{\mathrm{ph}} = \mathcal{Q}^{\mathrm{ph}} \cup \mathcal{P}^{\mathrm{ph}} \cup \mathcal{Z}^{\mathrm{ph}} \cup \mathcal{E}^{\mathrm{ph}} \cup \mathcal{C}^{\mathrm{ph}}$ is made up of five disjoint sets standing for the physical sites (known or among which locations are to be decided) of suppliers $\mathcal{Q}^{\mathrm{ph}}$, potential platform sites $\mathcal{P}^{\mathrm{ph}}$, possible satellite sites $\mathcal{Z}^{\mathrm{ph}}$, vehicle garages $\mathcal{E}^{\mathrm{ph}}$, and customers $\mathcal{C}^{\mathrm{ph}}$. A fixed selection (opening) cost F_p and a capacity Θ_p are defined for each possible platform location $p \in \mathcal{P}^{\mathrm{ph}}$. A fixed selection (opening) cost F_z is also defined for each potential satellite site.

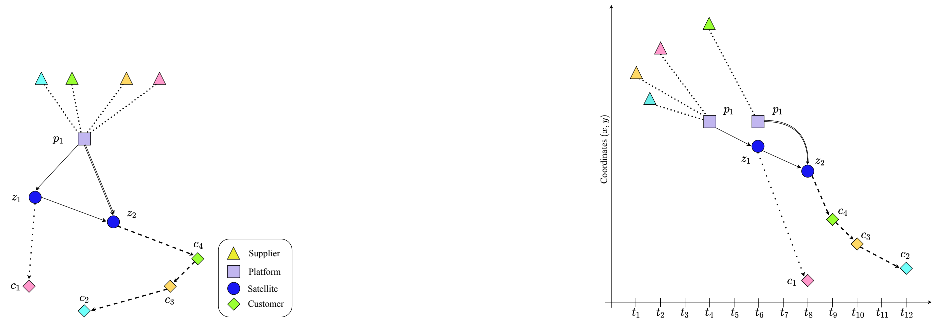
The arc-set $\mathcal{A}^{\mathrm{ph}} = \mathcal{A}_1^{\mathrm{ph}} \cup \mathcal{A}_2^{\mathrm{ph}}$ represents the direct links between locations, i.e., the vertices in $\mathcal{V}^{\text{ph}}$. A non-negative unit cost ζ_{ij} and a travel time τ_{ij} are associated with each arc $(i,j) \in \mathcal{A}^{\text{ph}}$. The set $\mathcal{A}_1^{\text{ph}}$ includes the arcs of the first echelon, corresponding to the connections between suppliers $\mathcal{Q}^{\text{ph}}$ and platforms $\mathcal{P}^{\text{ph}}$, between the latter and satellites $\mathcal{Z}^{\text{ph}}$, as well as the arcs connecting pairs of satellites and the first-echelon garage to platforms and satellites. The set $\mathcal{A}_2^{\text{ph}}$ includes the arcs of the second echelon, that is, the connections between satellites $\mathcal{Z}^{\text{ph}}$ and the final customers $\mathcal{C}^{\text{ph}}$, and the arcs connecting pairs of customers, and the second-echelon garage to satellites and customers.

Due to the lack of storage capacity at satellites and the time dependency of demand, interacting vehicles from the first and second echelons must be synchronized at satellites, at certain points in time, where first echelon vehicles may wait for a maximum time W^2_{max}. Moreover, it is assumed that each customer $c \in \mathcal{C}^{\text{ph}}$ has a (hard) time window $[a_c, b_c]$ (the time interval in which service must start at the node) and a service time σ_c. The distribution plan and the corresponding time-sensitive service network are built for a given schedule length Ψ (e.g., a day or a week). The system, and the distribution plan, follow a cyclic and repetitive logistics operation over a certain planning horizon (e.g., a month or a season), during which demand and the temporal properties of the system do not change. Therefore, all transportation activities take place between time 0 and the given schedule length Ψ.

Let $\mathcal{K}$ denote the set of OD demands that must be transported from suppliers to customers. For each commodity $k \in \mathcal{K}$, let $vol(k)$ be its volume, $O(k) \in \mathcal{Q}^{\text{ph}}$ the associated supplier node, $D(k) \in \mathcal{C}^{\text{ph}}$ the associated customer node, and α^{pk} the time when commodity k would become ready for transportation if assigned to be shipped from platform $p \in \mathcal{P}^{\text{ph}}$. This parameter takes into consideration the time required for the transportation of each commodity from the supplier to the given platform. An *itinerary* for a given commodity specifies a possible scheduled journey from the moment when it becomes available at the supplier node, until its delivery at its final destination, through a platform and a satellite, including the specific time instances associated to each arrival and departure at each site. More formally, an itinerary r for commodity $k \in \mathcal{K}$ is a tuple $\{(v_i, \mu_i, \nu_i) : i \in r\}$, where $v_i \in \mathcal{V}^{\text{ph}}$ is the i-th node visited, μ_i the arrival time to v_i, and ν_i the departure time from the node.

Two homogeneous fleets of vehicles $\mathcal{H} = \mathcal{H}^1 \cup \mathcal{H}^2$, with limited load capacities cap_1 and cap_2, are available for the first and second echelon, respectively. Vehicle capacities are fixed. Vehicles can deliver any demand and are parked in strategically-located garages, $\mathcal{E}_1^{\text{ph}}$ for vehicles operating in the first echelon, and $\mathcal{E}_2^{\text{ph}}$ for vehicles operating in the second echelon.

The 2E-MALRPS consists in the selection of platform and satellite facilities, the allocation of demand from suppliers to platforms and of customers to satellites, as well as the construction of a limited set of routes for the first and second echelons in such a way that: (i) all the customer demands are satisfied on time; (ii) the load capacity of each vehicle is not exceeded; (iii) each customer is visited by only one vehicle; (iv) the total demand assigned to a facility (platforms and satellites) does not exceed its capacity at any time moment; and (vi) the sum of the fixed selection costs and the variable routing costs is minimized.



(a) (b)

Figure 2: Example of a feasible solution for the 2E-MALRPS. (a) spatial layout of the feasible solution for the 2E-MALRPS. (b) time-space representation of the feasible solution for the 2E-MALRPS.

Figure 2 illustrates the dynamics of the system from a physical and a temporal point of view. Figure 2a shows a feasible solution where four OD demands are dispatched to their destinations by means of platform facility p_1 and satellites z_1 and z_2, the full and dotted lines illustrating the first- and second-echelon vehicle movements, respectively. Operations are illustrated from a temporal point of view in Figure 2b, starting with the three OD demands, each with its own availability time, all being assigned to platform p_1 and ready to be shipped at time t_4. The fourth OD demand, available at time t_4, is also assigned to platform p_1, but it is ready to be shipped at time t_6. A first-echelon vehicle (single full line) arrives at platform p_1, picks-up part of the available demand, and proceeds to visit satellite z_1 at time t_6 and satellite z_2 at time t_8. A different first-echelon vehicle (thick full line), then picks-up the remaining demand at a later time, t_6, and arrives at satellite z_2 at time t_8. Two second-echelon vehicles leave their garage to arrive on time at satellites z_1 and z_2 to enable the freight transfer from the first-echelon vehicles and, then, deliver on time the freight to the appropriate customers. Multiple fleet synchronization activities take thus place at the two satellites. A first synchronization at satellite z_1, at time t_6 and a second synchronization takes places at satellite z_2, at time t_8. Vehicles returns to their respective garages once their routes are completed.

4.2 Time-space network

Time is a key aspect of the system. Time-dependent OD demands constrain the timing of operations by means of availability restrictions at platforms, the need to pass through satellites of limited capacity (if any), and customer time windows, creating an interdependency in time throughout the whole distribution process. Excess in travelling or waiting times may thus become prohibitive or result in operational infeasibilities. One must therefore carefully model time.

There are two major modelling alternatives to represent these timing decisions. The first type considers an implicit representation by focusing on the time of operations, e.g., when vehicles arrive and depart facilities and customers to pick up or drop freight. This representation leads to a compact, *continuous-time* representation with a polynomial number of variables (indexed by the arcs and nodes of the network). The itinerary of each commodity can be constructed by matching the vehicle flow (visits of vehicles to sites), allocation (what commodity is allocated to what vehicle), and time variables (the time a site is visited).

The second modelling alternative is the "classic" *discretization* approach according to a Δ granularity. The schedule length Ψ is partitioned into Δ time periods, each physical node $i \in \mathcal{V}^{\text{ph}}$ being duplicated at each time period. For simplicity of presentation, but without loss of generality, we assume in the following that all time periods defined by the discretization granularity Δ are of equal length. This mechanism leads to a *time-space network*, where every node is a pair (i, t) with i representing a physical node in $\mathcal{V}^{\text{ph}}$ and $0 \leq t \leq \Psi$ a moment in time. Physical arcs (i, j) in the original system now take the form $((i, t_i), (j, t_j))$ meaning that travel is performed between nodes i and j departing at time t_i and arriving at time t_j. In contrast to the continuous-time representation, the discrete representation is explicit, in the sense that the nodes and arcs in the time-space network encode the timing decisions explicitly. Synchronization and other timing actions and requirements are thus expressed as decisions and constraints in the resulting formulation.

The proposed formulation is thus built on the 2E-MALRPS time-space network $\mathcal{G} = (\mathcal{V}, \mathcal{A})$, with the sets $\mathcal{V} = \mathcal{Q} \cup \mathcal{E} \cup \mathcal{P} \cup \mathcal{Z} \cup \mathcal{C}$ standing for the nodes in time and space for suppliers $\mathcal{Q}$, vehicle garages $\mathcal{E} = \mathcal{E}^1 \cup \mathcal{E}^2$, platforms $\mathcal{P}$, satellites $\mathcal{Z}$, and customers $\mathcal{C}$.

Let $\mathcal{T}(\Delta)$ be the (ordered) set of time periods given by discretizing the schedule length Ψ according to the granularity Δ. Let also $\mathcal{T}_i(\Delta) \subseteq \mathcal{T}(\Delta)$ represent the set of time periods at which node $i \in \mathcal{V}^{\text{ph}}$ is *relevant* in $\mathcal{G}$ because vehicles or commodity flows may access it at that time. Each system component has its own set of relevant periods: suppliers appear once only, the time realizations of customers $i \in \mathcal{C}^{\text{ph}}$ must satisfy $\mathcal{T}_i(\Delta) \subseteq [a_i, b_i]$, and copies in time are made at all periods for satellites and platforms, as these must be available for the complete schedule length.

Figure 3 illustrates this hybrid time-space network structure for two OD demands, from the same supplier to two customers, passing through a platform and a satellite. All periods are relevant for the platform and satellite, while two nodes only are relevant for each of the two customers. This mechanism allows to reduce the cardinality of $\mathcal{V}$ by considering the spatial and time positions (i, t) of each node $i \in \mathcal{V}$ at time periods $t \in \mathcal{T}_i(\Delta)$ only. Let $\mathcal{V}_i$ stand for the set of time-space nodes $\{(i, t) : i \in \mathcal{V}^{\text{ph}},\ t \in \mathcal{T}_i(\Delta)\}$, and $[a_i, b_i]$ be the time interval during which node $i \in \mathcal{V}$ is relevant in $\mathcal{G}$, i.e., $a_i = \min\{t : t \in \mathcal{T}_i(\Delta)\}$ and $b_i = \max\{t : t \in \mathcal{T}_i(\Delta)\}$.

Similar to the physical network, the set of arcs $\mathcal{A} = \mathcal{A}^1 \cup \mathcal{A}^2$ stands for connections between time-space nodes representing the various system components. At the first echelon, $\mathcal{A}^1$, one finds the connections between suppliers and platforms, platforms and satellites, pairs of satellites, as well as from the first-echelon garage to platforms and from satellites to the former. The second echelon arcs in $\mathcal{A}^2$ stand for the connections from satellites to customers, between pairs of the latter, as well as from the second-echelon garage to satellites and from customer to the former. An arc $((i, t), (j, t')) \in \mathcal{A}$ is then defined for arc $(i, j) \in \mathcal{A}^{\text{ph}}$ with $t \in \mathcal{T}_i(\Delta)$ and $t' = t + \tau_{ij} \in \mathcal{T}_j(\Delta)$. To simplify the notation, the travel time of inbound arcs to customer nodes are considered to embed the service time at customers. Recall that commodities must be assigned to a single platform and satellite and the flow should not be split. Consequently, let $\mathcal{P}_0(k) = \{(p, t), (j, t') : p \in \mathcal{P}, j \in \mathcal{Z}, t \geq \alpha^{pk}\}$ be the set of platform-to-satellite arcs commodity k can be assigned to if passing through platform p to travel to a reachable satellite.

4.3 The 2E-MALRPS formulation

This section presents a mixed-integer formulation for the 2E-MALRPS that combines continuous and discrete time modelling strategies to represent time. The formulation consists of a *standalone* time-space formulation, determined by constraints ((2)-(22)), and a series of continuous-time constraints ((23)-(29)). In this formulation, waiting times at nodes $v_i \in \mathcal{V}$ are implicitly represented by the time difference between the departure of a vehicle and its prior arrival at the node, and enforced by the set of continuous-time constraints. Although these continuous-time constraints may appear redundant, they serve to preserve the precision of time-related decisions, compensating for any potential loss of accuracy caused by coarse discretization granularity.

Define the following decision variables on $\mathcal{G}$:
$y_i = 1$, if facility i is open, 0 otherwise (location);
$x_{ij} = 1$ if arc (i, j) is selected, 0 otherwise (vehicle routing);
$f^k_{ijh} = 1$, if commodity k goes through arc (i, j) with vehicle h, 0 otherwise;
$\gamma^k_{ij} = 1$, if commodity k goes through arc (i, j), 0 otherwise;
μ^1_{ih}: Arrival time of first-echelon vehicle h at vertex i;
μ^2_{ik}: Arrival time of commodity k at vertex i;
ν^1_{ih}: Departure time of first-echelon vehicle h from vertex i;

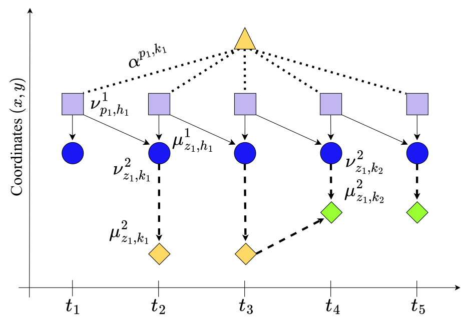


Figure 3: Hybrid representation of time.

ν^2_{ik}: Departure time of commodity k from vertex i .

The continuous-time variables needed to track vehicle schedules over the time-space network in order to deliver the demand of customer c_1 are illustrated in Figure 3, which also shows that only customer-to-customer connections are not handled by continuous time variables. Let M be a large integer number. The hybrid 2E-MALRPS formulation then becomes:

$$\min \sum_{i\in\mathcal{P}^{\text{ph}}} F_i y_i + \sum_{i\in\mathcal{Z}^{\text{ph}}} F_i y_i + \sum_{(i,j)\in\mathcal{A}^1} \zeta_{ij} x_{ij} + \sum_{(i,j)\in\mathcal{A}^2} \zeta_{ij} x_{ij} \tag{1}$$

Subject to

$$\sum_{j\in\mathcal{V}_c} \sum_{i\in((\mathcal{C}\setminus\mathcal{V}_c)\cup\mathcal{Z})} x_{ij} = 1 \quad \forall c \in \mathcal{C}^{\text{ph}} \tag{2}$$

$$\sum_{j\in\mathcal{V}_c} \sum_{i\in((\mathcal{C}\setminus\mathcal{V}_c)\cup\mathcal{Z})} x_{ij} = \sum_{j\in\mathcal{V}_c} \sum_{i\in((\mathcal{C}\setminus\mathcal{V}_c)\cup\mathcal{E}^2)} x_{ji} \quad \forall c \in \mathcal{C}^{\text{ph}} \tag{3}$$

$$\sum_{i\in((\mathcal{C}\setminus\mathcal{V}_c)\cup\mathcal{Z})} x_{ij} \geq \sum_{l\in\mathcal{V}_c} \sum_{i\in((\mathcal{C}\setminus\mathcal{V}_c)\cup\mathcal{E}^2)} x_{li} \quad \forall j \in \mathcal{V}_c, c \in \mathcal{C}^{\text{ph}} \tag{4}$$

$$\sum_{i\in\mathcal{E}^2} x_{ij} \leq \sum_{i\in\mathcal{C}} x_{ji} \quad \forall j \in \mathcal{Z} \tag{5}$$

$$\sum_{j\in\mathcal{V}_z} \sum_{i\in\mathcal{E}^2} x_{ij} = \sum_{j\in\mathcal{V}_z} \sum_{i\in\mathcal{C}} x_{ji} \quad \forall z \in \mathcal{Z}^{\text{ph}} \tag{6}$$

$$\sum_{i\in((\mathcal{Z}\setminus\mathcal{V}_z)\cup\mathcal{P})} x_{ij} \leq y_z \quad \forall j \in \mathcal{V}_z, z \in \mathcal{Z}^{\text{ph}} \tag{7}$$

$$\sum_{i\in((\mathcal{Z}\setminus\mathcal{V}_z)\cup\mathcal{P})} x_{ij} \leq \sum_{i\in((\mathcal{Z}\setminus\mathcal{V}_z)\cup\mathcal{E}^1)} x_{ji} \quad \forall j \in \mathcal{V}_z, z \in \mathcal{Z}^{\text{ph}} \tag{8}$$

$$\sum_{i\in\mathcal{E}^1} x_{ij} = \sum_{i\in\mathcal{Z}} x_{ji} \quad \forall j \in \mathcal{P} \tag{9}$$

$$\sum_{i\in\mathcal{V}_p} \sum_{j\in\mathcal{Z}} x_{ij} \leq |\mathcal{H}^1|\, y_p \quad \forall p \in \mathcal{P}^{\text{ph}} \tag{10}$$

$$\sum_{h\in\mathcal{H}^1}\sum_{i\in P_0(k)} f^k_{ijh} = 1 \quad \forall k \in \mathcal{K}, j \in \mathcal{Z} \tag{11}$$

$$\sum_{h\in\mathcal{H}^1}\sum_{i\in\mathcal{V}_p}\sum_{j\in\mathcal{Z}} f^k_{ijh} \le y_p \quad \forall k \in \mathcal{K}, p \in \mathcal{P}^{\mathrm{ph}} \tag{12}$$

$$\sum_{i\in((\mathcal{C}\setminus\mathcal{V}_c)\cup\mathcal{Z})}\sum_{j\in\mathcal{V}_c}\gamma^k_{ij} - \sum_{i\in((\mathcal{C}\setminus\mathcal{V}_c)\cup\mathcal{E}^2)}\sum_{j\in\mathcal{V}_c}\gamma^k_{ji} = 0$$
$$\forall c \in \mathcal{C}^{\mathrm{ph}}, k \in \mathcal{K}, D(k) \ne c \tag{13}$$

$$\sum_{i\in((\mathcal{C}\setminus\mathcal{V}_c)\cup\mathcal{Z})}\sum_{j\in\mathcal{V}_c}\gamma^k_{ij} - \sum_{i\in((\mathcal{C}\setminus\mathcal{V}_c)\cup\mathcal{E}^2)}\sum_{j\in\mathcal{V}_c}\gamma^k_{ji} \le 1 - \sum_{i\in((\mathcal{C}\setminus\mathcal{V}_c)\cup\mathcal{E}^2)}\sum_{j\in\mathcal{V}_c} x_{ji}$$
$$\forall c \in \mathcal{C}^{\mathrm{ph}}, k \in \mathcal{K}, D(k) \ne c \tag{14}$$

$$\sum_{i\in((\mathcal{C}\setminus\mathcal{V}_c)\cup\mathcal{Z})}\sum_{j\in\mathcal{V}_c}\gamma^k_{ij} - \sum_{i\in((\mathcal{C}\setminus\mathcal{V}_c)\cup\mathcal{E}^2)}\sum_{j\in\mathcal{V}_c}\gamma^k_{ji} = \sum_{i\in((\mathcal{C}\setminus\mathcal{V}_c)\cup\mathcal{Z})}\sum_{j\in\mathcal{V}_c} x_{ij}$$
$$\forall c \in \mathcal{C}^{\mathrm{ph}}, k \in \mathcal{K}, D(k) = c \tag{15}$$

$$\sum_{h\in\mathcal{H}^1}\sum_{j\in\mathcal{V}_z}\sum_{i\in((\mathcal{Z}\setminus\mathcal{V}_z)\cup\mathcal{P})} f^k_{ijh} = \sum_{h\in\mathcal{H}^1}\sum_{j\in\mathcal{V}_z}\sum_{i\in((\mathcal{Z}\setminus\mathcal{V}_z)\cup\mathcal{E}^1)} f^k_{jih} + \sum_{j\in\mathcal{V}_z}\sum_{l\in\mathcal{C}}\gamma^k_{jl}$$
$$\forall z \in \mathcal{Z}^{\mathrm{ph}}, k \in \mathcal{K} \tag{16}$$

$$\sum_{h\in\mathcal{H}^1}\sum_{i\in((\mathcal{Z}\setminus\mathcal{V}_z)\cup\mathcal{P})} f^k_{ijh} \le \sum_{h\in\mathcal{H}^1}\sum_{i\in((\mathcal{Z}\setminus\mathcal{V}_z)\cup\mathcal{E}^1)} f^k_{jih} + \sum_{l\in\mathcal{C}}\gamma^k_{jl}$$
$$\forall j \in \mathcal{V}_z, z \in \mathcal{Z}^{\mathrm{ph}}, k \in \mathcal{K} \tag{17}$$

$$\sum_{k\in\mathcal{K}} vol(k) \sum_{i\in((\mathcal{Z}\setminus\mathcal{V}_z)\cup\mathcal{P})} f^k_{ijh} \le cap_1 \quad \forall j \in \mathcal{V}_z, z \in \mathcal{Z}^{\mathrm{ph}}, h \in \mathcal{H}^1 \tag{18}$$

$$\sum_{k\in\mathcal{K}} vol(k) \sum_{i\in\mathcal{Z}} \gamma^k_{ij} \le cap_2 \quad \forall j \in \mathcal{C}, \tag{19}$$

$$\sum_{k\in\mathcal{K}} vol(k) \sum_{h\in\mathcal{H}^1}\sum_{i\in\mathcal{V}_p}\sum_{j\in\mathcal{Z}} f^k_{ijh} \le \Theta_p\, y_p \quad \forall p \in \mathcal{P}^{\mathrm{ph}} \tag{20}$$

$$\sum_{h\in\mathcal{H}^1} f^k_{ijh} \le x_{ij} \quad \forall k \in \mathcal{K}, (i,j) \in \mathcal{A}^1 \tag{21}$$

$$\gamma^k_{ij} \le x_{ij} \quad \forall k \in \mathcal{K}, (i,j) \in \mathcal{A}^2 \tag{22}$$

$$\nu^1_{ih} \ge \alpha^{ik} \sum_{j\in\mathcal{Z}} f^k_{ijh} \quad \forall h \in \mathcal{H}^1, k \in \mathcal{K}, i \in \mathcal{P} \tag{23}$$

$$\nu^2_{jk} \ge \mu^1_{jh} - (2 - \gamma^k_{jD(k)} - \sum_{i\in(\mathcal{Z}\cup\mathcal{P}), i\ne j} f^k_{ijh}) M \quad \forall k \in \mathcal{K}, h \in \mathcal{H}^1, j \in \mathcal{Z} \tag{24}$$

$$\nu^1_{jh} \ge \mu^2_{jk} - (2 - \gamma^k_{jD(k)} - \sum_{i\in(\mathcal{Z}\cup\mathcal{P}), i\ne j} f^k_{ijh}) M \quad \forall k \in \mathcal{K}, h \in \mathcal{H}^1, j \in \mathcal{Z} \tag{25}$$

$$\mu^1_{ih} + \tau_{ij} - \mu^1_{jh} \leq (1 - f^k_{ijh})M \quad \forall k \in \mathcal{K}, h \in \mathcal{H}^1, (i,j) \in \mathcal{A}^1 \tag{26}$$
$$\nu^1_{ih} + \tau_{ij} - \mu^1_{jh} \leq (1 - f^k_{ijh})M \quad \forall h \in \mathcal{H}^1, k \in \mathcal{K}, (i,j) \in \mathcal{A}^1 \tag{27}$$
$$\mu^2_{ik} + \tau_{ij} - \mu^2_{jk} \leq (1 - \gamma^k_{ij})M \quad \forall k \in \mathcal{K}, (i,j) \in \mathcal{A}^2 \tag{28}$$
$$\nu^2_{ik} + \tau_{ij} - \mu^2_{jk} \leq (1 - f^k_{ijh})M \quad \forall k \in \mathcal{K}, h \in \mathcal{H}^2, (i,j) \in \mathcal{A}^2 \tag{29}$$
$$\nu^1_{ih} - \mu^1_{ih} \leq W^2_{max} \quad \forall h \in \mathcal{H}^1, i \in \mathcal{Z} \tag{30}$$
$$\nu^2_{ik} - \mu^2_{ik} \leq W^2_{max} \quad \forall k \in \mathcal{K}, i \in \mathcal{Z} \tag{31}$$
$$a_{D(k)} \leq \mu^2_{D(k)k} \leq b_{D(k)} \quad \forall k \in \mathcal{K} \tag{32}$$

The objective function (1) minimizes the total cost of the system, which includes the fixed cost of selecting (opening) facilities on both echelons and the variable travel costs of the vehicles for demand transportation. Constraints (2) ensure that each customer is visited by a second-echelon vehicle exactly once. Constraints (3) represent vehicle-flow conservation at customer locations. Constraints (4) ensure that departure times of second-echelon vehicles from customers occur after the service time at the current customer. Constraints (5) enforce the requirement that for each outbound connection of a second-echelon vehicle from a satellite to a customer, there must be an inbound connection from a second-echelon garage to the satellite, including the waiting times at the satellite. Constraints (6) represent conservation constraints on routing variables for each satellite facility in the second echelon. Constraints (7) enable multiple visits to satellites in different time periods by ensuring that each open satellite is visited at most once for each time period, while constraints (8) ensure that for each outbound connection of a first-echelon vehicle from a satellite to another satellite or garage, there is an inbound connection from a platform or a different satellite. Constraints (9) and (10) enforce the routing conservation of first-echelon routing variables and restrict the maximum number of outbound connections from platform facilities in terms of the fleet size, respectively.

Constraints (11) and (12) ensure that each demand is not split and departs from the assigned open platform after its availability time, respectively. Constraints (13) and (14) impose flow conservation for commodities in nodes different to their destination customer. Constraints (15) guarantee that each commodity flow reaches its destination customer. Constraints (16) and (17) enforce flow conservation and spatial and temporal synchronization at time-space satellites, considering waiting times. Constraints (18) and (19) ensure that the total flow assigned to each route does not exceed the vehicle capacity for the first and second echelons, respectively. Similarly, constraints (20) impose that the assigned routes to each platform do not exceed the facility capacity. Constraints (21) and (22) establish the relationship between flow and routing variables.

Constraints (23) guarantee the feasibility of the schedule with respect to demand availability. Constraints (24) and (25) relate the arrival times at satellites of first- and second-echelon vehicles to ensure fleet synchronization at satellite facilities. Constraints (26) and (27) handle the arrival and departure times of first-echelon vehicles, while constraints (28) and (29) handle the arrival and departure times of second-echelon vehicles. Constraints (30) and (31) ensure that waiting times at satellite facilities respect the maximum permitted waiting time at each echelon. Constraints (32) ensure that second-echelon vehicles arrive within the customer time windows.

5 Dynamic Discretization Discovery for the 2E-MALRPS

The temporal dimension of the time-space 2E-MALRPS model provides a detailed and precise representation of the problem setting (Ford and Fulkerson, 1962). On the other hand, the pseudo-polynomial size of the integer formulation also makes head-on solution methods less scalable as the time granularity gets finer. We therefore propose a *Dynamic Discretization Discovery (DDD)* solution method for the time-space model to address this methodological challenge, building on the method introduced by Boland et al. (2017) for service network design problems. Notice that, demand in the 2E-MALRPS generates compulsory time moments, which must be explicitly included in the time-space network. While this appears to somehow facilitate the discretization of time, it greatly increases the difficulty of the problem and precludes a straightforward application of the method proposed in the service network design literature (Crainic and Hewitt, 2021).

The DDD algorithm we propose is illustrated in Figure 4. It iteratively refines a reduced time-space network, solving at each iteration the integer program defined by the hybrid formulation on that sparse network to extract lower and upper bounds for the 2E-MALRPS. The process is repeated until the problem is solved to optimality, or up to a specified tolerance ϵ. This tolerance is defined as the bound on the optimality gap percentage, which is utilized to determine when to stop the algorithm.

This section provides first the notation and foundations of the proposed DDD (Section 5.1), followed by the descriptions of the main algorithmic components as shown in Figure 4. To simplify the notation, we refer to the solution of the hybrid formulation as HTF($\mathcal{G}_{\Delta}$) for a reduced time-space network $\mathcal{G}_{\Delta}$.

5.1 Preliminary notation and problem analysis

Without loss of generality, let $\mathcal{G}_{\bar{\Delta}}$ be the *complete 2E-MALRPS time-space network*. Here, $\mathcal{T}(\bar{\Delta})$ denotes the full discretization of the length of the planning time span (i.e., schedule length), with $\mathcal{T}(\bar{\Delta}) = \bigcup_{i \in \mathcal{V}^{\text{ph}}} \mathcal{T}_i(\bar{\Delta}_i)$ standing for the set of the temporal time periods required to capture the relevant time moments $\mathcal{T}_i(\bar{\Delta}_i)$ for each $i \in \mathcal{V}^{\text{ph}}$. Similarly, let $\bar{\Delta}$ and $\bar{\Delta}_i$ denote the maximum number of time periods needed to encompass all relevant time moments across the system and for each $i \in \mathcal{V}^{\text{ph}}$, respectively. The proposed DDD

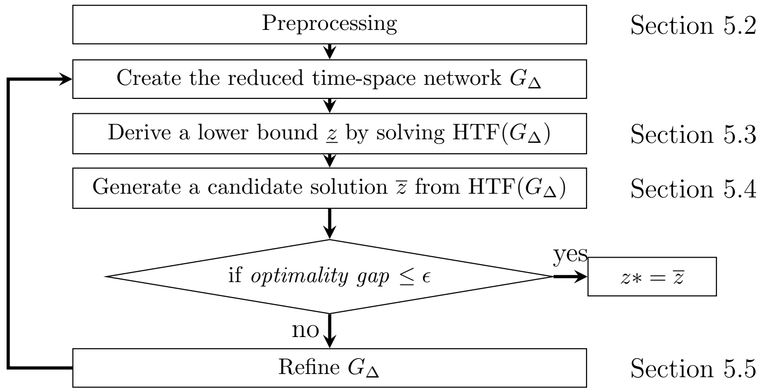


Figure 4: Dynamic discretization discovery for the 2E-MALRPS

solution method relaxes $\mathcal{T}(\bar{\Delta})$ through a granularity parameter $\Delta, 1 < \Delta < \bar{\Delta}$, and decreasing the number of relevant time periods for each vertex.

Let $\mathcal{G}_\Delta = (\mathcal{V}_\Delta, \mathcal{A}_\Delta)$, be the *reduced time-space network* defined by a reduced set of relevant time instants $\mathcal{T}_i(\Delta) \cap [a_i, b_i]$, for each node $i \in \mathcal{V}^{\text{ph}}$, where $[a_i, b_i]$ stands for the original interval of relevance of the node. Recall (Section 4.2) that, $[a_i, b_i]$ is the time window of each customer $i \in \mathcal{C}^{\text{ph}}$, the complete schedule length $[0, \Psi]$ for satellites and garages $i \in \mathcal{Z}^{\text{ph}} \cup \mathcal{E}^{\text{ph}}$, and the interval between the earliest potential commodity arrival $a_i = min_{k \in K}\{\alpha^{ik}\}$ and the schedule length $b_i = \Psi$ for platforms $i \in \mathcal{P}^{\text{ph}}$.

Inbound arcs to customers no longer embed waiting times in a reduced time-space network $\mathcal{G}_\Delta$. We rather consider a set of time-space nodes before each time window to represent early arrivals and waiting times at customers. The reduced time-space network is thus an aggregated network derived from $\mathcal{G}_{\bar{\Delta}}$, where $|\mathcal{G}_\Delta| \leq |\mathcal{G}_{\bar{\Delta}}|$. Consequently, the length τ_{ij} of arc $(i,j) \in \mathcal{A}^{\text{ph}}$ is also "aggregated" in terms of Δ, ensuring that there is a time-space arc $((i,t),(j,t')) \in \mathcal{A}_\Delta$ for each arc $(i,j) \in \mathcal{A}^{\text{ph}}$, with $t' \leq t + \tau_{ij}$. The aggregation does not change the arc travel costs, but it does impact travel times. Arc $((i,t),(j,t')) \in \mathcal{A}_\Delta$ is then considered to be *too short* when $t' < t + \tau_{ij}$, as it might model negative ($t' < t$) or zero ($t' = t$) travel times. These considerations are summarized in the following four properties:

- **Property 1.** A set of time-space nodes $(i,t) \in \mathcal{V}_\Delta$ exists for each node $i \in \mathcal{V}^{\text{ph}}$ and set of relevant time instants $\mathcal{T}_i(\Delta) \cap [a_i, b_i]$.
- **Property 2.** For each node $(i,t) \in \mathcal{V}_\Delta$ of a reduced time-space network and for each arc $(i,j) \in \mathcal{A}^{\text{ph}}$, there is a time-space arc $((i,t),(j,t')) \in \mathcal{A}_\Delta$ with $t' \leq t + \tau_{ij}$. If arc $((i,t),(j,t')) \in \mathcal{A}_\Delta$, there is no time-space node $(j,t'') \in \mathcal{V}_\Delta$ with $t' < t'' \leq t + \tau_{ij}$.
- **Property 3.** A *waiting-time* arc exists out of each time representation of a satellite $(i,t) \in \mathcal{Z}_\Delta$ towards a later node $(i,t') \in \mathcal{Z}_\Delta$ with $t' \leq t + W^2_{max}$.

- **Property 4.** For each customer $i \in \mathcal{C}^{\text{ph}}$, at least one time-space node (i, t) with $t < a_i$ exists in $\mathcal{C}_\Delta$ to handle early inbound connections exclusively.

Turning to the commodity flows through time and space, define *itinerary* r for commodity $k \in \mathcal{K}$ from its origin $O(k)$ to its destination $D(k)$ through the network $\mathcal{G}$ as a path $r = (v_i, t_i)_{i=1}^{l}$ connecting the node of the initial vehicle arrival and the departure of the commodity from a platform (i.e., $v_1 \in \mathcal{P}^{\text{ph}}$), to the node of the arrival and departure time at its destination (i.e., $v_l = D(k)$), including the times of arrival and departure at each intermediary node in $\mathcal{G}$. Let $R_{\mathcal{G}}^k$ be the set of feasible itineraries for commodity $k \in \mathcal{K}$, and $R_{\mathcal{G}} = \cup_k R_{\mathcal{G}}^k$.

The transfer between the first and second echelons of itinerary $r \in R^k$ occurs at a satellite $j \in \mathcal{Z}^{\text{ph}}$, where $v_j \in \mathcal{Z}, 1 < j < l-1$. The transfer timing is then framed by the four nodes $v_{j+w} \in \mathcal{Z}^{\text{ph}}$ with $w = \{0, 1, 2, 3\}$, i.e., t_j, t_{j+1} and t_{j+2}, t_{j+3} represent the arrival time to and departure time from the transfer satellite at each echelon, respectively. Hence, an itinerary $r = (v_i, t_i)_{i=1}^{l}$ in $\mathcal{G}$ is made up of the sequence of arcs $((v_i, t_i), (v_{i+1}, t_{i+1})) \in \mathcal{A}$ for every $i = 1, \ldots, l$, except for $i = j+1$, as the arc $((v_{j+1}, t_{j+1}), (v_{j+2}, t_{j+2}))$ would connect the departure and arrival times of first and second echelon routes, respectively.

The proposed DDD solution method relies on addressing the formulation (1) - (32) defined on a sequence of reduced time-space networks $\mathcal{G}_\Delta$ for various granularity levels $1 < \Delta < \bar{\Delta}$. The following lemmas ensure that 1) the 2E-MALRPS formulated on a reduced time-space network is a relaxation for the original problem and 2) the solutions obtained for the hybrid formulation on a reduced time-space network represent lower bounds for the 2E-MALRPS, regardless of the granularity of the discretization. The proofs of the lemmas may be found in the supplementary material Appendix A.

Lemma 1. Let $\mathcal{G}_\Delta$ be a reduced time-space network that satisfies properties 1, 2, 3, and 4. Then, for each commodity $k \in \mathcal{K}$ and itinerary $r = (v_i, t_i)_{i=1}^{l} \in R_{\mathcal{G}}^k$, there exists an itinerary $r' = (v_i, t'_i)_{i=1}^{l} \in R_{\mathcal{G}_\Delta}^k$ such that $t'_i \leq t_i$ for every $i = 1 \ldots l$.

Lemma 2. If a reduced time-space network $\mathcal{G}_\Delta$ satisfies properties 1, 2, 3, and 4, then the optimal solution of the 2E-MALRPS formulated on the reduced time-space network $\mathcal{G}_\Delta$ is a lower bound for the solution of the 2E-MALRPS on the complete time-space network $\mathcal{G}$.

Lemma 3. The proposed DDD algorithm ends with an optimal solution for the MA-2ELRPS.

Note that, the set of properties address fundamental aspects of the time-space representation of the problem. Therefore, the lemmas and proofs hold not only for the hybrid formulation (constraints (2) -(32)), but also for a "classic" time-space formulation that can be defined by isolating constraints (2) -(21).

5.2 Initial reduced network

The first step of the DDD algorithm tries to prune arcs and tighten the possible relevance-period sets, and then creates an initial reduced network $\mathcal{G}_\Delta$ satisfying Properties 1-4.

The time dependency of demand may imply that particular couples of locations and time instants might not be reachable when going from the origin to the destination of a demand. This may be used to tighten both the availability time at platforms and the customer time window of the particular demand and, thus, the set of relevant periods. The preprocessing analysis is performed on the physical network ($\mathcal{G}^{\text{ph}}$), through a breadth-first search strategy. The procedure iteratively explores the set of commodities, enumerating the possible platform-satellite combinations that could link the corresponding origin and destination at all time periods when the commodity could be available at the platform. The resulting set of feasible partial paths (including garage connections at each echelon) for each commodity defines the possible unreachable time periods for platforms and customers, which can then be excluded from the network.

The initial reduced time-space network $\mathcal{G}_\Delta$ is then generated by duplicating each node and arc in $\mathcal{G}^{\text{ph}}$ at each relevant time period, verifying that Properties 1-4 are satisfied.

5.3 A 2E-MALRPS lower bound on $\mathcal{G}_\Delta$

The definition of good quality lower bounds is fundamental for the DDD, as it guides the search through the solution space. The specifics of 2E-MALRPS makes this process challenging.

As previously indicated, lower bounds are obtained by solving the integer program defined by the hybrid formulation HTF($\mathcal{G}_\Delta$) (Section 4.3) on the reduced time-space network $\mathcal{G}_\Delta$. This solution may not be feasible for the original 2E-MALRPS, however. Indeed, even though it is feasible in terms of capacity constraints and location/allocation decisions, vehicle schedules might be infeasible when evaluated with the original travel times, due to the “aggregation” of the arc travel times. Compounding the challenge, the routing aspect of the problem involves a high number of interconnection between nodes of the time-space network, as each arc in the physical network may be represented by multiple time-space arcs, the multiplicity depending on the granularity of the time discretization. This tends to increase the size of the reduced time-space networks one has to address. Moreover, together with the presence of short arcs, it also results in numerous candidate solutions on the reduced time-space network to share the same solution structure, with similar objective function values and location/routing decisions while differing in vehicle schedules. Hence, although the reduced network is refined multiple times, one might still derive the same solution structure and costs with different sets of time-space arcs. This solution degeneracy increases the complexity of the optimization problem addressed at each iteration. The issue is particularly serious for the second tier of the system, first-echelon route degeneracy being avoided due to the presence of additional variables and constraints to keep track of time.

We therefore propose a procedure to handle degenerate solutions (second-echelon routes) within the reduced time-space networks, in order to mitigate the computational impact arising from the growth of the continuous refinement of the network (Algorithm 1). The fundamental idea when observing a degenerate solution is to dive towards a level of granularity where the degeneracy is no longer present, without solving the HTF($\mathcal{G}_\Delta$) for each intermediate value of Δ.

Algorithm 1: Degeneracy($\mathcal{G}_\Delta, Sol$)

```
   input: G_Δ, Sol = (z, x_{G_Δ}, f_{G_Δ}, y_{G_Δ})
1  if Sol ≠ ∅ then
2  |   Sol_local ← HTF(G_Δ, Sol) ;
3  |   if z(Sol_local) ≠ z(Sol) then
4  |   |   Sol ← HTF(G_Δ) ;
5  |   else
6  |   |   Sol ← Sol_local ;
7  |   end
8  else
9  |   Sol ← HTF(G_Δ) ;
10 end
```

Let us define a *solution structure* as the location, allocation, and routing decisions (but not the schedules) of the solution. A solution structure is then considered degenerate when two consecutive iterations of the DDD may identify it, even though the reduced network has been refined at least once. The procedure searches for degeneracy at each iteration of the DDD. It first aims to identify whether the structure of the current solution is degenerate by comparing it to that of the previous iteration (granularity level). If the answer is yes, the degeneracy mitigation procedure is activated by fixing the location and allocation decisions (line 2 and line 6) for the following iterations (the DDD continues normally otherwise). Thus, solving the HTF($\mathcal{G}_\Delta$) at the next granularity refinement level with fixed nodes and arcs focuses on routes and schedules only, which is significantly less computationally heavy compared to addressing the complete formulation. The solution structures are compared again and either degeneracy is still present, i.e., the same routes are found with the refined granularity, or is no longer observed. In the latter case, the fixing of location and allocations decisions is removed and the DDD continues normally. The diving with fixed location and allocation decisions continues otherwise until either degeneracy is no longer observed, in which case the current solution structure is feasible for the MA-2ELRPS, or the HTF($\mathcal{G}_\Delta$) returns an "infeasible solution" for the current level of granularity. The latter message indicates that the current structure cannot generate a feasible solution for the time-space network at the current or more refined level of granularity. Hence, the procedure has identified the granularity level where, with unfrozen location and allocation decisions, a new solution structure is found, without solving the HTF($\mathcal{G}_\Delta$) on the full networks for all the intermediate granularity levels.

5.4 A 2E-MALRPS upper bound

To determine a feasible upper bound for the 2E-MALRPS, the optimal solution obtained on the reduced time-space network $\mathcal{G}_\Delta$ must remain feasible when evaluated with the original travel times for all arcs. As indicated above, the variables tracking the vehicle schedules ensure the feasibility of the temporal decisions of the first-echelon routes, despite the presence of short arcs. Hence, a *feasibility-analysis* procedure is employed to evaluate the routes of the second echelon with the original travel times and verify the feasibility of customer time windows. This procedure utilizes the vehicle schedules defined by the optimal solution obtained from the reduced time-space network $\mathcal{G}_\Delta$ as the starting point for second-echelon routes. The procedure then proceeds by traversing the sequence of nodes for each second-echelon route while evaluating each connection using the original travel times. This iterative evaluation serves to identify whether all routes are feasible for the 2E-MALRPS or if any of the customer time windows are unreachable and, thus, infeasible for the 2E-MALRPS. It is important to note that the evaluation of each route acts as a means to map the routes generated by the reduced time-space network into the complete time-space formulation, thereby producing a feasible solution for the 2E-MALRPS when the schedules of both fleets are combined. When the feasibility of the solution obtained on $\mathcal{G}_\Delta$ is verified, then 1) the solution is feasible for the 2E-MALRPS and its value is an upper bound on the optimal-solution value of the original problem, and 2) if the solution value of HTF($\mathcal{G}_\Delta$) is better (lower) than the current best upper bound (if any), then the value of the best upper bound is updated and the procedure terminates.

When the HTF($\mathcal{G}_\Delta$) solution is found to be infeasible for the MA-2ELRPS, the algorithm stops and forwards the solution structure to the refinement step of the DDD. Notice that, in this case, the DDD procedure will iteratively refine the reduced network until the solution is proven infeasible or a DDD stopping criteria is reached. Notice also that, the upper-bounding procedure we propose provides the means to the DDD algorithm to identify potentially good-quality upper bounds even when the reduced time-space network is not well-refined.

Algorithm 2: Refinement($\mathcal{G}_\Delta, Sol$)

input: $\mathcal{G}_\Delta, Sol = (z^*, (\mathcal{V}^*_\Delta, \mathcal{A}^*_\Delta))$
for $((i,t),(j,t')) \in \mathcal{A}^*_\Delta$ **do**
    if $t' \leq t + \tau_{ij}$ ***and*** $i, j \in \mathcal{C}^{ph} \cup \mathcal{E}^{ph}$ **then**
        if $isFeasible(\mathcal{G}_\Delta, (i, t' - \tau_{ij}))$ **then**
            AddNode($\mathcal{G}_\Delta, (i, t' - \tau_{ij})$);
            Restore($\mathcal{G}_\Delta, (i, t' - \tau_{ij})$);
        end
        if $isFeasible(\mathcal{G}_\Delta, (j, t + \tau_{ij}))$ **then**
            AddNode($\mathcal{G}_\Delta, (j, t + \tau_{ij})$);
            Restore($\mathcal{G}_\Delta, (j, t + \tau_{ij})$);
        end
    end
end

5.5 Refinement

The final step of the proposed DDD solution method refines the reduced time-space network. Refinement is performed whenever the HTF($\mathcal{G}_\Delta$) solution, obtained on the reduced time-space network $\mathcal{G}_\Delta$, is found to have short arcs which violate one or more 2E-MALRPS temporal constraints when evaluated with the original travel times. Recall, Lemma 2 and Section 5.4, that a HTF($\mathcal{G}_\Delta$) solution is a lower bound for the 2E-MALRPS and, thus, insights on how to refine the reduced time-space network $\mathcal{G}_\Delta$ may be derived from the short arcs found in that solution. Moreover, given that the hybrid time representation provides the means to avoid refining the arcs involving platforms and satellite facilities, one may focus the refinement on arcs connecting customers to other customers or garages. Refining the reduced network in terms of these short arcs, extracted from the lower bound obtained at each iteration, strengthens the reduced time-space network and improves the precision of the 2E-MALRPS lower bounds in future iterations.

The proposed refinement procedure extends such short arcs in $\mathcal{G}_\Delta$, while ensuring that Properties 1-4 are valid at the nodes involved in the extension (Algorithm 2). Recall that, short arcs $((i,t),(j,t'))$ within the integer solution of HTF($\mathcal{G}_\Delta$) display short travel times $t' < t + \tau_{ij}$ relative to the pair of nodes $((i,t),(j,t+\tau_{ij}))$ in the original problem. The refinement procedure aims to extend each short arc from both of its extreme points (line 3 and line 7), thereby adding two new time-space customer nodes and arcs to the reduced network (Algorithm 3). (Notice that, due to the presence of solution degeneracy, refining a short arc in terms of one of its extreme points does not exclude its counterpart to potentially appear in the following iterations of the DDD.) Subsequently, Algorithm 3 is employed to incorporate each time-space node and its corresponding valid time-space arcs into the reduced network. The original travel time of each arc is utilized in this process, and the waiting times are updated accordingly (Algorithm 4), along with the arc connections of the reduced network (Algorithm 5) for each newly added node. To uphold Property 4, Algorithm 4 ensures that any newly introduced time-space customer node is added within the customer's time window and can connect with other time-space customers within and prior to (but never after) their respective time windows. Finally, Algorithm 5 guarantees the preservation of Property 2 once a new time-space node and its corresponding time-space arcs are added to the reduced time-space network.

Algorithm 3: AddNode((i,t'))

input: $\mathcal{G}_\Delta, (i,t')$ *with* $t_b < t' < t_{b+1}$
if $(i,t') \notin \mathcal{G}_\Delta$ **then**
  $\mathcal{G}_\Delta \leftarrow \mathcal{G}_\Delta \cup (i,t')$;
  for $((i,t_b),(j,t)) \in \mathcal{A}_\Delta$ **do**
    AddArc$(((i,t'),(j,t)))$;
  end
  UpdateWaitingArcs$((i,t'))$;
end

Algorithm 4: UpdateWaitingArcs((i, t'))

input: $\mathcal{G}_\Delta, (i, t')$ *with* $t' < a_i$
if $i \in \mathcal{C}^{ph}$ **then**
    for $(i, t) \in \mathcal{T}_i(\Delta)$ **do**
        if $t \geq a_i$ **then**
            $AddArc(((i, t'), (i, t)))$;
        end
    end
end

Algorithm 5: Restore($\mathcal{G}_\Delta, (i, t')$)

input: $\mathcal{G}_\Delta, (i, t')$ *with* $t_b < t' < t_{b+1}$
if $(i, t') \notin \mathcal{G}_\Delta$ **then**
    for $((i, t_b), (j, t)) \in \mathcal{A}_\Delta$ **do**
        $t" \leftarrow \arg\max\{d \in \mathcal{T}_j(\Delta) \mid d \leq t' + \tau_{ij}\}$;
        if $t" \neq t$ **then**
            $Delete(((i, t'), (j, t)))$;
            $AddArc(((i, t'), (j, t")))$;
        end
    end
    for $((j, t), (i, t_b)) \in \mathcal{A}_\Delta$ *with* $t + \tau_{ij} \geq t'$ **do**
        $Delete(((j, t), (i, t_b)))$;
        $AddArc(((j, t), (i, t')))$;
    end
end

6 Computational results

This section presents and discusses the results of experiments conducted to asses the performance of the proposed mathematical formulation and solution method for the 2E-MALRPS. We first introduce the instances used in the computational study in Section 6.1. We then present the performance of the proposed hybrid formulation using CPLEX in Section 6.2, followed by the performance analysis of our DDD algorithm in Section 6.3. The results of a series of experiments illustrating the sensitivity analysis of the DDD and the effects of problem instance characteristics are then analyzed in Section 6.4.

The experiments were conducted on a single machine with Intel(R) Core(TM) i7-7800X with 128 GB of RAM running Linux. The mathematical formulation and the proposed solution method are implemented in C++ using IBM ILOG CPLEX concert technology 20.1. The formulation is solved to optimality or to an optimality gap tolerance of less than or equal to 1% as the stopping criterion for instances with 50 OD demands.

The tables of this section display average results for the instance sets (detailed results in the supplementary material Appendix B) used in the associated experiments, indicating the numbers of potential platforms ($|\mathcal{P}^{\text{ph}}|$), satellites ($|\mathcal{Z}^{\text{ph}}|$), OD demands (OD), and instances in the set (NI). For the formulations considered in each experiment, the tables provide at least the number of instances for which feasible (FUB) and optimal (OUB) solutions were found within the given time limit, the run time in seconds (CPUsec), and the optimality gap (OG). The lower bound values provided by the proposed solution method correspond to, either the optimal solution of the hybrid model on the reduced network or, when the optimally gap tolerance is not reached within the given time limit, the best linear-relaxation value obtained throughout the optimization process.

6.1 Instances

No instances are available in the literature involving the integrated treatment of the attributes considered in the 2E-MALRPS, time-dependent origin-destination demands, fleet synchronization, and vehicle garages, in particular We thus define 5 new sets of instances, extending those with 15, 30, and 50 OD-demands (ODs in the following) introduced by Dellaert et al. (2019) for the 2EVRPTW, while instances with 5 and 10 ODs are created by selecting customers with the minimum distance to satellites from the 100-customer instances of Dellaert et al. (2019).

Each set of instances consists of 60 instances, which are divided into four categories (CA, CB, CC, and CD) to examine the behavior of the proposed DDD solution method under different variations of time and capacity. These categories introduce diverse distinctions by varying the time window width and demand values, thereby capturing the influence of the temporal component on the system and solution method. The instances represent a circular urban area, segmented into three concentric sections for platforms, satellites, and customers. OD demands are generated by randomly placing supplier points within the platform section (see Figure 1), assigning a unique customer (OD) and demand to each supply point. The availability time for each OD at each platform is determined by rounding up the Euclidean distance between the supply point and the platform. These availability times are incorporated into the temporal components of the system as a global temporal offset ρ based on the latest availability time. This offset ρ is added to each customer's time window to account for the additional time when the demand is available at the platforms, resulting in the time window range $[a_i + \rho, b_i + \rho]$. The maximum waiting time at satellites is set to 4, while the service time at customers is set to 2. Load capacities for vehicles given in Dellaert et al. (2019) are considered to be fixed, where first-level vehicles have a capacity of $cap_1 = 200$ and second-echeclon vehicles have a capacity of $cap_2 = 50$. Travel costs are computed as the ceiling of Euclidean distances. Schedule lengths are set to $\Psi = 100$ for small-sized instances with 5 and 10 ODs, and $\Psi = 200$ for medium- and large-sized instances with 15 ODs or more. The new 2E-MALRPS instances are available at `https://github.com/davescovar/2emalrpslib`

6.2 Hybrid formulation performance

We first focus on the effectiveness of the hybrid formulation, as well as on the impact of the granularity of the time discretization on its behaviour in terms of solution quality and computational efficiency. The sets of small- and medium-sized instances (5, 10, and 15 ODs) are used for benchmarking and the formulations are solved directly using CPLEX. The hybrid formulation is compared to a distinct standalone time-space formulation defined by isolating constraints (2) - (22) and the objective function (1). The hybrid and standalone time-space formulations are solved for a complete time-space network with $\bar{\Delta}$ time periods, as well as for different granularity values.

Tables 1 and 2 display the comparative performance results on the complete time-space network ($\Delta = \bar{\Delta}$) and the reduced time-space networks with granularity values $\Delta = 50$ and $\Delta = 25$. The run-time limit equals 2.5 hours. In addition to the information described earlier on, each table provides the average root gap (RG) computed as the percentage difference between the initial lower bound, obtained by the LP relaxation of the respective model at the root of the branch-and-bound tree, and the best integer solution obtained for the instance. Two additional performance measures present the average cost increment percentage (Dif UB) and CPU time reduction percentage (Dif CPUsec) obtained by each formulation using the reduced time-space network compared with the results of using the complete time-space network. (Recall that, we obtain reduced time-space networks with coarse discretizations by aggregating nodes and arcs of the complete time-space network. While this significantly reduces computation times, it also very often yields solutions which are infeasible with respect to the initial instance with a finer discretization. We did not consider these results when computing the last two performance measures.)

Instances				Hybrid								
				$\bar{\Delta}$					$\Delta = 50$		$\Delta = 25$	
$\lvert\mathcal{P}^{\mathbf{ph}}\rvert$	$\lvert\mathcal{Z}^{\mathbf{ph}}\rvert$	**OD**	**NI**	**FUB**	**OUB**	**CPUsec**	**RG(%)**	**OG(%)**	**Dif UB**	**Dif CPUsec**	**Dif UB**	**Dif CPUsec**
2	3	5	20	20	20	4561.33	16.38	0.00	0.71	49.70	1.54	83.94
3	5	5	20	20	20	3759.97	18.34	0.00	0.31	39.91	0.84	72.72
6	4	5	20	20	20	3118.99	21.99	0.00	1.38	40.05	3.19	75.45
2	3	10	20	20	15	4356.59	16.88	4.14	2.24	67.19	2.71	92.60
3	5	10	20	20	15	8586.08	17.86	9.13	2.79	64.20	3.28	87.36
6	4	10	20	20	15	8960.59	18.84	9.61	1.44	66.34	1.72	86.11
2	3	15	20	11	0	9000.00	36.43	29.30	10.41	4.31	11.78	36.83
3	5	15	20	0	0	9000.00	31.01	24.13	N.A	0.00	N.A	31.89
6	4	15	20	0	0	9000.00	37.85	31.19	N.A	0.00	N.A	33.19

Table 1: Performance of the hybrid time-space formulation

The results reported in Tables 1 and 2 show the expected performance similarity with respect to the upper bounds, but remarkable differences in the lower-bound and run-time values. When compared to the standalone formulation, the linear relaxation of the hybrid model provides much better lower bounds (average improvement of some 14%), but is usually slower at proving optimality. We observe that the overall lower bound

Instances				Standalone								
				Δ					$\Delta = 50$		$\Delta = 25$	
$\lvert\mathcal{P}^{\mathbf{ph}}\rvert$	$\lvert\mathcal{Z}^{\mathbf{ph}}\rvert$	**OD**	**NI**	**FUB**	**OUB**	**CPUsec**	**RG(%)**	**OG(%)**	**Dif UB**	**Dif CPUsec**	**Dif UB**	**Dif CPUsec**
2	3	5	20	20	20	1521.48	19.79	0.00	1.17	25.81	1.95	46.96
3	5	5	20	20	20	1290.38	21.55	0.00	0.86	26.36	1.22	45.24
6	4	5	20	20	20	2306.77	30.51	0.00	2.73	25.90	3.48	54.74
2	3	10	20	20	14	3208.01	21.57	3.19	2.54	71.70	3.07	81.01
3	5	10	20	20	14	4897.51	21.25	4.32	3.10	75.26	3.78	83.08
6	4	10	20	20	15	5255.78	21.94	5.03	1.64	74.93	2.14	84.85
2	3	15	20	17	1	9000.00	37.50	27.76	3.80	41.00	5.21	76.97
3	5	15	20	17	0	8961.78	31.59	23.06	3.71	39.48	4.66	79.89
6	4	15	20	16	0	9000.00	38.94	31.15	3.92	37.83	5.05	73.88

Table 2: Performance of the standalone time-space formulation

improvements by the hybrid formulation result from the combination of the vehicle index in the commodity-flow variables and the redundant continuous-time constraints to keep track of time, which help reduce noise in the mathematical model.

The experiments on 15-OD instances show a significant reduction in the number of feasible and optimal solutions for both formulations. Multiple factors contribute to this behavior. The use of time-space representation of the system clearly influences the quality of the solutions and makes it more challenging to tackle problems with larger numbers of ODs or longer schedule lengths with either of the two formulations. The standalone time-space model, despite providing good quality LP-relaxations, suffers from scalability issues provoked by the size of the time-space representation of the network, which, in turn, leads to larger and, thus, harder to solve, integer programs. The hybrid formulation shares a similar issue in terms of scalability due to the nature of its time-space structure, and the greater model size resulting from the inclusion of continuous-time constraints. The latter significantly increases the hybrid formulation size, which, when evaluated on the complete time-space network, results in longer exploration times of the solution space and, as a consequence, fewer optimal solutions being discovered for instances with 15 ODs. Nonetheless, the formulation usually gains from the redundancy that these continuous temporal constraints add, improving the root gap for all instance sets.

The discretization granularity plays a key role in the trade off between accuracy and performance of time-space models. On the one hand, as seen in the results of Tables 1 and 2, a finely discretized time-space network provides an accurate representation of the system, but at the price of a large integer problem. On the other hand, the sizes of the time-space representation and resulting integer problem, as well as the solution time, can be reduced by coarsening the discretization (see also Section 5), at the price of a decrease in solution quality or even feasibility. Thus, one observes average computing-time reductions of 77% and 50% for the hybrid and the standalone time-space formulation, respectively, with corresponding solution-quality losses of 1.3% and 1.9% in cost increments.

The results show the superiority of the hybrid model, even in the case a straightforward solution approach using a commercial solver. It consistently yields less-degraded solutions compared to the classical time-space formulation. Furthermore, it exhibits a robust behaviour when applied with coarser granularity values. This results from the redundancy induced through the continuous-time constraints, which, when paired with coarser granularity values, reduce model size while retaining accuracy on a significant number of arcs in the system. This, in turn, it improves the general performance of the hybrid formulation.

6.3 Performance of the dynamic discretization discovery solution method

We investigate the performance of the DDD solution method introduced in Section 5 for the hybrid 2E-MALRPS model, and compare it to that of the DDD adapted to address the standalone time-space formulation defined previously, by allowing the refinement and the degeneracy procedures to be executed on the complete time-space network (as opposed to being limited to the customer section, as designed when the hybrid formulation is used).

As discussed in Section 5, the use of continuous-time constraints helps the model to retain its precision under very coarse discretizations. Therefore, computational tests are performed using the coarsest discretization granularity ($\Delta = 2$) possible, to decrease the size of the underlying network and enable a further reduction in the time required to solve the integer program. The stopping criteria are a maximum run time of 2.5 hours for small (5 and 10 ODs) and medium-sized (15 ODs) instances, 5 hours fo 30-OD instances, and 8 hours for 50-OD instances and a an optimality gap of less or equal to 1% for instances with 50 OD demands.

Table 3 summarizes the results of the experiments for the proposed DDD algorithm using the hybrid model (H-DDD) and the DDD adapted for the standalone formulation (C-DDD). These results show that the DDD algorithm clearly outperforms the commercial solver (results in Section 6.2).

The H-DDD also presents a considerable general improvement in the solution quality and run times compared to the C-DDD. H-DDD identifies the optimal solution for 60 out of 60 instances with 15 ODs, compared with only 7 for the C-DDD. Furthermore, H-DDD is increasingly more robust as the problem size increases, providing feasible solutions for 120 out of 120 instances, as compared to only 80 for the C-DDD for the instances with 30 and 50 ODs. In terms of computational time, the H-DDD is on average 77% faster than the C-DDD in identifying optimal solutions. This notable performance gain is attributed to the use of continuous-time constraints along with the time-space representation of the problem. Despite the large size of the hybrid formulation (compared wit the standalone time-space model), the additional redundancy provided by these continuous-time

Instances			H-DDD					C-DDD				
$\|\mathcal{P}^{ph}\|$	$\|\mathcal{Z}^{ph}\|$	OD	NI	FUB	OUB	OG(%)	CPUsec	NI	FUB	OUB	OG(%)	CPUsec
2	3	5	20	20	20	0.00	1.95	20	20	20	0.00	947.99
3	5	5	20	20	20	0.00	1.64	20	20	20	0.00	1090.95
6	4	5	20	20	20	0.00	1.58	20	20	20	0.00	917.45
2	3	10	20	20	20	0.00	24.94	20	20	20	0.00	3502.23
3	5	10	20	20	20	0.00	176.99	20	20	20	0.00	3568.11
6	4	10	20	20	20	0.00	112.46	20	20	20	0.00	3948.30
2	3	15	20	20	20	0.00	238.17	20	20	4	17.86	15741.92
3	5	15	20	20	20	0.00	374.10	20	20	3	17.91	16296.16
6	4	15	20	20	20	0.00	545.73	20	20	0	27.43	18000.00
2	3	30	20	20	12	4.03	14506.63	20	20	0	36.45	18000.00
3	5	30	20	20	7	4.64	15381.70	20	20	1	33.23	17430.12
6	4	30	20	20	7	4.07	16156.92	20	20	0	39.07	18000.00
2	3	50	20	20	0	15.06	28800.00	20	20	0	47.57	28800.00
2	3	50	20	20	0	24.90	28800.00	20	0	0	N.A	28800.00
2	3	50	20	20	0	14.41	28800.00	20	0	0	N.A	28800.00

Table 3: Performance of DDD solution method for 5, 10, 15, 30, and 50 OD demands

constraints allows the model to prevent the degeneracy of the first-echelon and part of the second-echelon arcs and reduce the growth of the underlying network. This in turn accelerates the convergence of the H-DDD to optimal solutions.

Degeneracy is still present on the customer side of the problem, however, as the method relies on the time-space representation to derive a feasible integer solution, which reduces the lower-bound increase rate at each iteration. The proposed degeneracy procedure behaves as expected and considerably lowers the impact of degeneracy on the performance of the proposed DDD, especially for instances with broad time windows (which thus provide more scheduling-decision alternatives). Our analysis tends to show, however, that the degeneracy procedure primarily contributes to attaining good upper bounds faster, rather than improving the lower bound at a fast rate (see the supplementary material Appendix C). Consequently, while the proposed DDD solution method yields provably high-quality upper bounds for larger instances, the slow incremental rate of the lower bounds makes assessing the true quality of the solution more difficult, in particular as the number of ODs increase. This is illustrated on instances with 50 ODs, where the optimality of most solutions obtained by the H-DDD remains unproven.

6.4 Sensitivity analysis

We performed a sensitivity analysis of two main time-related components of the 2E-MALRPS, the availability times of demands and the synchronization at satellites, with respect to the behaviour of the proposed H-DDD solution method. These two characteristics directly determine the degree of tightness of vehicle operations on both echelons and, thus, the performance of the system in servicing the end customers. Our goal is to study the impact of these interacting attributes and gain insights into the system behavior in less time-sensitive environments. The study was performed on a subset of instances with 5, 10, and 15 ODs that the H-DDD solves to optimality (reducing the noise due to optimality gaps). Tables 4 and 5 summarize the results of the experiments performed with H-DDD using the coarsest discretization granularity ($\Delta = 2$) possible.

$\lvert\mathcal{P}^{\mathbf{ph}}\rvert$	$\lvert\mathcal{Z}^{\mathbf{ph}}\rvert$	OD	NI	FUB	OUB	OG(%)	CPUsec
2	3	5	20	20	20	0.00	0.77
3	5	5	20	20	20	0.00	2.71
6	4	5	20	20	20	0.00	6.52
2	3	10	20	20	20	0.00	30.13
3	5	10	20	20	20	0.00	260.04
6	4	10	20	20	20	0.00	573.43
2	3	15	20	20	20	0.00	102.23
3	5	15	20	20	20	0.00	993.43
6	4	15	20	20	20	0.00	1729.48

Table 4: H-DDD performance on instances without availability times

$\lvert\mathcal{P}^{\mathbf{ph}}\rvert$	$\lvert\mathcal{Z}^{\mathbf{ph}}\rvert$	OD	NI	FUB	OUB	OG(%)	CPUsec
2	3	5	20	20	20	0.00	0.74
3	5	5	20	20	20	0.00	3.72
6	4	5	20	20	20	0.00	9.89
2	3	10	20	20	20	0.00	1015.592
3	5	10	20	20	19	0.17	1770.973
6	4	10	20	20	15	1.90	4075.948
2	3	15	20	20	20	0.00	124.01
3	5	15	20	20	17	0.50	1557.42
6	4	15	20	20	18	0.68	3931.11

Table 5: H-DDD performance on instances without availability times and synchronization

The figures in Table 4 show that the H-DDD algorithm is able to provide good quality solutions, even when the demand availability time is not binding or is not important/considered in the problem setting. The H-DDD achieves optimality for all instances considered with or without demand availability times. In terms of computational efficiency, the solution method presents a better performance when availability times are present, finding the optimal solution some 60% faster than when these temporal elements are not considered. The lower performance in this latter case follows from the absence of precise time moments (due the larger number of availability options) which results in larger solution spaces and more complex integer problems at each iteration of the H-DDD. This impacts the quality of lower bounds and increases the runtime needed to achieve optimality (see Section 6.3). It is worth mentioning that the multi-commodity aspect limits the performance improvement when demand availability time is considered. In single commodity cases, prioritizing the earliest availability times for each platform allows for greater flexibility in synchronizing satellite operations and customer arrivals. In the multi-commodity case with multiple availability times for different OD demands, however, flexibility in other temporal aspects of the system is limited (see the supplementary material Appendix D).

Note that, tight availability times at the origin of demand and (narrow) time windows at destination imposes a certain *de facto* level of synchronization, in particular for small instances. Hence, to better study the impact of synchronization requirements on the H-DDD performance, we disabled both synchronization and availability times. Table 5 sums up the results of this analysis.

The results indicate that omitting also synchronization increases significantly the computation times, not only with respect to the full problem setting (by some 70%), but also compared to when availability times are disabled only. This follows from the fact that the lack of significant temporal constraints delimiting the area of relevance of satellite facilities yields a large number of second-echelon route alternatives and, thus, the generation of more degenerate solutions. The even larger performance degradation for the 10-OD instances comes from the additional impact of large time windows on the behaviour just explained.

In terms of cost, solutions on instances with disabled availability times with/without disabled synchronization are cheaper than solutions on instances with all temporal aspects enabled, with an average cost reduction of some 4% and 12%, respectively. This reduction in operational costs can be attributed to a series of low-cost routes that can only be used at certain times of the schedule length, but are unavailable when time limits are tight. Moreover, disabling the availability times opens up a lot more possibilities for demand consolidation at satellite facilities. The system fixed costs related to facility usage are reduced as a result of the higher level of consolidation, which results in a smaller number of facilities selected. This is even more significant for instances with late availability times and early due times (e.g., the instance types CA and CB), where a larger number of adjacent platforms and satellites must be opened and connected to satisfy demand on time.

7 Conclusions and Perspectives

This paper introduces the multi-attribute two-echelon location-routing problem with synchronization constraints, MA-2ELRPS, and presents a mixed-integer programming formulation on a hybrid time-space network combining continuous and discrete time representations We also present an exact solution method that iteratively refines a reduced time-space network, solving the MA-2ELRPS formulation defined on the reduced network to extract bounds and temporal granularity refinements, in order to guide the method towards to optimal solution of the original problem. The paper generalizes the dynamic discretization discovery method to complex problem settings involving several levels of location, routing, and synchronization decisions.

The computational study demonstrates the effectiveness of the mathematical formulation and the DDD solution method. Comparative analyses reveal that the proposed hybrid formulation outperforms a classic time-space formulation (which only considers discrete time modelling) in all performance measures. This superiority is observed when utilizing a commercial solver directly as well as when applying the DDD solution method. The results highlight the effectiveness of the proposed DDD in handling large instances, achieved through the degeneracy mitigation procedure and the integration of discrete and continuous-time representations to maintain the accuracy of time-related decisions.

To conclude, it is observed that neglecting availability times and synchronization requirements in time-driven applications can result in an inaccurate depiction of the distribution system. Sensitivity to time is evident, as lower distribution costs do not always guarantee feasible operations due to the inclusion of routes that may be infeasible when availability times or synchronization are not taken into account. The hybrid MA-2ELRPS formulation and DDD solution method offer high-quality solutions for decision-makers and improve performance for time-critical applications. Future research can focus on heuristic methods and focused optimization models to enhance the convergence of the DDD by improving upper and/or lower bounds in each iteration, as well as investigating the impact of variations in the properties defining the reduced network.

Acknowledgments

Partial funding for this project was provided by the Natural Sciences and Engineering Council of Canada (NSERC) through its Discovery Grants Program. We also thank the Centre Sur l'Intelligence^2 en Gestion de Systèmes complexes (CRI^{2}GS), ESG UQAM, for partial funding through the Excellence Student Fellowship program. While working on this paper, the second author was Adjunct Professor, Department of Computer Science and Operations Research, Université de Montréal, Montréal, Canada. The authors are grateful to Ms. Diane Larin for her invaluable linguistic and editorial assistance.

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A Mathematical Proofs of Lemmas of Section 5.1

Lemma 1. Let $\mathcal{G}_\Delta$ be a reduced time-space network that satisfies properties 1, 2, 3, and 4. Then, for each commodity $k \in \mathcal{K}$ and itinerary $r = (v_i, t_i)_{i=1}^{l} \in R_{\mathcal{G}}^k$, there exists an itinerary $r' = (v_i, t'_i)_{i=1}^{l} \in R_{\mathcal{G}_\Delta}^k$ such that $t'_i \leq t_i$ for every $i = 1 \dots l$.

Proof Lemma 1. We conduct the proof by induction on i for the itinerary $r = (v_i, t_i)_{i=1}^{l}$ in $R_{\mathcal{G}}^k$. For $i = 1$, let $t'_1 = a_{v_1}$ be the temporal lower bound of v_1. By Property 1, we have that the time-space node (v_1, t'_1) with $t'_1 = a_{v_1}$ yields to $t'_1 \leq t_1$, as there is no $(v_1, t''_1) \in \mathcal{G}_\Delta$ with $t''_1 < a_{v_1}$. From the time-space node (v_1, t'_1), we can map the remainder of the itinerary $r = (v_i, t_i)_{i=1}^{l} \in R_{\mathcal{G}}^k$ by defining an *equivalent* time-space node (v_i, t'_i) in $\mathcal{G}_\Delta$, for each $(v_i, t_i) \in R_{\mathcal{G}}^k$ with $t'_i = argmax\{d \in \mathcal{T}_i(\Delta) \mid d \leq t_i\}$. By Property 2, there is an arc $((v_i, t'_i), (v_{i+1}, t'_{i+1})) \in \mathcal{A}_\Delta$ with $t'_{i+1} \leq t'_i + \tau_{v_i v_{i+1}} \leq t_i + \tau_{v_i v_{i+1}}$, while Property 3 and 4, enables early waiting times at satellites as well early arrival at customers, respectively.

Assuming that $i = w$ is true, we can prove that our condition holds for $i = w + 1$. Hence, by the inductive assumption, there is an itinerary $r = [(v_1, t'_1), (v_2, t'_2), ..., (v_w, t'_w)]$ with $(v_w, t'_w) \in \mathcal{G}_\Delta$ and $t'_w \leq t_w$. By Property 1, the set of integer time points $\mathcal{T}_{v_i(\Delta)}$ representing the time moments at which node $i = w + 1$ becomes relevant must exist in $\mathcal{V}_\Delta$. By Property 2, 3 and 4, arc $((v_w, t'_w), (v_{w+1}, t'_{w+1})) \in \mathcal{A}_\Delta$ with $t'_{w+1} \leq t'_w + \tau_{v_w v_{w+1}} \leq t_w + \tau_{v_w v_{w+1}}$. By Property 3, there must exist a waiting time at satellites with a lesser of equal value to the original waiting time, while Property 4, ensures that there is an early arrival point in time for customers. Consequently, we can ensure that the defined conditions can be verified for each connection within $r = (v_i, t_i)_{i=1}^{l} \in R_{\mathcal{G}}^k$, and thus for each itinerary in $R_{\mathcal{G}}$. □

Lemma 2. If a reduced time-space network $\mathcal{G}_\Delta$ satisfies properties 1, 2, 3, and 4, then the optimal solution of the 2E-MALRPS formulated on the reduced time-space network $\mathcal{G}_\Delta$ is a lower bound for the solution of the 2E-MALRPS on the complete time-space network $\mathcal{G}$.

Proof Lemma 2. To prove this lemma, we will show that each time-space arc representing the optimal solution for the 2E-MALRPS in a complete time-space network, can be mapped onto a reduced time-space network, with an equal or lesser operational cost.

Consider $Z_{\mathcal{G}}^* = (x_{\mathcal{G}}^*, f_{\mathcal{G}}^*, y_{\mathcal{G}}^*)$ an optimal integer solution of the 2E-MALRPS in a complete time-space network $(\mathcal{G})$, with $A_{\mathcal{G}}^* = \{((v_i, t_i), (v_j, t_j)) \in \mathcal{A}_{\mathcal{G}} \mid x_{(v_i,t_i),(v_j,t_j)} = 1\}$. Let $R_{\mathcal{G}}$ be the set of itineraries $r \in R_{\mathcal{G}}$ dispatching each commodity $k \in \mathcal{K}$ from its origin $O(k)$ to its destination $D(k)$ throughout the system with the arcs in $A_{\mathcal{G}}^*$. In what follows, we will show that each arc in $A_{\mathcal{G}}^*$ can be mapped to a unique set of arcs $A_{\mathcal{G}_\Delta}$ of a reduced time-space network $(\mathcal{G}_\Delta)$, so we can construct $Z_{\mathcal{G}_\Delta} = (x_{\mathcal{G}_\Delta}, f_{\mathcal{G}_\Delta}, y_{\mathcal{G}_\Delta})$ in respect to each arc in $A_{\mathcal{G}_\Delta}$.

By Lemma 1, for each arc $((v_i, t_i)(v_j, t_j)) \in A^*_{\mathcal{G}}$ in $R_{\mathcal{G}}$ (excluding garages connections), there exists $(v_i, t'_i) \in \mathcal{G}_\Delta$ with $t'_i \leq t_i$ and a t'_j such that $((v_i, t'_i)(v_j, t'_j)) \in A_{\mathcal{G}_\Delta}$. Hence, for each itinerary $r = (v_i, t_i)_{i=1}^{l} \in R_{\mathcal{G}}$, there is an *equivalent* itinerary $r' = (v_i, t'_i)_{i=1}^{l} \in R_{\mathcal{G}_\Delta}$ such that $t'_i \leq t_i$. Because the number of both platform and satellite facilities must hold for each $r \in R_{\mathcal{G}}$ mapped to $\mathcal{G}_\Delta$, we have that $y_{\mathcal{G}_\Delta} = y^*_{\mathcal{G}}$. Now we can track each commodity flow from its origin to its destination in $R_{\mathcal{G}_\Delta}$ to derive both routing and flow decisions to $x_{\mathcal{G}_\Delta}$ and $f_{\mathcal{G}_\Delta}$. Notice that by Lemma 1 and Property 3, fleet synchronization within each $r \in R_{\mathcal{G}_\Delta}$ holds, but takes place at the same or earlier point in time on the same satellite.

Recall that every route in the first and second echelon must start and end at a vehicle garage. We have that for each path $r \in R_{\mathcal{G}}$, every origin, satellite serving as the transfer point and destination of each commodity $k \in \mathcal{K}$ are known. Notice that, for some of these time-space nodes, there exists an unique time-space arc in $A^*_{\mathcal{G}}$ that represents the leg used for each route as the *start point* after a vehicle leaves the garage or the *end point* before the vehicle returns to the garage. By Lemma 1 and Properties 1 and 2, there must exist a time-space node $(e_1, t'_{e_1}) \in \mathcal{E}^1_\Delta$ and $(e_2, u'_{e_2}) \in \mathcal{E}^2_\Delta$, such that $((z, t'_z), (e_1, t'_{e_1}))$ and $((c, t'_c), (e_2, t'_{e_2}))$ exists in $A_{\mathcal{G}_\Delta}$ for each *end point* at the first and second echelon, with (z, t'_z) and (c, t'_c) in $R_{\mathcal{G}_\Delta}$, $z \in \mathcal{Z}^{\text{ph}}$ and $c \in \mathcal{C}^{\text{ph}}$. Similarly, there are time-space nodes $(e'_1, t'_{e'_1}) \in \mathcal{E}^1_\Delta$ and $(e'_2, u'_{e'_2}) \in \mathcal{E}^2_\Delta$, such that $((e'_1, t'_{e'_1}), (p, t'_p))$ and $((e'_2, t'_{e'_2}), (z', t''_{z'}))$ exists in $A_{\mathcal{G}_\Delta}$ for each *start point* at the first and second echelon, with (p, t'_p) and $(z', t''_{z'})$ in $R_{\mathcal{G}_\Delta}$, $p \in \mathcal{P}^{\text{ph}}$ and $z' \in \mathcal{Z}^{\text{ph}}$. Thus, we can then derive the routing decisions to $x_{\mathcal{G}_\Delta}$ for the resulting inbound and outbound for each garage.

Observe that, for a given reduced network $\mathcal{G}_\Delta$, there is a unique set of time-space arcs that satisfy properties 1 and 2. This implies that first-echelon routes mapped from the complete time-space network would have the same cost but reach each destination in the reduced time-space network at the same or an earlier time moment. By property 3, we know that waiting times follow a similar behavior, wherein vehicles can wait until the same or a lesser time moment than the one defined in the complete time-space network. Furthermore, property 2 guarantees that there are no time-space arcs $((i, t), (j, t'))$ and $((i, t), (j, t"))$ where $t' < t"$. Consequently, for each time-space node $(i, t) \in \mathcal{V}_\Delta$ and each $(i, j) \in \mathcal{A}^{ph}$, there can be at most one time-space arc of the form $((i, t), (j, t'))$ that satisfies properties 1 and 2. Combining properties 2 and 3, it follows that $t' = \text{argmax}\{d \mid d \leq t + \tau_{ij}, (i, j) \in \mathcal{A}^{ph}, (j, d) \in \mathcal{V}_\Delta\}$. Therefore, for each time moment in which fleet synchronization takes place in the complete time-space network, there exists a time moment with an equal or lesser time value in the reduced network.

Now, the solution $Z_{\mathcal{G}_\Delta} = (x_{\mathcal{G}_\Delta}, f_{\mathcal{G}_\Delta}, y_{\mathcal{G}_\Delta})$ constructed in this way is feasible for the 2E-MALRPS onto the reduced time-space network ($\mathcal{G}_\Delta$) while routing and location costs holds. Therefore, we have that $Z_{\mathcal{G}_\Delta}$ is identical to $Z^*_{\mathcal{G}}$ with arrival and departure times taking earlier or equal values than the ones on the complete time-space network, but with the same operational cost. Consequently, the optimal solution for the 2E-MALRPS on a reduced time-space network ($\mathcal{G}_\Delta$) is a lower bound of the optimal solution obtained in a complete time-space network ($\mathcal{G}$). □

Lemma 3. The proposed DDD algorithm ends with an optimal solution for the MA-2ELRPS.

Proof Lemma 3.

The DDD algorithm terminates when the optimal integer solution of the hybrid formulation HTF($\mathcal{G}_\Delta$) on a reduced time-space network $\mathcal{G}_\Delta$ can be mapped onto a complete time-space network with the same cost. By Lemma 2, the solutions derived from HTF($\mathcal{G}_\Delta$) are lower bounds for the 2E-MALRPS on the complete time-space network $\mathcal{G}$. Therefore, the solution obtained from HTF($\mathcal{G}_\Delta$), which remains feasible when mapped to the complete time-space network, is the optimal solution for the MA-2ELRPS.

During each iteration of the DDD algorithm, there may be situations where the solution obtained by HTF($\mathcal{G}_\Delta$) is not feasible for the MA-2ELRPS when evaluated using the original travel times, due to the presence of at least one short arc in the solution. When such a situation occurs, a refinement procedure is applied, which extends these short arcs found in the solution while ensuring that Properties 1-4 remain valid at the nodes involved in the extension. Given that the system comprises a finite number of time points and arcs, the DDD algorithm will eventually reach an iteration where all arcs in the reduced network have travel times corresponding to the actual travel time of the system. In this specific scenario, the solution obtained from HTF($\mathcal{G}_\Delta$) effectively satisfies the temporal requirements of the 2E-MALRPS and leads to the optimal solution, ultimately resulting in the termination of the algorithm.

B Complete Result Tables

Tables 6 - 11 showcase the results of the proposed formulations under different granularity values. Each table display the comparative performance results on the complete time-space network ($\Delta = \bar{\Delta}$) and the reduced time-space networks with granularity values $\Delta = 50$ and $\Delta = 25$ with a time limit of 2.5 hours. Table 6, Table 8 and Table 10 presents experiments of the hybrid formulation with different granularity values on instances with 5, 10 and 15 OD, respectively. Results presented in Table 7, Table 9 and Table 11 extend the same experiments using the standalone time-space formulation. The tables display the instance **ID**, the schedule length (Ψ), the best upper bound (**UB**), the run-time (CPUsec), the lower bound (**LB**), the root gap (**RG(%)**), and cost increment percentage (**Dif UB**).

Tables 12 - 17 display the detailed results of the set of experiments focusing on the performance of the dynamic discretization discovery (DDD) solution algorithm for the 2E-MALRPS. Test results are shown for the instances with 5, 10 and 15 OD demands in Table 12. Results for the same instances with disabled availability times in Table 16 as well as disabled availability times and synchronization in Table 17. The experiments are performed using a coarse discretization granularity $\Delta = 2$. The stopping criteria are an optimality gap of less or equal to 1% and a maximum run time of 2.5 hours for small-sized instances (5 and 10 OD demands), 5 hours for instances with 30 OD demands and 8 hours for instances with 50 OD demands. The tables display the instance **ID**, the schedule length (Ψ), the best upper bound (**UB**), the run-time (CPUsec), the lower bound (**LB**), and the optimality gap (**OG(%)**).

ID	Ψ	Hybrid formulation ($\Delta = \bar{\Delta}$)				Hybrid formulation ($\Delta = 50$)			Hybrid formulation ($\Delta = 25$)		
		UB	**CPU (sec)**	**LB**	**RG (%)**	**UB**	**CPU (sec)**	**Dif. Ub (%)**	**UB**	**CPU (sec)**	**Dif. Ub (%)**
Ca1-2,3,5	100	280	1.73	191.38	31.65	280	1.05	0.00	285	0.75	1.61
Ca1-3,5,5	100	222	82.53	199.00	10.36	222	32.55	0.00	223	22.52	0.54
Ca1-6,4,5	100	271	328.45	213.03	21.39	282	73.03	3.87	282	70.75	3.87
Ca2-2,3,5	100	152	7.81	132.86	12.59	152	5.53	0.00	152	3.29	0.13
Ca2-3,5,5	100	284	9.16	216.74	23.68	284	6.70	0.00	293	3.68	3.10
Ca2-6,4,5	100	150	87.36	112.00	25.33	156	67.39	4.00	156	24.61	4.13
Ca3-2,3,5	100	287	8.47	238.33	16.96	299	1.64	4.14	299	1.48	4.15
Ca3-3,5,5	100	220	72.43	173.73	21.03	224	29.99	2.00	224	29.54	2.00
Ca3-6,4,5	100	171	42.30	164.50	3.80	173	39.66	1.29	173	38.72	1.29
Ca4-2,3,5	100	358	6.66	252.23	29.55	358	6.32	0.00	374	0.84	4.27
Ca4-3,5,5	100	168	9.36	128.78	23.35	170	5.06	1.19	170	0.44	1.37
Ca4-6,4,5	100	161	99.82	125.28	22.19	164	43.65	1.98	164	25.31	1.99
Ca5-2,3,5	100	199	39.11	189.30	4.87	200	19.69	0.25	202	1.46	1.31
Ca5-3,5,5	100	186	134.80	161.93	12.94	186	104.69	0.00	186	96.60	0.22
Ca5-6,4,5	100	159	2620.27	152.67	3.98	165	1765.59	3.65	167	25.92	4.84
Cb1-2,3,5	100	152	2.49	140.00	7.89	153	1.83	0.39	153	1.20	0.53
Cb1-3,5,5	100	164	645.87	117.74	28.21	164	629.06	0.00	165	1.63	0.30
Cb1-6,4,5	100	305	581.10	186.03	39.01	305	412.05	0.00	311	5.93	1.87
Cb2-2,3,5	100	129	1.75	117.95	8.57	131	1.61	1.86	131	0.44	1.86
Cb2-3,5,5	100	154	4.12	134.00	12.99	154	2.04	0.00	155	1.46	0.84
Cb2-6,4,5	100	143	454.68	117.53	17.81	143	25.37	0.00	149	14.55	4.27
Cb3-2,3,5	100	332	0.38	248.84	25.05	343	0.37	3.10	343	0.37	3.10
Cb3-3,5,5	100	160	52.57	129.00	19.37	160	40.04	0.00	161	6.86	0.62
Cb3-6,4,5	100	198	158.15	141.85	28.36	201	54.56	1.46	204	8.71	2.78
Cb4-2,3,5	100	280	9000.00	171.05	38.91	280	2047.83	0.00	286	4.73	2.21
Cb4-3,5,5	100	142	27.91	122.00	14.08	142	20.69	0.28	144	13.48	1.34
Cb4-6,4,5	100	188	193.07	136.05	27.63	192	173.47	1.97	195	22.69	3.56
Cb5-2,3,5	100	129	17.97	115.31	10.61	130	12.31	0.78	131	1.82	1.32
Cb5-3,5,5	100	179	429.57	139.65	21.98	179	131.93	0.00	180	16.65	0.50
Cb5-6,4,5	100	199	44.91	137.67	30.82	206	41.15	3.37	207	35.11	3.82
Cc1-2,3,5	100	129	9000.00	114.69	11.10	129	3934.41	0.00	131	163.17	1.24
Cc1-3,5,5	100	135	9000.00	109.99	18.53	136	5153.75	0.89	136	261.11	0.89
Cc1-6,4,5	100	150	9000.00	108.20	27.86	150	5115.29	0.00	155	460.03	3.40
Cc2-2,3,5	100	122	9000.00	99.62	18.34	122	8764.65	0.00	122	21.64	0.00
Cc2-3,5,5	100	175	9000.00	120.46	31.17	175	3660.59	0.23	176	3355.92	0.29
Cc2-6,4,5	100	140	9000.00	114.84	17.97	140	7960.76	0.00	144	4019.37	2.86
Cc3-2,3,5	100	136	9000.00	116.66	14.22	136	2030.28	0.00	137	513.69	0.37
Cc3-3,5,5	100	142	9000.00	128.80	9.30	142	7102.57	0.00	144	323.32	1.06
Cc3-6,4,5	100	157	9000.00	110.13	29.86	157	5963.72	0.00	161	1371.77	2.48
Cc4-2,3,5	100	171	9000.00	120.69	29.42	171	2234.24	0.00	171	546.19	0.23
Cc4-3,5,5	100	154	9000.00	114.87	25.41	154	7494.61	0.19	154	2255.48	0.26
Cc4-6,4,5	100	138	9000.00	122.31	11.37	138	7090.72	0.00	142	2201.86	3.04
Cc5-2,3,5	100	123	9000.00	114.54	6.88	124	1568.97	0.81	124	151.12	0.89
Cc5-3,5,5	100	124	9000.00	104.60	15.64	124	4568.95	0.00	124	3832.12	0.00
Cc5-6,4,5	100	138	9000.00	119.21	13.61	138	6978.99	0.00	144	2600.04	3.84
Cd1-2,3,5	100	155	1140.22	136.21	12.13	155	239.64	0.00	156	29.06	0.90
Cd1-3,5,5	100	170	24.13	140.45	17.38	170	23.17	0.18	171	12.65	0.53
Cd1-6,4,5	100	188	870.99	127.01	32.44	188	407.19	0.00	191	198.22	1.70
Cd2-2,3,5	100	140	9000.00	127.24	9.11	142	2619.41	1.21	145	30.92	3.29
Cd2-3,5,5	100	158	1707.02	141.04	10.73	158	857.78	0.00	159	1204.89	0.57
Cd2-6,4,5	100	142	692.04	100.52	29.21	142	478.78	0.34	148	152.25	3.73
Cd3-2,3,5	100	142	9000.00	119.98	15.51	142	1312.74	0.28	145	89.65	1.83
Cd3-3,5,5	100	147	9000.00	118.75	19.22	148	2994.43	0.95	149	1324.92	1.09
Cd3-6,4,5	100	157	133.33	125.88	19.82	160	47.84	1.72	162	47.06	3.31
Cd4-2,3,5	100	202	9000.00	172.63	14.54	204	4954.89	0.84	204	102.22	1.04
Cd4-3,5,5	100	162	9000.00	138.26	14.65	163	3916.45	0.31	164	552.18	1.11
Cd4-6,4,5	100	147	2073.23	109.56	25.47	147	1149.21	0.00	151	397.82	2.38
Cd5-2,3,5	100	178	9000.00	160.73	9.70	179	2681.85	0.50	179	56.07	0.56
Cd5-3,5,5	100	178	9000.00	148.29	16.69	178	4878.72	0.00	178	442.24	0.22
Cd5-6,4,5	100	157	9000.00	138.29	11.92	163	2445.97	3.95	165	549.26	4.65
Averages			***3813.43***		***18.90***		***1907.12***	***0.80***		***462.46***	***1.86***

Table 6: Direct solving the hybrid formulation on instances with 5 OD demands and multiple Δ time periods

ID	Ψ	Time-Space formulation ($\Delta = \bar{\Delta}$)				Time-Space formulation ($\Delta = 50$)			Time-Space formulation ($\Delta = 25$)		
		UB	CPU (sec)	LB	RG (%)	UB	CPU (sec)	Dif. Ub (%)	UB	CPU (sec)	Dif. Ub (%)
Ca1-2,3,5	100	280	0.14	168.08	39.97	287	0.06	2.50	286	0.01	2.10
Ca1-3,5,5	100	222	0.88	190.89	14.01	225	0.48	1.35	224	0.47	0.89
Ca1-6,4,5	100	271	0.37	209.51	22.69	281	0.32	3.69	282	0.22	3.90
Ca2-2,3,5	100	152	0.27	120.73	20.57	152	0.27	0.00	153	0.26	0.65
Ca2-3,5,5	100	284	0.15	173.40	38.95	294	0.14	3.52	295	0.14	3.73
Ca2-6,4,5	100	150	1.26	90.82	39.45	156	1.23	4.00	157	0.71	4.46
Ca3-2,3,5	100	287	0.05	235.01	18.12	299	0.05	4.18	301	0.05	4.65
Ca3-3,5,5	100	220	0.46	160.73	26.94	224	0.28	1.82	225	0.27	2.22
Ca3-6,4,5	100	171	0.64	120.30	29.65	173	0.56	1.17	174	0.33	1.72
Ca4-2,3,5	100	358	0.07	189.88	46.96	373	0.09	4.19	376	0.09	4.79
Ca4-3,5,5	100	168	0.42	128.26	23.66	170	0.30	1.19	171	0.28	1.75
Ca4-6,4,5	100	161	1.55	123.71	23.16	164	1.48	1.86	165	1.45	2.42
Ca5-2,3,5	100	199	0.48	175.45	11.84	199	0.36	0.00	202	0.18	1.49
Ca5-3,5,5	100	186	0.32	120.45	35.24	189	0.27	1.61	187	0.19	0.53
Ca5-6,4,5	100	159	54.79	91.36	42.54	165	54.65	3.77	168	54.59	5.36
Cb1-2,3,5	100	152	0.23	131.27	13.64	152	0.12	0.00	153	0.16	0.65
Cb1-3,5,5	100	164	1.98	116.89	28.72	167	1.62	1.83	165	1.51	0.61
Cb1-6,4,5	100	305	0.31	178.02	41.63	314	0.24	2.95	312	0.14	2.24
Cb2-2,3,5	100	129	0.14	117.44	8.96	131	0.12	1.55	132	0.14	2.27
Cb2-3,5,5	100	154	0.22	133.35	13.41	155	0.17	0.65	156	0.11	1.28
Cb2-6,4,5	100	143	1.04	105.47	26.25	149	0.86	4.20	150	0.78	4.67
Cb3-2,3,5	100	332	0.06	236.65	28.72	342	0.04	3.01	344	0.00	3.49
Cb3-3,5,5	100	160	0.52	128.32	19.80	161	0.23	0.63	162	0.19	1.23
Cb3-6,4,5	100	198	0.35	136.20	31.21	201	0.23	1.52	204	0.16	2.94
Cb4-2,3,5	100	280	9000	161.05	42.48	290	1642.28	3.57	288	19.91	2.78
Cb4-3,5,5	100	142	0.82	118.46	16.58	142	0.61	0.00	144	0.47	1.39
Cb4-6,4,5	100	188	0.64	129.68	31.02	191	0.62	1.60	196	0.30	4.08
Cb5-2,3,5	100	129	0.47	111.73	13.38	130	0.43	0.78	131	0.07	1.53
Cb5-3,5,5	100	179	0.71	131.95	26.28	181	0.57	1.12	180	0.46	0.56
Cb5-6,4,5	100	199	0.43	136.02	31.65	205	0.41	3.02	208	0.35	4.33
Cc1-2,3,5	100	129	9000	112.13	13.08	130	4854.27	0.78	131	35.47	1.53
Cc1-3,5,5	100	135	3622.1	109.11	19.18	136	2544.38	0.74	137	22.82	1.46
Cc1-6,4,5	100	150	9000	81.69	45.54	155	3838.39	3.33	156	19.61	3.85
Cc2-2,3,5	100	122	51	99.08	18.79	123	50.91	0.82	123	50.75	0.81
Cc2-3,5,5	100	175	9000	119.59	31.66	175	4816.26	0.00	176	9.56	0.57
Cc2-6,4,5	100	140	9000	107.85	22.96	144	2401.17	2.86	145	84.68	3.45
Cc3-2,3,5	100	136	208.75	115.10	15.37	136	186.73	0.00	137	25.37	0.73
Cc3-3,5,5	100	142	3906.78	127.89	9.94	144	823.62	1.41	144	30.25	1.39
Cc3-6,4,5	100	157	9000	109.02	30.56	160	3907.48	1.91	161	9.49	2.48
Cc4-2,3,5	100	171	3160.12	120.89	29.31	171	1291.09	0.00	172	14.53	0.58
Cc4-3,5,5	100	154	9000	114.98	25.34	154	4762.04	0.00	155	259.42	0.65
Cc4-6,4,5	100	138	9000	88.36	35.97	142	335.82	2.90	142	5.56	2.82
Cc5-2,3,5	100	123	9000	114.32	7.06	124	2641.20	0.81	125	75.83	1.60
Cc5-3,5,5	100	124	246.22	103.09	16.86	124	142.84	0.00	124	20.03	0.00
Cc5-6,4,5	100	138	9000	104.58	24.22	143	7828.48	3.62	144	89.96	4.17
Cd1-2,3,5	100	155	1.31	134.59	13.17	155	1.27	0.00	157	1.25	1.27
Cd1-3,5,5	100	170	0.75	139.12	18.16	170	0.74	0.00	172	0.56	1.16
Cd1-6,4,5	100	188	99.91	121.25	35.50	191	99.41	1.60	191	99.38	1.57
Cd2-2,3,5	100	140	1.72	126.35	9.75	141	1.24	0.71	145	1.19	3.45
Cd2-3,5,5	100	158	6.96	136.47	13.62	158	6.84	0.00	160	6.83	1.25
Cd2-6,4,5	100	142	25.38	94.76	33.27	147	25.21	3.52	148	25.12	4.05
Cd3-2,3,5	100	142	3.05	117.30	17.40	142	3.01	0.00	145	2.93	2.07
Cd3-3,5,5	100	147	11.45	118.29	19.53	148	11.28	0.68	149	11.27	1.34
Cd3-6,4,5	100	157	600.47	121.23	22.78	159	282.06	1.27	163	20.45	3.68
Cd4-2,3,5	100	202	0.58	168.87	16.40	203	0.53	0.50	205	0.41	1.46
Cd4-3,5,5	100	162	1.7	137.76	14.96	162	1.70	0.00	165	1.69	1.82
Cd4-6,4,5	100	147	156.39	108.88	25.93	150	113.33	2.04	151	4.87	2.65
Cd5-2,3,5	100	178	1.13	158.80	10.79	178	0.84	0.00	180	0.72	1.11
Cd5-3,5,5	100	178	5.14	145.71	18.14	179	5.07	0.56	179	4.95	0.56
Cd5-6,4,5	100	157	191.79	134.67	14.22	163	146.44	3.82	165	84.29	4.85
Averages			***1706.21***		***23.95***		***713.89***	***1.59***		***18.40***	***2.22***

Table 7: Direct solving the classic time-space formulation on instances with 5 OD demands and multiple Δ time periods

ID	Ψ	Hybrid Formulation ($\Delta = \bar{\Delta}$)				Hybrid Formulation ($\Delta = 50$)			Hybrid Formulation ($\Delta = 25$)		
		UB	CPU (sec)	LB	RG (%)	UB	CPU (sec)	Dif. Ub (%)	UB	CPU (sec)	Dif. Ub (%)
Ca1-2,3,10	100	256	1683.71	225.71	11.83	258	732.01	0.78	261	17.53	1.92
Ca1-3,5,10	100	215	9000	181.20	15.72	216	3640.05	0.46	217	252.25	0.92
Ca1-6,4,10	100	305	9000	266.85	12.51	320	1605.60	4.69	320	707.46	4.69
Ca2-2,3,10	100	161	1904.23	126.71	21.30	162	1278.83	0.62	162	32.92	0.62
Ca2-3,5,10	100	340	6601.6	289.82	14.76	355	3600.04	4.23	356	768.02	4.49
Ca2-6,4,10	100	179	8211.97	149.64	16.40	182	4260.06	1.65	182	460.61	1.65
Ca3-2,3,10	100	344	7.26	303.32	11.82	363	3.56	5.23	363	1.81	5.23
Ca3-3,5,10	100	318	9000	274.63	13.64	333	4391.81	4.50	333	295.38	4.50
Ca3-6,4,10	100	236	9000	184.54	21.80	240	3018.73	1.67	240	787.23	1.67
Ca4-2,3,10	100	437	48.51	408.79	6.46	462	27.39	5.41	462	8.35	5.41
Ca4-3,5,10	100	213	9000	166.95	21.62	220	3908.61	3.18	220	994.43	3.18
Ca4-6,4,10	100	238	9000	204.74	13.97	243	4560.37	2.06	243	1530.76	2.06
Ca5-2,3,10	100	277	9000	245.16	11.49	282	819.25	1.77	284	14.58	2.46
Ca5-3,5,10	100	264	9000	240.84	8.77	272	3829.65	2.94	275	965.99	4.00
Ca5-6,4,10	100	187	9000	135.19	27.71	190	3427.89	1.58	192	592.42	2.60
Cb1-2,3,10	100	181	419.36	155.28	14.21	183	195.68	1.09	183	41.96	1.09
Cb1-3,5,10	100	211	9000	179.52	14.92	215	1429.47	1.86	219	1062.87	3.65
Cb1-6,4,10	100	261	9000	227.46	12.85	262	2925.71	0.38	267	1593.28	2.25
Cb2-2,3,10	100	199	1184.1	163.46	17.86	204	618.57	2.45	204	14.39	2.45
Cb2-3,5,10	100	268	3120.03	215.18	19.71	278	1980.90	3.60	282	544.55	4.96
Cb2-6,4,10	100	185	9000	145.09	21.58	187	4463.33	1.07	187	1485.47	1.07
Cb3-2,3,10	100	337	8.02	304.17	9.74	348	3.47	3.16	348	1.37	3.16
Cb3-3,5,10	100	202	9000	173.86	13.93	207	3557.87	2.42	207	368.58	2.42
Cb3-6,4,10	100	283	9000	257.07	9.16	286	2836.46	1.05	289	870.77	2.08
Cb4-2,3,10	100	251	37.24	213.21	15.06	254	13.10	1.18	258	7.34	2.71
Cb4-3,5,10	100	245	9000	190.39	22.29	254	3525.42	3.54	256	1348.33	4.30
Cb4-6,4,10	100	288	9000	238.55	17.17	291	2928.67	1.03	294	1268.66	2.04
Cb5-2,3,10	100	197	7056.22	168.90	14.26	202	144.08	2.48	203	18.16	2.96
Cb5-3,5,10	100	232	9000	205.91	11.24	236	3597.15	1.69	240	166.47	3.33
Cb5-6,4,10	100	337	9000	306.20	9.14	343	2600.04	1.75	344	1511.27	2.03
Cc1-2,3,10	100	200	9000	159.41	20.30	206	3559.44	2.91	206	163.17	2.91
Cc1-3,5,10	100	199	9000	129.07	35.14	204	3184.53	2.45	204	1601.11	2.45
Cc1-6,4,10	100	255	9000	212.97	16.48	261	2523.50	2.30	261	1400.03	2.30
Cc2-2,3,10	100	201	9000	141.67	29.52	209	3486.52	3.83	209	21.64	3.83
Cc2-3,5,10	100	244	9000	200.18	17.96	252	2611.12	3.17	252	1355.92	3.17
Cc2-6,4,10	100	188	9000	120.23	36.05	189	2489.66	0.53	189	1019.37	0.53
Cc3-2,3,10	100	211	9000	147.37	30.15	218	2244.13	3.21	218	513.69	3.21
Cc3-3,5,10	100	241	9000	198.54	17.62	249	2951.10	3.21	249	2203.32	3.21
Cc3-6,4,10	100	215	9000	164.31	23.58	217	2554.78	0.92	217	2371.77	0.92
Cc4-2,3,10	100	235	9000	188.90	19.62	241	2684.68	2.49	241	546.19	2.49
Cc4-3,5,10	100	236	9000	190.70	19.20	245	3183.95	3.67	245	2255.48	3.67
Cc4-6,4,10	100	246	9000	195.04	20.72	253	3443.32	2.77	253	2001.86	2.77
Cc5-2,3,10	100	194	9000	137.50	29.12	200	2610.25	3.00	200	151.12	3.00
Cc5-3,5,10	100	182	9000	135.97	25.29	188	3110.36	3.19	188	2332.12	3.19
Cc5-6,4,10	100	187	9000	151.80	18.82	187	2775.97	0.00	187	1300.04	0.00
Cd1-2,3,10	100	197	189.76	173.95	11.70	200	87.32	1.50	201	29.06	1.99
Cd1-3,5,10	100	215	9000	172.93	19.57	219	2100.18	1.83	220	712.65	2.27
Cd1-6,4,10	100	248	9000	218.19	12.02	251	3532.77	1.20	251	1298.22	1.20
Cd2-2,3,10	100	193	9000	142.18	26.33	194	951.20	0.52	197	30.92	2.03
Cd2-3,5,10	100	236	9000	199.03	15.66	243	1466.72	2.88	244	1204.89	3.28
Cd2-6,4,10	100	213	9000	161.68	24.09	215	3505.38	0.93	215	1252.25	0.93
Cd3-2,3,10	100	186	8423.65	158.57	14.75	188	458.37	1.06	190	89.65	2.11
Cd3-3,5,10	100	224	9000	180.00	19.64	230	3073.16	2.61	230	1324.92	2.61
Cd3-6,4,10	100	196	9000	161.72	17.49	197	856.91	0.51	197	757.06	0.51
Cd4-2,3,10	100	257	189.76	249.57	2.89	260	51.36	1.15	264	42.22	2.65
Cd4-3,5,10	100	241	9000	220.19	8.63	247	2001.15	2.43	250	1252.18	3.60
Cd4-6,4,10	100	189	9000	134.55	28.81	190	2300.01	0.53	190	1197.82	0.53
Cd5-2,3,10	100	207	2980	167.13	19.26	209	12.21	0.96	211	6.07	1.90
Cd5-3,5,10	100	213	9000	166.35	21.90	217	2255.85	1.84	218	442.24	2.29
Cd5-6,4,10	100	184	9000	153.48	16.59	188	3568.37	2.13	189	1549.26	2.65
Averages			***7301.09***		***17.86***		***2325.97***	***2.15***		***802.66***	***2.57***

Table 8: Direct solving the hybrid formulation on instances with 10 OD demands and multiple Δ time periods

ID	Ψ	Time-Space formulation ($\Delta = \bar{\Delta}$)				Time-Space formulation ($\Delta = 50$)			Time-Space formulation ($\Delta = 25$)		
		UB	**CPU (sec)**	**LB**	**RG (%)**	**UB**	**CPU (sec)**	**Dif. Ub (%)**	**UB**	**CPU (sec)**	**Dif. Ub (%)**
Ca1-2,3,10	100	256	2.81	198.06	22.63	260	1.39	1.54	261	0.87	1.92
Ca1-3,5,10	100	215	84.24	163.75	23.84	217	46.13	0.92	219	25.35	1.83
Ca1-6,4,10	100	305	7.62	266.12	12.75	320	2.71	4.69	320	2.62	4.69
Ca2-2,3,10	100	161	2210.6	120.84	24.94	162	1044.25	0.62	163	466.77	1.23
Ca2-3,5,10	100	340	1.21	289.54	14.84	355	0.68	4.23	359	0.42	5.29
Ca2-6,4,10	100	179	2537.13	147.69	17.49	182	1074.93	1.65	183	692.36	2.19
Ca3-2,3,10	100	344	0.3	303.28	11.84	363	0.13	5.23	365	0.08	5.75
Ca3-3,5,10	100	318	4.29	272.62	14.27	333	2.24	4.50	333	1.00	4.50
Ca3-6,4,10	100	236	4.15	175.41	25.67	240	2.38	1.67	241	1.18	2.07
Ca4-2,3,10	100	437	0.96	395.44	9.51	462	0.34	5.41	465	0.39	6.02
Ca4-3,5,10	100	213	485.53	162.78	23.58	220	219.65	3.18	220	171.13	3.18
Ca4-6,4,10	100	238	9.32	187.07	21.40	243	5.17	2.06	244	2.62	2.46
Ca5-2,3,10	100	277	8.18	241.47	12.83	284	4.74	2.46	287	1.86	3.48
Ca5-3,5,10	100	264	13.21	234.69	11.10	274	4.94	3.65	276	4.50	4.35
Ca5-6,4,10	100	187	9000	116.30	37.81	192	4539.17	2.60	192	2356.60	2.60
Cb1-2,3,10	100	181	3.35	118.59	34.48	183	1.67	1.09	184	1.26	1.63
Cb1-3,5,10	100	211	3913.71	163.51	22.51	218	1806.98	3.21	221	1020.80	4.52
Cb1-6,4,10	100	261	4.71	223.75	14.27	266	2.17	1.88	269	1.94	2.97
Cb2-2,3,10	100	199	0.99	149.31	24.97	204	0.56	2.45	204	0.41	2.45
Cb2-3,5,10	100	268	2.54	215.04	19.76	280	1.26	4.29	284	1.01	5.63
Cb2-6,4,10	100	185	1017	144.74	21.76	187	484.26	1.07	188	158.54	1.60
Cb3-2,3,10	100	337	0.7	302.22	10.32	348	0.31	3.16	349	0.16	3.44
Cb3-3,5,10	100	202	1061.83	170.30	15.69	207	562.08	2.42	209	430.68	3.35
Cb3-6,4,10	100	283	11.74	250.45	11.50	288	5.72	1.74	290	4.30	2.41
Cb4-2,3,10	100	251	1.04	212.99	15.14	258	0.59	2.71	259	0.36	3.09
Cb4-3,5,10	100	245	806.72	185.77	24.18	255	300.39	3.92	258	202.18	5.04
Cb4-6,4,10	100	288	2519.29	234.85	18.45	293	1067.52	1.71	296	549.69	2.70
Cb5-2,3,10	100	197	30.08	159.27	19.15	203	12.41	2.96	203	11.62	2.96
Cb5-3,5,10	100	232	1579.01	194.55	16.14	237	554.47	2.11	241	540.96	3.73
Cb5-6,4,10	100	337	4.77	304.82	9.55	344	2.18	2.03	347	1.35	2.88
Cc1-2,3,10	100	197	9000	153.53	23.23	203	282.33	2.96	203	179.71	2.96
Cc1-3,5,10	100	189	9000	118.86	40.27	194	285.40	2.58	194	186.83	2.58
Cc1-6,4,10	100	246	9000	210.99	17.26	251	329.03	1.99	253	153.20	2.77
Cc2-2,3,10	100	191	9000	140.21	30.25	199	325.79	4.02	199	182.08	4.02
Cc2-3,5,10	100	232	9000	184.16	24.52	239	213.75	2.93	241	91.92	3.73
Cc2-6,4,10	100	173	9000	117.67	37.41	174	321.26	0.57	174	170.18	0.57
Cc3-2,3,10	100	196	9000	143.42	32.03	203	271.74	3.45	204	159.23	3.92
Cc3-3,5,10	100	237	9000	198.00	17.84	246	261.09	3.66	246	115.71	3.66
Cc3-6,4,10	100	207	9000	157.89	26.56	209	233.51	0.96	210	99.00	1.43
Cc4-2,3,10	100	231	9000	179.82	23.48	237	267.75	2.53	238	206.18	2.94
Cc4-3,5,10	100	228	9000	175.35	25.70	237	248.87	3.80	237	100.82	3.80
Cc4-6,4,10	100	232	9000	189.96	22.78	238	276.99	2.52	240	127.53	3.33
Cc5-2,3,10	100	183	9000	127.46	34.30	189	326.35	3.17	190	134.12	3.68
Cc5-3,5,10	100	174	9000	120.60	33.74	180	239.61	3.33	181	170.28	3.87
Cc5-6,4,10	100	179	9000	147.09	21.34	179	235.01	0.00	180	181.83	0.56
Cd1-2,3,10	100	199	9000	167.26	15.09	202	209.24	1.49	203	106.09	1.97
Cd1-3,5,10	100	219	9000	170.03	20.92	224	212.45	2.23	225	149.05	2.67
Cd1-6,4,10	100	260	9000	214.67	13.44	263	237.26	1.14	265	123.12	1.89
Cd2-2,3,10	100	193	3145.09	140.47	27.22	194	88.04	0.52	197	23.99	2.03
Cd2-3,5,10	100	236	9000	191.03	19.05	244	214.38	3.28	246	86.34	4.07
Cd2-6,4,10	100	213	9000	156.23	26.65	215	303.55	0.93	216	139.53	1.39
Cd3-2,3,10	100	186	4709.4	152.16	18.19	190	143.60	2.11	191	81.38	2.62
Cd3-3,5,10	100	224	9000	178.49	20.32	230	257.46	2.61	231	142.98	3.03
Cd3-6,4,10	100	196	9000	138.59	29.29	197	229.62	0.51	197	89.97	0.51
Cd4-2,3,10	100	257	5.25	212.23	17.42	261	1.73	1.53	266	1.68	3.38
Cd4-3,5,10	100	241	9000	210.43	12.68	248	253.67	2.82	251	184.33	3.98
Cd4-6,4,10	100	189	9000	132.67	29.81	190	256.44	0.53	191	128.71	1.05
Cd5-2,3,10	100	207	41.39	156.58	24.36	210	10.43	1.43	211	7.01	1.90
Cd5-3,5,10	100	213	9000	161.67	24.10	218	329.92	2.29	219	128.80	2.74
Cd5-6,4,10	100	184	9000	140.24	23.78	189	222.80	2.65	189	154.58	2.65
Averages			***4453.80***		***21.59***		***314.01***	***2.43***		***174.32***	***2.99***

Table 9: Direct solving the classic time-space formulation on instances with 10 OD demands and multiple Δ time periods

ID	Ψ	Hybrid formulation ($\Delta = \bar{\Delta}$)				Hybrid formulation ($\Delta = 50$)			Hybrid formulation ($\Delta = 25$)		
		UB	CPU (sec)	LB	RG (%)	UB	CPU (sec)	Dif. Ub (%)	UB	CPU (sec)	Dif. Ub (%)
Ca1-2,3,15	200	338	9000.00	177.75	35.36	441	9000.00	23.36	432	4130.20	21.76
Ca1-3,5,15	200	N.A	9000.00	236.04	28.04	N.A	9000.00	N.A	342	1438.47	N.A
Ca1-6,4,15	200	N.A	9000.00	185.08	37.05	N.A	9000.00	N.A	300	4379.18	N.A
Ca2-2,3,15	200	330	9000.00	188.28	41.71	417	9000.00	20.86	419	3261.55	21.24
Ca2-3,5,15	200	N.A	9000.00	223.03	36.28	532	9000.00	N.A	350	9000.00	N.A
Ca2-6,4,15	200	N.A	9000.00	169.65	40.68	N.A	9000.00	N.A	384	9000.00	N.A
Ca3-2,3,15	200	366	9000.00	239.38	26.34	401	9000.00	8.73	401	4282.41	8.73
Ca3-3,5,15	200	N.A	9000.00	1.21	0.00	441	9000.00	N.A	441	1366.57	N.A
Ca3-6,4,15	200	N.A	9000.00	153.30	44.25	N.A	9000.00	N.A	383	9000.00	N.A
Ca4-2,3,15	200	420	9000.00	224.95	30.78	517	9000.00	18.76	447	9000.00	6.04
Ca4-3,5,15	200	N.A	9000.00	234.36	18.34	N.A	9000.00	N.A	418	9000.00	N.A
Ca4-6,4,15	200	N.A	9000.00	188.65	33.81	N.A	9000.00	N.A	568	4828.49	N.A
Ca5-2,3,15	200	N.A	9000.00	3.72	0.00	518	9000.00	N.A	547	9000.00	N.A
Ca5-3,5,15	200	N.A	9000.00	226.55	17.92	N.A	9000.00	N.A	384	4823.17	N.A
Ca5-6,4,15	200	N.A	9000.00	132.55	45.45	684	9000.00	N.A	687	9000.00	N.A
Cb1-2,3,15	200	N.A	9000.00	2.70	0.00	387	9000.00	N.A	422	9000.00	N.A
Cb1-3,5,15	200	N.A	9000.00	236.76	24.84	N.A	9000.00	N.A	406	673.45	N.A
Cb1-6,4,15	200	N.A	9000.00	171.47	44.86	N.A	9000.00	N.A	424	9000.00	N.A
Cb2-2,3,15	200	N.A	9000.00	2.20	0.00	487	1240.03	N.A	530	523.20	N.A
Cb2-3,5,15	200	N.A	9000.00	196.89	44.38	N.A	9000.00	N.A	359	9000.00	N.A
Cb2-6,4,15	200	N.A	9000.00	187.48	39.52	N.A	9000.00	N.A	415	4239.02	N.A
Cb3-2,3,15	200	402	9000.00	242.17	30.81	501	9000.00	19.76	550	9000.00	26.91
Cb3-3,5,15	200	N.A	9000.00	240.51	35.52	N.A	9000.00	N.A	665	9000.00	N.A
Cb3-6,4,15	200	N.A	9000.00	135.31	50.25	N.A	9000.00	N.A	546	5620.25	N.A
Cb4-2,3,15	200	380	9000.00	186.06	40.55	387	9000.00	1.81	593	6000.65	35.92
Cb4-3,5,15	200	N.A	9000.00	210.87	22.48	N.A	9000.00	N.A	562	7643.49	N.A
Cb4-6,4,15	200	N.A	9000.00	165.94	52.86	N.A	9000.00	N.A	443	5611.26	N.A
Cb5-2,3,15	200	N.A	9000.00	175.07	35.16	N.A	9000.00	N.A	688	6780.86	N.A
Cb5-3,5,15	200	N.A	9000.00	160.85	36.92	N.A	9000.00	N.A	558	5467.55	N.A
Cb5-6,4,15	200	N.A	9000.00	97.19	58.46	N.A	9000.00	N.A	611	5934.99	N.A
Cc1-2,3,15	200	N.A	9000.00	143.35	48.80	N.A	9000.00	N.A	728	7322.75	N.A
Cc1-3,5,15	200	N.A	9000.00	154.43	37.48	N.A	9000.00	N.A	549	7938.73	N.A
Cc1-6,4,15	200	N.A	9000.00	2.85	0.00	N.A	9000.00	N.A	637	6295.60	N.A
Cc2-2,3,15	200	N.A	9000.00	146.36	50.05	N.A	9000.00	N.A	702	5276.32	N.A
Cc2-3,5,15	200	N.A	9000.00	4.58	0.00	N.A	9000.00	N.A	617	7192.03	N.A
Cc2-6,4,15	200	N.A	9000.00	19.02	93.62	N.A	9000.00	N.A	608	6348.15	N.A
Cc3-2,3,15	200	N.A	9000.00	97.58	64.77	N.A	9000.00	N.A	699	5284.95	N.A
Cc3-3,5,15	200	N.A	9000.00	154.90	32.36	N.A	9000.00	N.A	644	6750.50	N.A
Cc3-6,4,15	200	N.A	9000.00	3.24	0.00	N.A	9000.00	N.A	575	5247.18	N.A
Cc4-2,3,15	200	N.A	9000.00	126.74	61.59	N.A	9000.00	N.A	436	5729.75	N.A
Cc4-3,5,15	200	N.A	9000.00	57.45	78.48	N.A	9000.00	N.A	458	5167.62	N.A
Cc4-6,4,15	200	N.A	9000.00	3.43	0.00	N.A	9000.00	N.A	570	6517.16	N.A
Cc5-2,3,15	200	N.A	9000.00	148.45	43.12	N.A	9000.00	N.A	581	5496.01	N.A
Cc5-3,5,15	200	N.A	9000.00	5.22	0.00	N.A	9000.00	N.A	617	6322.83	N.A
Cc5-6,4,15	200	N.A	9000.00	3.94	0.00	N.A	9000.00	N.A	601	6169.12	N.A
Cd1-2,3,15	200	330	9000.00	190.60	39.49	445	9000.00	25.84	445	5702.43	25.84
Cd1-3,5,15	200	N.A	9000.00	194.57	40.13	303	9000.00	N.A	303	5657.12	N.A
Cd1-6,4,15	200	N.A	9000.00	171.62	43.17	544	9000.00	N.A	544	1864.64	N.A
Cd2-2,3,15	200	303	9000.00	143.88	44.45	429	9000.00	29.37	429	5386.76	29.37
Cd2-3,5,15	200	N.A	9000.00	225.45	34.27	417	9000.00	N.A	417	6077.70	N.A
Cd2-6,4,15	200	N.A	9000.00	183.72	37.72	409	9000.00	N.A	408	2894.30	N.A
Cd3-2,3,15	200	305	9000.00	138.14	48.84	504	9000.00	39.48	504	2417.37	39.48
Cd3-3,5,15	200	N.A	9000.00	136.41	44.55	333	9000.00	N.A	333	6124.06	N.A
Cd3-6,4,15	200	N.A	9000.00	175.03	40.26	413	9000.00	N.A	430	6959.73	N.A
Cd4-2,3,15	200	400	9000.00	179.28	46.64	502	9000.00	20.32	502	5736.13	20.32
Cd4-3,5,15	200	N.A	9000.00	169.17	44.17	N.A	9000.00	N.A	465	6474.81	N.A
Cd4-6,4,15	200	N.A	9000.00	155.54	44.84	N.A	9000.00	N.A	586	4454.65	N.A
Cd5-2,3,15	200	388	9000.00	169.24	40.20	388	9000.00	0.00	388	4375.93	0.00
Cd5-3,5,15	200	N.A	9000.00	150.00	43.82	N.A	9000.00	N.A	586	7474.07	N.A
Cd5-6,4,15	200	N.A	9000.00	111.93	50.25	466	9000.00	N.A	625	6901.60	N.A
Averages			***9000.00***		***35.10***	***N.A***	***8870.67***	***18.94***		***5942.75***	***21.42***

Table 10: Direct solving the hybrid formulation on instances with 15 OD demands and multiple Δ time periods

ID	Ψ	Time-Space formulation ($\Delta = \bar{\Delta}$)				Time-Space formulation ($\Delta = 50$)			Time-Space formulation ($\Delta = 25$)		
		UB	CPU (sec)	LB	RG (%)	UB	CPU (sec)	Dif. Ub (%)	UB	CPU (sec)	Dif. Ub (%)
Ca1-2,3,15	200	275	9000	175.58	36.15	285	5545.51	3.51	290	2743.71	5.17
Ca1-3,5,15	200	328	9000	229.32	30.09	347	5817.79	5.48	351	716.29	6.55
Ca1-6,4,15	200	294	9000	179.08	39.09	307	4260.08	4.23	312	3498.33	5.77
Ca2-2,3,15	200	323	9000	180.22	44.20	330	5522.43	2.12	346	3421.68	6.65
Ca2-3,5,15	200	350	9000	219.94	37.16	374	5197.28	6.42	374	2261.60	6.42
Ca2-6,4,15	200	286	9000	161.71	43.46	298	4206.75	4.03	298	2936.00	4.03
Ca3-2,3,15	200	325	9000	234.42	27.87	361	4141.41	9.97	361	3081.85	9.97
Ca3-3,5,15	200	N.A	9000	N.A	N.A	370	5298.97	N.A	370	1033.74	N.A
Ca3-6,4,15	200	275	9000	148.21	46.10	292	5012.42	5.82	295	743.37	6.78
Ca4-2,3,15	200	325	9000	218.81	32.67	335	6356.93	2.99	354	1907.54	8.19
Ca4-3,5,15	200	287	8235.57	234.09	18.43	299	2549.68	4.01	309	967.52	7.12
Ca4-6,4,15	200	285	9000	186.46	34.58	299	6152.51	4.68	303	1079.68	5.94
Ca5-2,3,15	200	N.A	9000	N.A	N.A	282	5985.84	N.A	300	2829.34	N.A
Ca5-3,5,15	200	276	9000	223.43	19.05	291	4415.18	5.15	292	2161.30	5.48
Ca5-6,4,15	200	243	9000	124.65	48.70	254	5047.90	4.33	254	2877.62	4.33
Cb1-2,3,15	200	N.A	9000	N.A	N.A	330	4216.92	N.A	335	1066.85	N.A
Cb1-3,5,15	200	315	9000	232.01	26.35	322	3619.18	2.17	331	1055.96	4.83
Cb1-6,4,15	200	311	9000	162.25	47.83	330	4437.30	5.76	332	1433.31	6.33
Cb2-2,3,15	200	N.A	9000	N.A	N.A	400	4924.76	N.A	400	797.08	N.A
Cb2-3,5,15	200	354	9000	188.23	46.83	363	4341.31	2.48	365	2786.88	3.01
Cb2-6,4,15	200	310	9000	189.34	38.92	324	5781.35	4.32	332	2763.45	6.63
Cb3-2,3,15	200	350	9000	250.12	28.54	364	4353.17	3.85	368	1646.80	4.89
Cb3-3,5,15	200	373	9000	235.42	36.88	394	4503.81	5.33	394	3081.23	5.33
Cb3-6,4,15	200	272	9000	131.81	51.54	287	3874.09	5.23	289	2529.11	5.88
Cb4-2,3,15	200	313	9000	185.83	40.63	333	6398.45	6.01	333	2143.75	6.01
Cb4-3,5,15	200	272	9000	211.23	22.34	287	6419.74	5.23	289	2935.72	5.88
Cb4-6,4,15	200	352	9000	167.40	52.44	368	3816.80	4.35	383	1880.40	8.09
Cb5-2,3,15	200	270	9000	180.19	33.26	280	3521.03	3.57	283	2464.00	4.59
Cb5-3,5,15	200	255	9000	154.02	39.60	266	9000.00	4.14	266	799.29	4.14
Cb5-6,4,15	200	234	9000	95.36	59.25	246	5572.18	4.88	246	852.20	4.88
Cc1-2,3,15	200	280	9000	130.72	53.32	286	4487.22	2.10	294	1500.38	4.76
Cc1-3,5,15	200	247	9000	157.25	36.33	261	5303.19	5.36	264	2736.41	6.44
Cc1-6,4,15	200	N.A	9000	N.A	N.A	315	4340.11	N.A	385	2960.93	N.A
Cc2-2,3,15	200	293	9000	146.72	49.92	299	6345.32	2.01	301	1343.04	2.66
Cc2-3,5,15	200	N.A	9000	N.A	N.A	300	5651.23	N.A	310	1229.22	N.A
Cc2-6,4,15	200	298	9000	13.10	95.60	317	5551.68	5.99	320	2028.28	6.88
Cc3-2,3,15	200	277	9000	94.16	66.01	292	5678.36	5.14	294	1574.91	5.78
Cc3-3,5,15	200	229	9000	151.77	33.73	234	5964.98	2.14	249	2264.92	8.03
Cc3-6,4,15	200	N.A	9000	N.A	N.A	333	4792.36	N.A	401	1903.99	N.A
Cc4-2,3,15	200	330	9000	117.27	64.46	343	3523.23	3.79	352	2330.30	6.25
Cc4-3,5,15	200	267	9000	N.A	80.14	278	5726.73	3.96	281	1041.89	4.98
Cc4-6,4,15	200	N.A	9000	N.A	0.00	N.A	9000.00	N.A	312	3169.75	N.A
Cc5-2,3,15	200	261	9000	145.64	44.20	275	5711.14	5.09	283	3114.25	7.77
Cc5-3,5,15	200	N.A	9000	N.A	N.A	N.A	9000.00	N.A	222	1125.20	N.A
Cc5-6,4,15	200	N.A	9000	N.A	N.A	N.A	9000.00	N.A	320	3094.40	N.A
Cd1-2,3,15	200	315	9000	188.39	40.19	335	5458.08	5.97	335	1173.86	5.97
Cd1-3,5,15	200	325	9000	196.65	39.49	346	5721.14	6.07	347	2221.93	6.34
Cd1-6,4,15	200	302	9000	170.41	43.57	320	5998.57	5.63	325	3351.01	7.08
Cd2-2,3,15	200	259	9000	135.81	47.57	269	9000.00	3.72	275	1728.63	5.82
Cd2-3,5,15	200	343	9000	221.52	35.42	350	5918.94	2.00	351	1425.62	2.28
Cd2-6,4,15	200	295	9000	181.65	38.42	311	4737.80	5.14	320	2921.11	7.81
Cd3-2,3,15	200	270	9000	131.55	51.28	288	4225.03	6.25	289	1734.90	6.57
Cd3-3,5,15	200	246	9000	137.15	44.25	257	3614.86	4.28	258	3076.52	4.65
Cd3-6,4,15	200	293	9000	171.98	41.30	301	5991.25	2.66	319	2732.01	8.15
Cd4-2,3,15	200	336	9000	184.15	45.19	353	4518.30	4.82	358	2614.95	6.15
Cd4-3,5,15	200	303	9000	172.97	42.91	318	4534.98	4.72	324	1145.43	6.48
Cd4-6,4,15	200	282	9000	158.66	43.74	302	9000.00	6.62	303	1270.97	6.93
Cd5-2,3,15	200	283	9000	157.03	44.51	298	6278.18	5.03	304	2244.60	6.91
Cd5-3,5,15	200	267	9000	152.71	42.80	282	6093.68	5.32	282	2049.28	5.32
Cd5-6,4,15	200	225	9000	103.12	54.17	236	5337.94	4.66	238	2991.61	5.46
Averages			***8987.26***		***42.36***		***5446.58***	***4.57***		***2076.60***	***5.97***

Table 11: Direct solving the classic time-space formulation on instances with 15 OD demands and multiple Δ time periods

ID	Ψ	UB	CPU (sec)	LB	OG (%)	ID	Ψ	UB	CPU (sec)	LB	OG (%)	ID	Ψ	UB	CPU (sec)	LB	OG (%)
Ca1-2,3,5	100	280	2.55	280	0	Ca1-2,3,10	100	256	36.36	256	0	Ca1-2,3,15	200	275	257.86	275	0
Ca1-3,5,5	100	222	0.48	222	0	Ca1-3,5,10	100	215	21.38	215	0	Ca1-3,5,15	200	309	288.40	309	0
Ca1-6,4,5	100	271	2.72	271	0	Ca1-6,4,10	100	305	24.14	305	0	Ca1-6,4,15	200	274	1427.18	274	0
Ca2-2,3,5	100	152	1.86	152	0	Ca2-2,3,10	100	161	14.07	161	0	Ca2-2,3,15	200	306	29.53	306	0
Ca2-3,5,5	100	284	2.30	284	0	Ca2-3,5,10	100	340	5.56	340	0	Ca2-3,5,15	200	335	442.19	335	0
Ca2-6,4,5	100	150	1.56	150	0	Ca2-6,4,10	100	179	14.42	179	0	Ca2-6,4,15	200	272	944.95	272	0
Ca3-2,3,5	100	287	2.05	287	0	Ca3-2,3,10	100	344	3.08	344	0	Ca3-2,3,15	200	319	36.86	319	0
Ca3-3,5,5	100	220	2.89	220	0	Ca3-3,5,10	100	318	20.26	318	0	Ca3-3,5,15	200	307	54.43	307	0
Ca3-6,4,5	100	171	1.61	171	0	Ca3-6,4,10	100	236	41.83	236	0	Ca3-6,4,15	200	267	5526.43	267	0
Ca4-2,3,5	100	358	1.24	358	0	Ca4-2,3,10	100	437	16.14	437	0	Ca4-2,3,15	200	309	31.34	309	0
Ca4-3,5,5	100	168	2.35	168	0	Ca4-3,5,10	100	213	462.45	213	0	Ca4-3,5,15	200	287	3585.06	287	0
Ca4-6,4,5	100	161	0.83	161	0	Ca4-6,4,10	100	238	31.8	238	0	Ca4-6,4,15	200	262	1147.66	262	0
Ca5-2,3,5	100	199	2.20	199	0	Ca5-2,3,10	100	277	27.64	277	0	Ca5-2,3,15	200	265	32.87	265	0
Ca5-3,5,5	100	186	2.16	186	0	Ca5-3,5,10	100	264	26.4	264	0	Ca5-3,5,15	200	262	25.99	262	0
Ca5-6,4,5	100	159	1.01	159	0	Ca5-6,4,10	100	187	18.09	187	0	Ca5-6,4,15	200	218	11.95	218	0
Cb1-2,3,5	100	152	1.03	152	0	Cb1-2,3,10	100	181	13.08	181	0	Cb1-2,3,15	200	305	280.14	305	0
Cb1-3,5,5	100	164	0.59	164	0	Cb1-3,5,10	100	211	20.3	211	0	Cb1-3,5,15	200	291	126.13	291	0
Cb1-6,4,5	100	305	1.57	305	0	Cb1-6,4,10	100	261	18.99	261	0	Cb1-6,4,15	200	295	98.39	295	0
Cb2-2,3,5	100	129	2.09	129	0	Cb2-2,3,10	100	199	13.42	199	0	Cb2-2,3,15	200	337	207.52	337	0
Cb2-3,5,5	100	154	1.07	154	0	Cb2-3,5,10	100	268	41.25	268	0	Cb2-3,5,15	200	292	53.31	292	0
Cb2-6,4,5	100	143	2.47	143	0	Cb2-6,4,10	100	185	234.99	185	0	Cb2-6,4,15	200	330	145.68	330	0
Cb3-2,3,5	100	332	2.85	332	0	Cb3-2,3,10	100	337	6.28	337	0	Cb3-2,3,15	200	354	1655.75	354	0
Cb3-3,5,5	100	160	1.22	160	0	Cb3-3,5,10	100	202	144.99	202	0	Cb3-3,5,15	200	266	74.13	266	0
Cb3-6,4,5	100	198	2.13	198	0	Cb3-6,4,10	100	283	1456.59	283	0	Cb3-6,4,15	200	298	44.46	298	0
Cb4-2,3,5	100	280	1.84	280	0	Cb4-2,3,10	100	251	106.53	251	0	Cb4-2,3,15	200	255	384.53	255	0
Cb4-3,5,5	100	142	0.96	142	0	Cb4-3,5,10	100	245	2355.73	245	0	Cb4-3,5,15	200	334	311.41	334	0
Cb4-6,4,5	100	188	1.97	188	0	Cb4-6,4,10	100	288	7.43	288	0	Cb4-6,4,15	200	252	51.74	252	0
Cb5-2,3,5	100	129	2.35	129	0	Cb5-2,3,10	100	197	42.39	197	0	Cb5-2,3,15	200	243	64.14	243	0
Cb5-3,5,5	100	179	1.90	179	0	Cb5-3,5,10	100	232	128.22	232	0	Cb5-3,5,15	200	223	699.52	223	0
Cb5-6,4,5	100	199	1.56	199	0	Cb5-6,4,10	100	337	50.75	337	0	Cb5-6,4,15	200	265	119.31	265	0
Cc1-2,3,5	100	129	2.03	129	0	Cc1-2,3,10	100	189	64.63	189	0	Cc1-2,3,15	200	233	594.93	233	0
Cc1-3,5,5	100	135	2.30	135	0	Cc1-3,5,10	100	180	41.67	180	0	Cc1-3,5,15	200	280	175.81	280	0
Cc1-6,4,5	100	150	2.40	150	0	Cc1-6,4,10	100	238	45.67	238	0	Cc1-6,4,15	200	284	131.55	284	0
Cc2-2,3,5	100	122	1.04	122	0	Cc2-2,3,10	100	187	29.35	187	0	Cc2-2,3,15	200	299	64.06	299	0
Cc2-3,5,5	100	175	0.94	175	0	Cc2-3,5,10	100	231	27.7	231	0	Cc2-3,5,15	200	273	378.17	273	0
Cc2-6,4,5	100	122	1.77	122	0	Cc2-6,4,10	100	163	18.8	163	0	Cc2-6,4,15	200	271	380.97	271	0
Cc3-2,3,5	100	136	2.11	136	0	Cc3-2,3,10	100	184	12.7	184	0	Cc3-2,3,15	200	223	670.41	223	0
Cc3-3,5,5	100	142	2.59	142	0	Cc3-3,5,10	100	225	79.81	225	0	Cc3-3,5,15	200	259	668.33	259	0
Cc3-6,4,5	100	157	0.90	157	0	Cc3-6,4,10	100	193	21.95	193	0	Cc3-6,4,15	200	307	90.63	307	0
Cc4-2,3,5	100	171	1.27	171	0	Cc4-2,3,10	100	225	24.11	225	0	Cc4-2,3,15	200	250	240.03	250	0
Cc4-3,5,5	100	154	1.25	154	0	Cc4-3,5,10	100	223	28.05	223	0	Cc4-3,5,15	200	268	69.52	268	0
Cc4-6,4,5	100	138	1.16	138	0	Cc4-6,4,10	100	230	25.04	230	0	Cc4-6,4,15	200	236	14.54	236	0
Cc5-2,3,5	100	123	1.66	123	0	Cc5-2,3,10	100	182	12.52	182	0	Cc5-2,3,15	200	247	69.91	247	0
Cc5-3,5,5	100	124	2.65	124	0	Cc5-3,5,10	100	168	27.72	168	0	Cc5-3,5,15	200	208	73.73	208	0
Cc5-6,4,5	100	138	1.41	138	0	Cc5-6,4,10	100	179	51.26	179	0	Cc5-6,4,15	200	293	12.48	293	0
Cd1-2,3,5	100	155	2.84	155	0	Cd1-2,3,10	100	197	13.29	197	0	Cd1-2,3,15	200	304	43.78	304	0
Cd1-3,5,5	100	170	1.68	170	0	Cd1-3,5,10	100	215	12.11	215	0	Cd1-3,5,15	200	277	47.97	277	0
Cd1-6,4,5	100	188	2.20	188	0	Cd1-6,4,10	100	248	44.06	248	0	Cd1-6,4,15	200	249	3.62	249	0
Cd2-2,3,5	100	140	1.21	140	0	Cd2-2,3,10	100	193	11.31	193	0	Cd2-2,3,15	200	319	4.06	319	0
Cd2-3,5,5	100	158	1.09	158	0	Cd2-3,5,10	100	236	12.85	236	0	Cd2-3,5,15	200	274	28.50	274	0
Cd2-6,4,5	100	142	0.97	142	0	Cd2-6,4,10	100	213	44.5	213	0	Cd2-6,4,15	200	257	2.57	257	0
Cd3-2,3,5	100	142	2.64	142	0	Cd3-2,3,10	100	186	20.14	186	0	Cd3-2,3,15	200	222	45.33	222	0
Cd3-3,5,5	100	147	2.16	147	0	Cd3-3,5,10	100	224	32.52	224	0	Cd3-3,5,15	200	268	100.99	268	0
Cd3-6,4,5	100	157	1.03	157	0	Cd3-6,4,10	100	196	25.17	196	0	Cd3-6,4,15	200	324	25.14	324	0
Cd4-2,3,5	100	202	2.73	202	0	Cd4-2,3,10	100	257	11.3	257	0	Cd4-2,3,15	200	289	31.08	289	0
Cd4-3,5,5	100	162	1.35	162	0	Cd4-3,5,10	100	241	20.37	241	0	Cd4-3,5,15	200	263	25.48	263	0
Cd4-6,4,5	100	147	1.11	147	0	Cd4-6,4,10	100	189	30.48	189	0	Cd4-6,4,15	200	271	10.09	271	0
Cd5-2,3,5	100	178	1.35	178	0	Cd5-2,3,10	100	207	20.54	207	0	Cd5-2,3,15	200	247	19.36	247	0
Cd5-3,5,5	100	178	0.82	178	0	Cd5-3,5,10	100	213	30.54	213	0	Cd5-3,5,15	200	208	252.93	208	0
Cd5-6,4,5	100	157	1.19	157	0	Cd5-6,4,10	100	184	43.23	184	0	Cd5-6,4,15	200	272	725.23	272	0
Averages			1.72		0	***Averages***			104.80		0	***Averages***			386.00		0

Table 12: DDD results on instances with 5, 10 and 15 OD demands

ID	Ψ	UB	CPU (sec)	LB	OG (%)	ID	Ψ	UB	CPU (sec)	LB	OG (%)	ID	Ψ	UB	CPU (sec)	LB	OG (%)
Ca1-2,3,5	100	280	78.14	280	0	Ca1-2,3,10	100	256	597.03	256	0	Ca1-2,3,15	200	277	18000.00	236.03	14.79
Ca1-3,5,5	100	222	92.85	222	0	Ca1-3,5,10	100	215	4068.76	215	0	Ca1-3,5,15	200	311	18000.00	283.20	8.94
Ca1-6,4,5	100	271	302.53	271	0	Ca1-6,4,10	100	305	2916.36	305	0	Ca1-6,4,15	200	289	18000.00	221.95	23.20
Ca2-2,3,5	100	152	191.19	152	0	Ca2-2,3,10	100	161	3620.48	161	0	Ca2-2,3,15	200	306	8160.277	306.00	0.00
Ca2-3,5,5	100	284	368.60	284	0	Ca2-3,5,10	100	340	4042.23	340	0	Ca2-3,5,15	200	339	18000.00	267.23	21.17
Ca2-6,4,5	100	150	439.69	150	0	Ca2-6,4,10	100	179	3685.74	179	0	Ca2-6,4,15	200	283	18000.00	210.13	25.75
Ca3-2,3,5	100	287	293.68	287	0	Ca3-2,3,10	100	344	1172.01	344	0	Ca3-2,3,15	200	319	4097.53	319.00	0.00
Ca3-3,5,5	100	220	134.13	220	0	Ca3-3,5,10	100	318	1678.52	318	0	Ca3-3,5,15	200	315	18000.00	298.27	5.31
Ca3-6,4,5	100	171	113.06	171	0	Ca3-6,4,10	100	236	2872.09	236	0	Ca3-6,4,15	200	286	18000.00	206.43	27.82
Ca4-2,3,5	100	358	136.83	358	0	Ca4-2,3,10	100	437	1076.81	437	0	Ca4-2,3,15	200	321	18000.00	275.13	14.29
Ca4-3,5,5	100	168	269.84	168	0	Ca4-3,5,10	100	213	969.23	213	0	Ca4-3,5,15	200	287	5999.64	287.00	0.00
Ca4-6,4,5	100	161	104.24	161	0	Ca4-6,4,10	100	238	3998.11	238	0	Ca4-6,4,15	200	272	18000.00	229.60	15.59
Ca5-2,3,5	100	199	406.72	199	0	Ca5-2,3,10	100	277	3044.70	277	0	Ca5-2,3,15	200	270	18000.00	266.46	1.31
Ca5-3,5,5	100	186	202.17	186	0	Ca5-3,5,10	100	264	948.10	264	0	Ca5-3,5,15	200	280	7166.95	280.00	0.00
Ca5-6,4,5	100	159	410.28	159	0	Ca5-6,4,10	100	187	768.66	187	0	Ca5-6,4,15	200	220	18000.00	170.64	22.44
Cb1-2,3,5	100	152	341.33	152	0	Cb1-2,3,10	100	181	2666.72	181	0	Cb1-2,3,15	200	305	8958.35	305.00	0.00
Cb1-3,5,5	100	164	381.53	164	0	Cb1-3,5,10	100	211	2702.84	211	0	Cb1-3,5,15	200	298	18000.00	264.82	11.14
Cb1-6,4,5	100	305	370.46	305	0	Cb1-6,4,10	100	261	2746.78	261	0	Cb1-6,4,15	200	301	18000.00	200.44	33.41
Cb2-2,3,5	100	129	283.59	129	0	Cb2-2,3,10	100	199	3759.89	199	0	Cb2-2,3,15	200	275	5621.68	275.00	0.00
Cb2-3,5,5	100	154	173.17	154	0	Cb2-3,5,10	100	268	2191.68	268	0	Cb2-3,5,15	200	348	18000.00	248.03	28.73
Cb2-6,4,5	100	143	162.15	143	0	Cb2-6,4,10	100	185	3813.43	185	0	Cb2-6,4,15	200	311	18000.00	237.86	23.52
Cb3-2,3,5	100	332	132.18	332	0	Cb3-2,3,10	100	337	3115.60	337	0	Cb3-2,3,15	200	337	18000.00	300.20	10.92
Cb3-3,5,5	100	160	355.92	160	0	Cb3-3,5,10	100	202	2510.81	202	0	Cb3-3,5,15	200	354	6756.64	354.00	0.00
Cb3-6,4,5	100	198	270.45	198	0	Cb3-6,4,10	100	283	4204.87	283	0	Cb3-6,4,15	200	278	18000.00	180.21	35.18
Cb4-2,3,5	100	280	399.33	280	0	Cb4-2,3,10	100	251	3211.56	251	0	Cb4-2,3,15	200	306	18000.00	249.17	18.57
Cb4-3,5,5	100	142	78.82	142	0	Cb4-3,5,10	100	245	4197.41	245	0	Cb4-3,5,15	200	270	18000.00	248.00	8.15
Cb4-6,4,5	100	188	294.48	188	0	Cb4-6,4,10	100	288	3570.49	288	0	Cb4-6,4,15	200	340	18000.00	231.25	31.98
Cb5-2,3,5	100	129	314.79	129	0	Cb5-2,3,10	100	197	4020.52	197	0	Cb5-2,3,15	200	265	18000.00	229.01	13.58
Cb5-3,5,5	100	179	314.78	179	0	Cb5-3,5,10	100	232	3327.24	232	0	Cb5-3,5,15	200	247	18000.00	197.69	19.96
Cb5-6,4,5	100	199	166.44	199	0	Cb5-6,4,10	100	337	3733.03	337	0	Cb5-6,4,15	200	238	18000.00	146.17	38.59
Cc1-2,3,5	100	129	3641.96	129	0	Cc1-2,3,10	100	189	6462.98	189	0	Cc1-2,3,15	200	271	18000.00	173.28	36.06
Cc1-3,5,5	100	135	5094.15	135	0	Cc1-3,5,10	100	180	7163.11	180	0	Cc1-3,5,15	200	245	18000.00	201.21	17.88
Cc1-6,4,5	100	150	1347.79	150	0	Cc1-6,4,10	100	238	6139.92	238	0	Cc1-6,4,15	200	302	18000.00	227.77	24.58
Cc2-2,3,5	100	122	2960.55	122	0	Cc2-2,3,10	100	187	6832.77	187	0	Cc2-2,3,15	200	296	18000.00	209.84	29.11
Cc2-3,5,5	100	175	1491.50	175	0	Cc2-3,5,10	100	231	6496.19	231	0	Cc2-3,5,15	200	315	18000.00	215.18	31.69
Cc2-6,4,5	100	122	5175.69	122	0	Cc2-6,4,10	100	163	6195.34	163	0	Cc2-6,4,15	200	301	18000.00	188.30	37.44
Cc3-2,3,5	100	136	1253.47	136	0	Cc3-2,3,10	100	184	6843.57	184	0	Cc3-2,3,15	200	290	18000.00	199.05	31.36
Cc3-3,5,5	100	142	3175.83	142	0	Cc3-3,5,10	100	225	6556.92	225	0	Cc3-3,5,15	200	230	18000.00	186.84	18.77
Cc3-6,4,5	100	157	2602.36	157	0	Cc3-6,4,10	100	193	5790.61	193	0	Cc3-6,4,15	200	284	18000.00	224.51	20.95
Cc4-2,3,5	100	171	5693.12	171	0	Cc4-2,3,10	100	225	5937.40	225	0	Cc4-2,3,15	200	326	18000.00	229.35	29.65
Cc4-3,5,5	100	154	2677.88	154	0	Cc4-3,5,10	100	223	5937.87	223	0	Cc4-3,5,15	200	269	18000.00	165.09	38.63
Cc4-6,4,5	100	138	2836.50	138	0	Cc4-6,4,10	100	230	5152.39	230	0	Cc4-6,4,15	200	276	18000.00	195.97	29.00
Cc5-2,3,5	100	123	1525.61	123	0	Cc5-2,3,10	100	182	6576.45	182	0	Cc5-2,3,15	200	243	18000.00	179.41	26.17
Cc5-3,5,5	100	124	5426.52	124	0	Cc5-3,5,10	100	168	7008.74	168	0	Cc5-3,5,15	200	261	18000.00	189.47	27.40
Cc5-6,4,5	100	138	2312.30	138	0	Cc5-6,4,10	100	179	5791.82	179	0	Cc5-6,4,15	200	226	18000.00	183.83	18.66
Cd1-2,3,5	100	155	126.76	155	0	Cd1-2,3,10	100	197	794.44	197	0	Cd1-2,3,15	200	301	18000.00	225.56	25.06
Cd1-3,5,5	100	170	221.75	170	0	Cd1-3,5,10	100	215	943.94	215	0	Cd1-3,5,15	200	311	18000.00	243.56	21.69
Cd1-6,4,5	100	188	289.63	188	0	Cd1-6,4,10	100	248	1027.98	248	0	Cd1-6,4,15	200	279	18000.00	219.46	21.34
Cd2-2,3,5	100	140	400.20	140	0	Cd2-2,3,10	100	193	1594.63	193	0	Cd2-2,3,15	200	259	18000.00	191.34	26.12
Cd2-3,5,5	100	158	431.44	158	0	Cd2-3,5,10	100	236	2202.01	236	0	Cd2-3,5,15	200	325	18000.00	285.64	12.11
Cd2-6,4,5	100	142	358.49	142	0	Cd2-6,4,10	100	213	3507.31	213	0	Cd2-6,4,15	200	280	18000.00	213.54	23.74
Cd3-2,3,5	100	142	91.05	142	0	Cd3-2,3,10	100	186	3333.83	186	0	Cd3-2,3,15	200	272	18000.00	195.50	28.12
Cd3-3,5,5	100	147	165.73	147	0	Cd3-3,5,10	100	224	3978.59	224	0	Cd3-3,5,15	200	227	18000.00	152.09	33.00
Cd3-6,4,5	100	157	435.13	157	0	Cd3-6,4,10	100	196	4341.92	196	0	Cd3-6,4,15	200	281	18000.00	212.89	24.24
Cd4-2,3,5	100	202	337.53	202	0	Cd4-2,3,10	100	257	1845.55	257	0	Cd4-2,3,15	200	338	18000.00	254.39	24.74
Cd4-3,5,5	100	162	325.12	162	0	Cd4-3,5,10	100	241	3545.78	241	0	Cd4-3,5,15	200	295	18000.00	206.79	29.90
Cd4-6,4,5	100	147	204.10	147	0	Cd4-6,4,10	100	189	4254.66	189	0	Cd4-6,4,15	200	266	18000.00	184.60	30.60
Cd5-2,3,5	100	178	351.91	178	0	Cd5-2,3,10	100	207	3537.62	207	0	Cd5-2,3,15	200	274	18000.00	198.78	27.45
Cd5-3,5,5	100	178	437.20	178	0	Cd5-3,5,10	100	213	892.31	213	0	Cd5-3,5,15	200	254	18000.00	193.76	23.72
Cd5-6,4,5	100	157	153.18	157	0	Cd5-6,4,10	100	184	4454.43	184	0	Cd5-6,4,15	200	214	18000.00	126.97	40.67
Averages			985.463		0	***Averages***			3672.88		0	***Averages***			16679.36		21.07

Table 13: DDD results on instances with 5, 10 and 15 OD demands using the time-space formulation

ID	Ψ	UB	CPU (sec)	LB	OG (%)	ID	Ψ	UB	CPU (sec)	LB	OG (%)
Ca1-2,3,30	200	530	2006.52	530	0.00	Ca1-2,3,50	200	808	28800	742.50	8.11
Ca1-3,5,30	200	490	12877.17	490	0.00	Ca1-3,5,50	200	649	28800	585.80	9.74
Ca1-6,4,30	200	417	18000	398.1389	4.52	Ca1-6,4,50	200	710	28800	606.11	14.63
Ca2-2,3,30	200	441	9001.31	441	0.00	Ca2-2,3,50	200	713	28800	410.33	42.45
Ca2-3,5,30	200	439	1214.24	439	0.00	Ca2-3,5,50	200	817	28800	510.96	37.46
Ca2-6,4,30	200	448	18000	436.5814	2.55	Ca2-6,4,50	200	784	28800	707.58	9.75
Ca3-2,3,30	200	570	15555.90	570	0.00	Ca3-2,3,50	200	770	28800	664.91	13.65
Ca3-3,5,30	200	428	12028.32	428	0.00	Ca3-3,5,50	200	573	28800	405.81	29.18
Ca3-6,4,30	200	445	12407.91	445	0.00	Ca3-6,4,50	200	745	28800	662.60	11.06
Ca4-2,3,30	200	508	15297.08	508	0.00	Ca4-2,3,50	200	685	28800	632.00	7.74
Ca4-3,5,30	200	423	18000	399.735	5.50	Ca4-3,5,50	200	650	28800	554.98	14.62
Ca4-6,4,30	200	392	10124.80	392	0.00	Ca4-6,4,50	200	793	28800	608.65	23.25
Ca5-2,3,30	200	429	11249.56	429	0.00	Ca5-2,3,50	200	812	28800	771.54	4.98
Ca5-3,5,30	200	499	18000	487.4232	2.32	Ca5-3,5,50	200	734	28800	488.00	33.51
Ca5-6,4,30	200	406	13173.54	406	0.00	Ca5-6,4,50	200	820	28800	710.29	13.38
Cb1-2,3,30	200	492	10787.85	492	0.00	Cb1-2,3,50	200	790	28800	683.60	13.47
Cb1-3,5,30	200	510	18000	454.869	10.81	Cb1-3,5,50	200	729	28800	578.56	20.64
Cb1-6,4,30	200	421	18000	389.26	7.54	Cb1-6,4,50	200	745	28800	597.25	19.83
Cb2-2,3,30	200	402	12971.62	402	0.00	Cb2-2,3,50	200	680	28800	440.46	35.23
Cb2-3,5,30	200	436	18000	421.65	3.29	Cb2-3,5,50	200	835	28800	518.71	37.88
Cb2-6,4,30	200	437	10177.10	437	0.00	Cb2-6,4,50	200	837	28800	742.92	11.24
Cb3-2,3,30	200	553	18000	494.76	10.53	Cb3-2,3,50	200	780	28800	588.23	24.59
Cb3-3,5,30	200	432	18000	409.66	5.17	Cb3-3,5,50	200	656	28800	535.00	18.45
Cb3-6,4,30	200	473	18000	453.13	4.20	Cb3-6,4,50	200	683	28800	614.19	10.07
Cb4-2,3,30	200	505	18000	433.44	14.17	Cb4-2,3,50	200	798	28800	432.00	45.86
Cb4-3,5,30	200	437	18000	410.73	6.01	Cb4-3,5,50	200	675	28800	522.44	22.60
Cb4-6,4,30	200	395	18000	378.37	4.21	Cb4-6,4,50	200	788	28800	670.66	14.89
Cb5-2,3,30	200	396	14560.44	396	0.00	Cb5-2,3,50	200	726	28800	639.87	11.86
Cb5-3,5,30	200	527	18000	505.02	4.17	Cb5-3,5,50	200	654	28800	514.33	21.36
Cb5-6,4,30	200	418	18000	370.18	11.44	Cb5-6,4,50	200	797	28800	717.83	9.93
Cc1-2,3,30	200	411	18000	411	0.00	Cc1-2,3,50	200	767	28800	663.57	13.49
Cc1-3,5,30	200	536	18000	485.83	9.36	Cc1-3,5,50	200	691	28800	629.23	8.94
Cc1-6,4,30	200	393	18000	361.20	8.09	Cc1-6,4,50	200	632	28800	562.32	11.03
Cc2-2,3,30	200	423	18000	374.56	11.45	Cc2-2,3,50	200	700	28800	547.60	21.77
Cc2-3,5,30	200	418	18000	375.69	10.12	Cc2-3,5,50	200	726	28800	543.50	25.14
Cc2-6,4,30	200	445	18000	411.58	7.51	Cc2-6,4,50	200	680	28800	573.80	15.62
Cc3-2,3,30	200	552	18000	506.62	8.22	Cc3-2,3,50	200	663	28800	521.80	21.30
Cc3-3,5,30	200	450	18000	409.81	8.93	Cc3-3,5,50	200	618	28800	458.83	25.75
Cc3-6,4,30	200	443	18000	404.85	8.61	Cc3-6,4,50	200	652	28800	493.58	24.30
Cc4-2,3,30	200	501	18000	454.30	9.32	Cc4-2,3,50	200	593	28800	498.80	15.89
Cc4-3,5,30	200	418	18000	370.64	11.33	Cc4-3,5,50	200	705	28800	544.39	22.78
Cc4-6,4,30	200	378	18000	341.14	9.75	Cc4-6,4,50	200	637	28800	371.41	41.69
Cc5-2,3,30	200	423	18000	402.61	4.82	Cc5-2,3,50	200	735	28800	472.97	35.65
Cc5-3,5,30	200	429	18000	396.69	7.53	Cc5-3,5,50	200	619	28800	461.00	25.53
Cc5-6,4,30	200	390	18000	364.45	6.55	Cc5-6,4,50	200	774	28800	689.63	10.9
Cd1-2,3,30	200	445	13823.36	445	0.00	Cd1-2,3,50	200	640	28800	560.32	12.45
Cd1-3,5,30	200	533	13043.42	533	0.00	Cd1-3,5,50	200	664	28800	554.97	16.42
Cd1-6,4,30	200	412	12720.21	412	0.00	Cd1-6,4,50	200	688	28800	604.82	12.09
Cd2-2,3,30	200	429	18000	388.24	9.50	Cd2-2,3,50	200	741	28800	606.21	18.19
Cd2-3,5,30	200	435	15479.58	435	0.00	Cd2-3,5,50	200	769	28800	562.75	26.82
Cd2-6,4,30	200	434	16873.03	434	0.00	Cd2-6,4,50	200	680	28800	607.92	10.6
Cd3-2,3,30	200	550	18000	481.25	12.50	Cd3-2,3,50	200	625	28800	542.50	13.2
Cd3-3,5,30	200	440	8435.23	440	0.00	Cd3-3,5,50	200	771	28800	651.42	15.51
Cd3-6,4,30	200	437	18000	420.65	3.74	Cd3-6,4,50	200	712	28800	603.70	15.21
Cd4-2,3,30	200	510	12095.64	510	0.00	Cd4-2,3,50	200	614	28800	438.82	28.53
Cd4-3,5,30	200	424	18000	389.02	8.25	Cd4-3,5,50	200	678	28800	605.38	10.71
Cd4-6,4,30	200	395	18000	384.57	2.64	Cd4-6,4,50	200	648	28800	583.91	9.89
Cd5-2,3,30	200	425	10783.34	425	0.00	Cd5-2,3,50	200	764	28800	697.99	8.64
Cd5-3,5,30	200	501	10556.08	501	0.00	Cd5-3,5,50	200	641	28800	556.51	13.18
Cd5-6,4,30	200	432	13661.84	432	0.00	Cd5-6,4,50	200	857	28800	767.53	10.44
Averages			15348.42		4.24	***Averages***			28800		18.88

Table 14: DDD results on instances with 30 and 50 OD demands

ID	Ψ	UB	CPU (sec)	LB	OG (%)	ID	Ψ	UB	CPU (sec)	LB	OG (%)
Ca1-2,3,30	200	542	18000	418.42	22.80	Ca1-2,3,50	200	1018	28800	607.13	40.36
Ca1-3,5,30	200	548	18000	409.38	25.29	Ca1-3,5,50	200	N.A	28800	N.A	N.A
Ca1-6,4,30	200	616	18000	336.00	45.45	Ca1-6,4,50	200	N.A	28800	N.A	N.A
Ca2-2,3,30	200	481	18000	353.93	26.42	Ca2-2,3,50	200	819	28800	409.14	50.04
Ca2-3,5,30	200	439	6602.54	439.00	0.00	Ca2-3,5,50	200	N.A	28800	N.A	N.A
Ca2-6,4,30	200	583	18000	433.39	25.66	Ca2-6,4,50	200	N.A	28800	N.A	N.A
Ca3-2,3,30	200	713	18000	354.52	50.28	Ca3-2,3,50	200	1010	28800	491.68	51.32
Ca3-3,5,30	200	557	18000	284.06	49.00	Ca3-3,5,50	200	N.A	28800	N.A	N.A
Ca3-6,4,30	200	594	18000	335.30	43.55	Ca3-6,4,50	200	N.A	28800	N.A	N.A
Ca4-2,3,30	200	573	18000	429.24	25.09	Ca4-2,3,50	200	890	28800	486.17	45.37
Ca4-3,5,30	200	488	18000	351.20	28.03	Ca4-3,5,50	200	N.A	28800	N.A	N.A
Ca4-6,4,30	200	520	18000	290.77	44.08	Ca4-6,4,50	200	N.A	28800	N.A	N.A
Ca5-2,3,30	200	518	18000	274.52	47.00	Ca5-2,3,50	200	1049	28800	577.54	44.94
Ca5-3,5,30	200	516	18000	375.33	27.26	Ca5-3,5,50	200	N.A	28800	N.A	N.A
Ca5-6,4,30	200	549	18000	375.75	31.56	Ca5-6,4,50	200	N.A	28800	N.A	N.A
Cb1-2,3,30	200	561	18000	345.97	38.33	Cb1-2,3,50	200	1050	28800	508.18	51.60
Cb1-3,5,30	200	702	18000	480.47	31.56	Cb1-3,5,50	200	N.A	28800	N.A	N.A
Cb1-6,4,30	200	528	18000	265.76	49.67	Cb1-6,4,50	200	N.A	28800	N.A	N.A
Cb2-2,3,30	200	582	18000	286.90	50.70	Cb2-2,3,50	200	849	28800	429.67	49.39
Cb2-3,5,30	200	553	18000	301.47	45.49	Cb2-3,5,50	200	N.A	28800	N.A	N.A
Cb2-6,4,30	200	576	18000	315.15	45.29	Cb2-6,4,50	200	N.A	28800	N.A	N.A
Cb3-2,3,30	200	616	18000	393.11	36.18	Cb3-2,3,50	200	1020	28800	508.25	50.17
Cb3-3,5,30	200	549	18000	387.74	29.37	Cb3-3,5,50	200	N.A	28800	N.A	N.A
Cb3-6,4,30	200	586	18000	373.50	36.26	Cb3-6,4,50	200	N.A	28800	N.A	N.A
Cb4-2,3,30	200	575	18000	428.42	25.49	Cb4-2,3,50	200	952	28800	423.85	55.48
Cb4-3,5,30	200	614	18000	434.47	29.24	Cb4-3,5,50	200	N.A	28800	N.A	N.A
Cb4-6,4,30	200	511	18000	342.67	32.94	Cb4-6,4,50	200	N.A	28800	N.A	N.A
Cb5-2,3,30	200	456	18000	241.95	46.94	Cb5-2,3,50	200	997	28800	554.78	44.36
Cb5-3,5,30	200	649	18000	433.26	33.24	Cb5-3,5,50	200	N.A	28800	N.A	N.A
Cb5-6,4,30	200	512	18000	297.06	41.98	Cb5-6,4,50	200	N.A	28800	N.A	N.A
Cc1-2,3,30	200	536	18000	330.78	38.29	Cc1-2,3,50	200	984	28800	592.02	39.84
Cc1-3,5,30	200	873	18000	517.15	40.76	Cc1-3,5,50	200	N.A	28800	N.A	N.A
Cc1-6,4,30	200	696	18000	380.85	45.28	Cc1-6,4,50	200	N.A	28800	N.A	N.A
Cc2-2,3,30	200	634	18000	400.56	36.82	Cc2-2,3,50	200	913	28800	500.54	45.18
Cc2-3,5,30	200	652	18000	417.32	35.99	Cc2-3,5,50	200	N.A	28800	N.A	N.A
Cc2-6,4,30	200	697	18000	360.73	48.24	Cc2-6,4,50	200	N.A	28800	N.A	N.A
Cc3-2,3,30	200	809	18000	455.79	43.66	Cc3-2,3,50	200	1097	28800	504.77	53.99
Cc3-3,5,30	200	669	18000	383.55	42.67	Cc3-3,5,50	200	N.A	28800	N.A	N.A
Cc3-6,4,30	200	681	18000	404.33	40.63	Cc3-6,4,50	200	N.A	28800	N.A	N.A
Cc4-2,3,30	200	832	18000	410.83	50.62	Cc4-2,3,50	200	806	28800	466.72	42.09
Cc4-3,5,30	200	574	18000	379.32	33.92	Cc4-3,5,50	200	N.A	28800	N.A	N.A
Cc4-6,4,30	200	521	18000	333.55	35.98	Cc4-6,4,50	200	N.A	28800	N.A	N.A
Cc5-2,3,30	200	516	18000	380.26	26.31	Cc5-2,3,50	200	908	28800	458.25	49.53
Cc5-3,5,30	200	576	18000	417.31	27.55	Cc5-3,5,50	200	N.A	28800	N.A	N.A
Cc5-6,4,30	200	644	18000	357.40	44.50	Cc5-6,4,50	200	N.A	28800	N.A	N.A
Cd1-2,3,30	200	588	18000	363.64	38.16	Cd1-2,3,50	200	902	28800	540.51	40.08
Cd1-3,5,30	200	655	18000	410.98	37.25	Cd1-3,5,50	200	N.A	28800	N.A	N.A
Cd1-6,4,30	200	544	18000	277.67	48.96	Cd1-6,4,50	200	N.A	28800	N.A	N.A
Cd2-2,3,30	200	637	18000	389.28	38.89	Cd2-2,3,50	200	1002	28800	606.04	39.52
Cd2-3,5,30	200	559	18000	348.86	37.59	Cd2-3,5,50	200	N.A	28800	N.A	N.A
Cd2-6,4,30	200	450	18000	393.57	12.54	Cd2-6,4,50	200	N.A	28800	N.A	N.A
Cd3-2,3,30	200	647	18000	468.87	27.53	Cd3-2,3,50	200	843	28800	376.43	55.35
Cd3-3,5,30	200	534	18000	348.58	34.72	Cd3-3,5,50	200	N.A	28800	N.A	N.A
Cd3-6,4,30	200	653	18000	403.99	38.13	Cd3-6,4,50	200	N.A	28800	N.A	N.A
Cd4-2,3,30	200	687	18000	497.36	27.60	Cd4-2,3,50	200	754	28800	405.51	46.22
Cd4-3,5,30	200	530	18000	352.39	33.51	Cd4-3,5,50	200	N.A	28800	N.A	N.A
Cd4-6,4,30	200	527	18000	303.49	42.41	Cd4-6,4,50	200	N.A	28800	N.A	N.A
Cd5-2,3,30	200	536	18000	365.00	31.90	Cd5-2,3,50	200	1044	28800	452.35	56.67
Cd5-3,5,30	200	646	18000	374.14	42.08	Cd5-3,5,50	200	N.A	28800	N.A	N.A
Cd5-6,4,30	200	565	18000	404.74	28.37	Cd5-6,4,50	200	N.A	28800	N.A	N.A
Averages			18000		36.25	***Averages***			28800		47.57

Table 15: DDD results on instances with 30 and 50 OD demands using the time-space formulation

ID	Ψ	UB	CPU (sec)	LB	OG (%)	ID	Ψ	UB	CPU (sec)	LB	OG (%)	ID	Ψ	UB	CPU (sec)	LB	OG (%)
Ca1-2,3,5	100	171	0.87	171	0	Ca1-2,3,10	100	226	24.25	226	0	Ca1-2,3,15	200	270	60.19	270	0
Ca1-3,5,5	100	185	2.34	185	0	Ca1-3,5,10	100	199	13.59	199	0	Ca1-3,5,15	200	279	106.11	279	0
Ca1-6,4,5	100	195	6.95	195	0	Ca1-6,4,10	100	247	19.67	247	0	Ca1-6,4,15	200	274	789.99	274	0
Ca2-2,3,5	100	136	1.04	136	0	Ca2-2,3,10	100	161	17.4	161	0	Ca2-2,3,15	200	306	18.54	306	0
Ca2-3,5,5	100	199	0.83	199	0	Ca2-3,5,10	100	260	10.76	260	0	Ca2-3,5,15	200	332	208.01	332	0
Ca2-6,4,5	100	150	4.96	150	0	Ca2-6,4,10	100	179	72.76	179	0	Ca2-6,4,15	200	272	724.68	272	0
Ca3-2,3,5	100	287	0.22	287	0	Ca3-2,3,10	100	344	0.88	344	0	Ca3-2,3,15	200	285	14.50	285	0
Ca3-3,5,5	100	198	1.89	198	0	Ca3-3,5,10	100	211	29.24	211	0	Ca3-3,5,15	200	226	182.90	226	0
Ca3-6,4,5	100	154	5.06	154	0	Ca3-6,4,10	100	220	19.04	220	0	Ca3-6,4,15	200	261	265.32	261	0
Ca4-2,3,5	100	197	0.46	197	0	Ca4-2,3,10	100	306	39.93	306	0	Ca4-2,3,15	200	309	36.50	309	0
Ca4-3,5,5	100	150	3.2	150	0	Ca4-3,5,10	100	213	31.29	213	0	Ca4-3,5,15	200	245	743.13	245	0
Ca4-6,4,5	100	154	6.32	154	0	Ca4-6,4,10	100	230	69.39	230	0	Ca4-6,4,15	200	249	128.39	249	0
Ca5-2,3,5	100	185	0.33	185	0	Ca5-2,3,10	100	256	2.85	256	0	Ca5-2,3,15	200	265	21.18	265	0
Ca5-3,5,5	100	141	2.1	141	0	Ca5-3,5,10	100	230	18.43	230	0	Ca5-3,5,15	200	262	92.58	262	0
Ca5-6,4,5	100	159	4.51	159	0	Ca5-6,4,10	100	187	46.47	187	0	Ca5-6,4,15	200	218	74.68	218	0
Cb1-2,3,5	100	139	0.8	139	0	Cb1-2,3,10	100	181	2.78	181	0	Cb1-2,3,15	200	275	54.05	275	0
Cb1-3,5,5	100	147	2.17	147	0	Cb1-3,5,10	100	207	24.51	207	0	Cb1-3,5,15	200	247	241.33	247	0
Cb1-6,4,5	100	199	3.83	199	0	Cb1-6,4,10	100	238	63.94	238	0	Cb1-6,4,15	200	295	1466.17	295	0
Cb2-2,3,5	100	129	0.35	129	0	Cb2-2,3,10	100	171	12.1	171	0	Cb2-2,3,15	200	268	15.24	268	0
Cb2-3,5,5	100	152	2.03	152	0	Cb2-3,5,10	100	241	30.76	241	0	Cb2-3,5,15	200	337	247.10	337	0
Cb2-6,4,5	100	143	5.88	143	0	Cb2-6,4,10	100	185	133.02	185	0	Cb2-6,4,15	200	290	432.00	290	0
Cb3-2,3,5	100	332	0.11	332	0	Cb3-2,3,10	100	337	6.46	337	0	Cb3-2,3,15	200	304	24.60	304	0
Cb3-3,5,5	100	133	2.51	133	0	Cb3-3,5,10	100	200	122.22	200	0	Cb3-3,5,15	200	226	9093.18	226	0
Cb3-6,4,5	100	198	3.56	198	0	Cb3-6,4,10	100	254	41.3	254	0	Cb3-6,4,15	200	266	972.32	266	0
Cb4-2,3,5	100	280	0.55	280	0	Cb4-2,3,10	100	251	25.72	251	0	Cb4-2,3,15	200	298	57.86	298	0
Cb4-3,5,5	100	135	3.71	135	0	Cb4-3,5,10	100	231	2651.79	231	0	Cb4-3,5,15	200	251	65.95	251	0
Cb4-6,4,5	100	188	7.16	188	0	Cb4-6,4,10	100	232	47.33	232	0	Cb4-6,4,15	200	264	348.13	264	0
Cb5-2,3,5	100	129	0.59	129	0	Cb5-2,3,10	100	184	6.35	184	0	Cb5-2,3,15	200	249	103.93	249	0
Cb5-3,5,5	100	146	1.57	146	0	Cb5-3,5,10	100	184	262.08	184	0	Cb5-3,5,15	200	243	120.09	243	0
Cb5-6,4,5	100	199	4.9	199	0	Cb5-6,4,10	100	224	39.64	224	0	Cb5-6,4,15	200	223	8273.80	223	0
Cc1-2,3,5	100	129	1.26	129	0	Cc1-2,3,10	100	189	181.9	189	0	Cc1-2,3,15	200	265	205.65	265	0
Cc1-3,5,5	100	135	4.2	135	0	Cc1-3,5,10	100	180	194.61	180	0	Cc1-3,5,15	200	233	4940.03	233	0
Cc1-6,4,5	100	150	7.94	150	0	Cc1-6,4,10	100	238	764.7	238	0	Cc1-6,4,15	200	280	2901.32	280	0
Cc2-2,3,5	100	122	0.78	122	0	Cc2-2,3,10	100	187	120.43	187	0	Cc2-2,3,15	200	284	165.56	284	0
Cc2-3,5,5	100	175	4.5	175	0	Cc2-3,5,10	100	231	173.82	231	0	Cc2-3,5,15	200	299	285.15	299	0
Cc2-6,4,5	100	122	8.8	122	0	Cc2-6,4,10	100	163	3248.49	163	0	Cc2-6,4,15	200	273	4266.17	273	0
Cc3-2,3,5	100	136	1.2	136	0	Cc3-2,3,10	100	184	32.61	184	0	Cc3-2,3,15	200	271	828.79	271	0
Cc3-3,5,5	100	142	3.89	142	0	Cc3-3,5,10	100	225	308.58	225	0	Cc3-3,5,15	200	223	651.09	223	0
Cc3-6,4,5	100	157	7.71	157	0	Cc3-6,4,10	100	193	700.1	193	0	Cc3-6,4,15	200	259	8204.47	259	0
Cc4-2,3,5	100	171	1.37	171	0	Cc4-2,3,10	100	225	62.13	225	0	Cc4-2,3,15	200	307	261.46	307	0
Cc4-3,5,5	100	154	4.56	154	0	Cc4-3,5,10	100	223	193.08	223	0	Cc4-3,5,15	200	250	1686.34	250	0
Cc4-6,4,5	100	138	8.23	138	0	Cc4-6,4,10	100	230	702	230	0	Cc4-6,4,15	200	268	1221.56	268	0
Cc5-2,3,5	100	123	1.09	123	0	Cc5-2,3,10	100	182	51.66	182	0	Cc5-2,3,15	200	236	51.99	236	0
Cc5-3,5,5	100	124	3.37	124	0	Cc5-3,5,10	100	168	1068.67	168	0	Cc5-3,5,15	200	247	348.89	247	0
Cc5-6,4,5	100	138	9.7	138	0	Cc5-6,4,10	100	179	5168.43	179	0	Cc5-6,4,15	200	208	767.43	208	0
Cd1-2,3,5	100	155	1.06	155	0	Cd1-2,3,10	100	197	3.03	197	0	Cd1-2,3,15	200	293	17.37	293	0
Cd1-3,5,5	100	170	1.18	170	0	Cd1-3,5,10	100	215	3.7	215	0	Cd1-3,5,15	200	304	260.80	304	0
Cd1-6,4,5	100	188	8.07	188	0	Cd1-6,4,10	100	248	125.57	248	0	Cd1-6,4,15	200	277	907.69	277	0
Cd2-2,3,5	100	140	0.56	140	0	Cd2-2,3,10	100	193	3.41	193	0	Cd2-2,3,15	200	241	10.60	241	0
Cd2-3,5,5	100	158	2.58	158	0	Cd2-3,5,10	100	231	19.82	231	0	Cd2-3,5,15	200	319	25.77	319	0
Cd2-6,4,5	100	142	8.59	142	0	Cd2-6,4,10	100	213	81.68	213	0	Cd2-6,4,15	200	274	597.98	274	0
Cd3-2,3,5	100	142	1.17	142	0	Cd3-2,3,10	100	186	4.79	186	0	Cd3-2,3,15	200	257	6.42	257	0
Cd3-3,5,5	100	147	2.78	147	0	Cd3-3,5,10	100	224	18.39	224	0	Cd3-3,5,15	200	222	248.68	222	0
Cd3-6,4,5	100	157	5.73	157	0	Cd3-6,4,10	100	196	29.91	196	0	Cd3-6,4,15	200	268	939.95	268	0
Cd4-2,3,5	100	202	0.82	202	0	Cd4-2,3,10	100	243	2.25	243	0	Cd4-2,3,15	200	324	57.97	324	0
Cd4-3,5,5	100	162	2.07	162	0	Cd4-3,5,10	100	241	15.96	241	0	Cd4-3,5,15	200	251	229.10	251	0
Cd4-6,4,5	100	147	7.59	147	0	Cd4-6,4,10	100	187	55.48	187	0	Cd4-6,4,15	200	263	537.24	263	0
Cd5-2,3,5	100	178	0.7	178	0	Cd5-2,3,10	100	207	1.59	207	0	Cd5-2,3,15	200	271	32.12	271	0
Cd5-3,5,5	100	178	2.67	178	0	Cd5-3,5,10	100	213	9.55	213	0	Cd5-3,5,15	200	247	92.34	247	0
Cd5-6,4,5	100	157	4.92	157	0	Cd5-6,4,10	100	184	39.76	184	0	Cd5-6,4,15	200	208	770.27	208	0
Averages			3.33		0	***Averages***			287.87		0	***Averages***			941.71		0

Table 16: DDD results on instances with 5, 10 and 15 OD demands and disabled availability time

ID	Ψ	UB	CPU (sec)	LB	OG (%)	ID	Ψ	UB	CPU (sec)	LB	OG (%)	ID	Ψ	UB	CPU (sec)	LB	OG (%)
Ca1-2,3,5	100	144	0.64	144	0	Ca1-2,3,10	100	214	4630.13	214	0	Ca1-2,3,15	200	267	63.59	267.00	0.00
Ca1-3,5,5	100	171	16.18	171	0	Ca1-3,5,10	100	201	3626.34	201	0	Ca1-3,5,15	200	276	222.42	276.00	0.00
Ca1-6,4,5	100	166	15.23	166	0	Ca1-6,4,10	100	225	9000	213.21	5.24	Ca1-6,4,15	200	272	934.90	272.00	0.00
Ca2-2,3,5	100	134	1.28	134	0	Ca2-2,3,10	100	208	9.22	208	0	Ca2-2,3,15	200	305	43.23	305.00	0.00
Ca2-3,5,5	100	181	3.87	181	0	Ca2-3,5,10	100	197	28.89	197	0	Ca2-3,5,15	200	328	286.45	328.00	0.00
Ca2-6,4,5	100	148	9.13	148	0	Ca2-6,4,10	100	204	9000.00	173.196	15.1	Ca2-6,4,15	200	270	758.88	270.00	0.00
Ca3-2,3,5	100	144	0.88	144	0	Ca3-2,3,10	100	160	22.62	160	0	Ca3-2,3,15	200	255	26.67	255.00	0.00
Ca3-3,5,5	100	171	2.42	171	0	Ca3-3,5,10	100	208	104.75	208	0	Ca3-3,5,15	200	224	692.66	224.00	0.00
Ca3-6,4,5	100	148	28.61	148	0	Ca3-6,4,10	100	171	7404.46	171	0	Ca3-6,4,15	200	260	963.44	260.00	0.00
Ca4-2,3,5	100	180	1.53	180	0	Ca4-2,3,10	100	202	67.71	202	0	Ca4-2,3,15	200	306	92.77	306.00	0.00
Ca4-3,5,5	100	147	2.21	147	0	Ca4-3,5,10	100	192	8053.48	192	0	Ca4-3,5,15	200	244	259.36	244.00	0.00
Ca4-6,4,5	100	150	10.61	150	0	Ca4-6,4,10	100	198	422.81	198	0	Ca4-6,4,15	200	218	8500.72	218.00	0.00
Ca5-2,3,5	100	173	0.60	173	0	Ca5-2,3,10	100	225	83.81	225	0	Ca5-2,3,15	200	263	138.70	263.00	0.00
Ca5-3,5,5	100	141	2.08	141	0	Ca5-3,5,10	100	194	455.02	194	0	Ca5-3,5,15	200	238	109.63	238.00	0.00
Ca5-6,4,5	100	159	8.52	159	0	Ca5-6,4,10	100	202	118.65	202	0	Ca5-6,4,15	200	201	8960.25	201.00	0.00
Cb1-2,3,5	100	136	0.53	136	0	Cb1-2,3,10	100	166	1418.14	166	0	Cb1-2,3,15	200	270	99.10	270.00	0.00
Cb1-3,5,5	100	149	14.15	149	0	Cb1-3,5,10	100	190	6046.12	190		Cb1-3,5,15	200	261	9000.00	237.80	8.89
Cb1-6,4,5	100	171	12.03	171	0	Cb1-6,4,10	100	203	9000	191.1448	5.84	Cb1-6,4,15	200	277	1211.64	277.00	0.00
Cb2-2,3,5	100	128	0.70	128	0	Cb2-2,3,10	100	170	111.28	170	0	Cb2-2,3,15	200	268	31.27	268.00	0.00
Cb2-3,5,5	100	135	1.92	135	0	Cb2-3,5,10	100	195	9000	188.292	3.44	Cb2-3,5,15	200	332	571.72	332.00	0.00
Cb2-6,4,5	100	142	8.86	142	0	Cb2-6,4,10	100	173	5035.45	173	0	Cb2-6,4,15	200	287	7900.56	287.00	0.00
Cb3-2,3,5	100	266	1.53	266	0	Cb3-2,3,10	100	286	558.57	286	0	Cb3-2,3,15	200	277	231.81	277.00	0.00
Cb3-3,5,5	100	129	5.32	129	0	Cb3-3,5,10	100	198	6481.42	198	0	Cb3-3,5,15	200	224	2767.83	224.00	0.00
Cb3-6,4,5	100	167	19.53	167	0	Cb3-6,4,10	100	237	9000	223.5621	5.67	Cb3-6,4,15	200	264	5908.05	264.00	0.00
Cb4-2,3,5	100	156	0.77	156	0	Cb4-2,3,10	100	220	5457.06	220	0	Cb4-2,3,15	200	297	75.03	297.00	0.00
Cb4-3,5,5	100	135	2.07	135	0	Cb4-3,5,10	100	215	810.03	215	0	Cb4-3,5,15	200	204	4248.78	204.00	0.00
Cb4-6,4,5	100	170	10.96	170	0	Cb4-6,4,10	100	262	9000	245.6774	6.23	Cb4-6,4,15	200	263	1304.74	263.00	0.00
Cb5-2,3,5	100	129	0.32	129	0	Cb5-2,3,10	100	185	7527.81	0	0	Cb5-2,3,15	200	229	38.39	229.00	0.00
Cb5-3,5,5	100	152	4.20	152	0	Cb5-3,5,10	100	176	180.39	0	0	Cb5-3,5,15	200	238	143.96	238.00	0.00
Cb5-6,4,5	100	158	10.2	158	0	Cb5-6,4,10	100	319	8244.13	0	0	Cb5-6,4,15	200	219	9000.00	206.82	5.56
Cc1-2,3,5	100	129	0.51	129	0	Cc1-2,3,10	100	189	33	189	0	Cc1-2,3,15	200	262	489.25	262.00	0.00
Cc1-3,5,5	100	135	2.15	135	0	Cc1-3,5,10	100	180	35.8	180	0	Cc1-3,5,15	200	233	348.71	233.00	0.00
Cc1-6,4,5	100	150	4.92	150	0	Cc1-6,4,10	100	238	99.58	238	0	Cc1-6,4,15	200	278	9000.00	265.05	4.66
Cc2-2,3,5	100	122	0.25	122	0	Cc2-2,3,10	100	187	23.43	187	0	Cc2-2,3,15	200	283	190.07	283.00	0.00
Cc2-3,5,5	100	174	2.89	174	0	Cc2-3,5,10	100	231	62.14	231	0	Cc2-3,5,15	200	259	307.38	259.00	0.00
Cc2-6,4,5	100	122	5.73	122	0	Cc2-6,4,10	100	163	869.12	163	0	Cc2-6,4,15	200	270	7602.87	270.00	0.00
Cc3-2,3,5	100	136	0.43	136	0	Cc3-2,3,10	100	184	15.29	184	0	Cc3-2,3,15	200	270	382.08	270.00	0.00
Cc3-3,5,5	100	142	2.12	142	0	Cc3-3,5,10	100	225	64.8	225	0	Cc3-3,5,15	200	222	1910.51	222.00	0.00
Cc3-6,4,5	100	157	5.87	157	0	Cc3-6,4,10	100	193	178.57	193	0	Cc3-6,4,15	200	258	9000.00	249.33	3.36
Cc4-2,3,5	100	171	0.71	171	0	Cc4-2,3,10	100	225	20.91	225	0	Cc4-2,3,15	200	307	398.04	307.00	0.00
Cc4-3,5,5	100	154	2.27	154	0	Cc4-3,5,10	100	223	72.43	223	0	Cc4-3,5,15	200	249	9000.00	246.09	1.17
Cc4-6,4,5	100	138	6.54	138	0	Cc4-6,4,10	100	230	230.58	230	0	Cc4-6,4,15	200	265	519.87	265.00	0.00
Cc5-2,3,5	100	123	0.46	123	0	Cc5-2,3,10	100	182	21.87	182	0	Cc5-2,3,15	200	218	20.72	218.00	0.00
Cc5-3,5,5	100	124	1.25	124	0	Cc5-3,5,10	100	168	56.6	168	0	Cc5-3,5,15	200	244	202.62	244.00	0.00
Cc5-6,4,5	100	138	7.90	138	0	Cc5-6,4,10	100	179	5088.73	179	0	Cc5-6,4,15	200	208	1732.72	208.00	0.00
Cd1-2,3,5	100	143	0.83	143	0	Cd1-2,3,10	100	179	19.88	179	0	Cd1-2,3,15	200	267	37.17	267.00	0.00
Cd1-3,5,5	100	165	0.57	165	0	Cd1-3,5,10	100	203	7.42	203	0	Cd1-3,5,15	200	278	264.90	278.00	0.00
Cd1-6,4,5	100	157	5.72	157	0	Cd1-6,4,10	100	211	176.35	211	0	Cd1-6,4,15	200	274	1929.13	274.00	0.00
Cd2-2,3,5	100	140	0.56	140	0	Cd2-2,3,10	100	176	16.62	176	0	Cd2-2,3,15	200	240	21.44	240.00	0.00
Cd2-3,5,5	100	150	2.11	150	0	Cd2-3,5,10	100	216	42	216	0	Cd2-3,5,15	200	275	150.03	275.00	0.00
Cd2-6,4,5	100	131	6.16	131	0	Cd2-6,4,10	100	190	7942.26	190	0	Cd2-6,4,15	200	273	647.53	273.00	0.00
Cd3-2,3,5	100	139	0.78	139	0	Cd3-2,3,10	100	171	14.85	171	0	Cd3-2,3,15	200	224	5.10	224.00	0.00
Cd3-3,5,5	100	144	2.00	144	0	Cd3-3,5,10	100	199	123.8	199	0	Cd3-3,5,15	200	221	394.04	221.00	0.00
Cd3-6,4,5	100	138	4.76	138	0	Cd3-6,4,10	100	175	206.05	175	0	Cd3-6,4,15	200	266	1460.48	266.00	0.00
Cd4-2,3,5	100	175	0.82	175	0	Cd4-2,3,10	100	223	19.12	223	0	Cd4-2,3,15	200	316	77.34	316.00	0.00
Cd4-3,5,5	100	160	2.05	160	0	Cd4-3,5,10	100	217	74.25	217	0	Cd4-3,5,15	200	247	174.70	247.00	0.00
Cd4-6,4,5	100	136	7.22	136	0	Cd4-6,4,10	100	186	215	186	0	Cd4-6,4,15	200	260	432.44	260.00	0.00
Cd5-2,3,5	100	144	0.78	144	0	Cd5-2,3,10	100	183	240.52	183	0	Cd5-2,3,15	200	270	18.32	270.00	0.00
Cd5-3,5,5	100	160	2.53	160	0	Cd5-3,5,10	100	191	93.78	191	0	Cd5-3,5,15	200	247	92.71	247.00	0.00
Cd5-6,4,5	100	157	9.31	157	0	Cd5-6,4,10	100	183	287.22	183	0	Cd5-6,4,15	200	208	853.88	208.00	0.00
Averages			4.78		0	***Averages***			2287.50		0.69	***Averages***			1870.84		0.394

Table 17: DDD results on instances with 5, 10 and 15 OD demands with disabled availability time and synchronization

C Degeneracy

Figure 5 illustrates the impact of the degeneracy procedure using a coarse granularity ($\Delta = 2$). The performance of the DDD is illustrated by contrasting the average optimality gap obtained by the DDD using the degeneracy procedure, for each instance type CA, CB, CC, and CD, to that of the DDD without the degeneracy procedure, identified as NCA, NCB, NCC, and NCD. The experimental results show that, using the degeneracy procedure leads to a general improvement of the optimality gap over the entire instance set. One also observes that, wider customer time windows, reduce the temporal preciseness of the reduced time-space network and provide broader refining options for short arcs, which, in turn drives degeneracy on the integer problem. Instances with broader and sparse time windows, most notably, instances of CC and CD types, tend to benefit from the degeneracy procedure, displaying optimality-gap improvements of 78% and 84%, respectively, compared to the results on same instances types without the degeneracy procedure. On the other hand, instances with tighter time windows, e.g., instances of CA and CB types, display improvements of 58% and 77%, respectively. The latter reflects the efficiency of the proposed degeneracy procedure in supporting the DDD to avoid being trapped on lower bounds values for several iterations and enabling tightening the general lower-bound values obtained by the DDD.

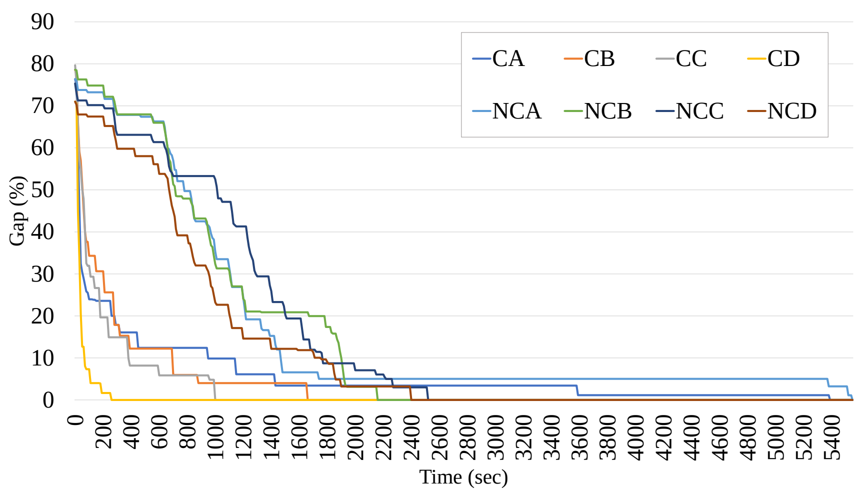

Figure 5: Degeneracy: Average Optimality gap (%) versus run time and instance type.

D Sensitivity analysis OD demands

We conducted a sensitivity analysis on the number of OD demands in the 2E-MALRPS to evaluate the performance of the proposed H-DDD solution algorithm. Our objective is to assess the impact of the number of OD demands on the problem and gain insights into system behavior. To conduct these experiments, we adapted a subset of instances originally defined with 5, 10, and 15 ODs to a single-commodity problem. This adaptation preserved the original number of destinations while assuming that all demands consist of the same commodity. Consequently, availability times are not differentiated by demand, implying that the single commodity is available on all platforms at the original availability times defined for each instance set.

Each instance set was solved to optimality using the H-DDD to compare the effects of the number of OD demands on solution quality and runtime. Table 18 summarizes the complete results (see Table 19) obtained using the H-DDD with the coarsest discretization granularity ($\Delta = 2$).

The results presented in Table 18 demonstrate a general decrease in solution quality as the average runtime of the H-DDD increases. Removing the multi-commodity aspect of the problem setting leads to an average cost reduction of 5% for the complete instance set. Despite the unchanged number of availability times, the larger number of departure options from platforms allows for solutions with earlier departure times. This enables the utilization of more cost-effective routes that are otherwise restricted in the original problem, where ODs have specific availability times. However, the runtime has an average increase of 16% due to the increased flexibility introduced by considering a single commodity. By allowing for more homogeneous departure times at platform facilities, more synchronization options are available. This, in turn, increases the number of possible synchronization alternatives and expands the feasible region defined by the integer problem.

The single-commodity case demonstrates a robust behavior of the H-DDD by obtaining the optimal solution for all instances considered. However, it is important to note that not considering the multi-commodity aspect may yield cheaper solutions that are not realistic or achievable when demand items are not substitutable.

$\lvert\mathcal{P}^{\mathbf{ph}}\rvert$	$\lvert\mathcal{Z}^{\mathbf{ph}}\rvert$	**OD**	**NI**	**FUB**	**OUB**	**OG(%)**	**CPUsec**
2	3	5	20	20	20	0.00	1.65
3	5	5	20	20	20	0.00	1.35
6	4	5	20	20	20	0.00	1.50
2	3	10	20	20	20	0.00	17.24
3	5	10	20	20	20	0.00	39.06
6	4	10	20	20	20	0.00	28.80
2	3	15	20	20	20	0.00	187.62
3	5	15	20	20	20	0.00	257.26
6	4	15	20	20	20	0.00	497.48

Table 18: H-DDD performance on instances with a single OD demand

ID	Ψ	UB	CPU (sec)	LB	OG (%)	ID	Ψ	UB	CPU (sec)	LB	OG (%)	ID	Ψ	UB	CPU (sec)	LB	OG (%)
Ca1-2,3,5	100	271	2.17	271	0	Ca1-2,3,10	100	245	34.33	245	0	Ca1-2,3,15	200	273	186.86	273	0
Ca1-3,5,5	100	214	1.45	214	0	Ca1-3,5,10	100	197	14.48	197	0	Ca1-3,5,15	200	301	203.37	301	0
Ca1-6,4,5	100	268	1.73	268	0	Ca1-6,4,10	100	295	6.95	295	0	Ca1-6,4,15	200	267	1344.67	267	0
Ca2-2,3,5	100	152	2.30	152	0	Ca2-2,3,10	100	139	3.89	139	0	Ca2-2,3,15	200	306	47.03	306	0
Ca2-3,5,5	100	264	2.36	264	0	Ca2-3,5,10	100	327	6.57	327	0	Ca2-3,5,15	200	329	388.78	329	0
Ca2-6,4,5	100	148	1.21	148	0	Ca2-6,4,10	100	179	10.40	179	0	Ca2-6,4,15	200	272	865.41	272	0
Ca3-2,3,5	100	276	0.68	276	0	Ca3-2,3,10	100	343	16.23	343	0	Ca3-2,3,15	200	310	41.01	310	0
Ca3-3,5,5	100	218	0.48	218	0	Ca3-3,5,10	100	300	20.18	300	0	Ca3-3,5,15	200	300	3.52	300	0
Ca3-6,4,5	100	155	0.46	155	0	Ca3-6,4,10	100	221	24.09	221	0	Ca3-6,4,15	200	255	5452.21	255	0
Ca4-2,3,5	100	346	0.51	346	0	Ca4-2,3,10	100	422	14.15	422	0	Ca4-2,3,15	200	302	37.63	302	0
Ca4-3,5,5	100	155	1.38	155	0	Ca4-3,5,10	100	194	180.35	194	0	Ca4-3,5,15	200	284	2310.80	284	0
Ca4-6,4,5	100	150	1.62	150	0	Ca4-6,4,10	100	221	11.68	221	0	Ca4-6,4,15	200	250	690.85	250	0
Ca5-2,3,5	100	179	2.49	179	0	Ca5-2,3,10	100	274	16.09	274	0	Ca5-2,3,15	200	253	44.13	253	0
Ca5-3,5,5	100	176	1.01	176	0	Ca5-3,5,10	100	242	14.68	242	0	Ca5-3,5,15	200	256	33.07	256	0
Ca5-6,4,5	100	147	1.65	147	0	Ca5-6,4,10	100	178	7.47	178	0	Ca5-6,4,15	200	206	73.70	206	0
Cb1-2,3,5	100	152	0.63	152	0	Cb1-2,3,10	100	179	7.42	179	0	Cb1-2,3,15	200	296	208.41	296	0
Cb1-3,5,5	100	148	1.56	148	0	Cb1-3,5,10	100	210	16.46	210	0	Cb1-3,5,15	200	284	73.38	284	0
Cb1-6,4,5	100	286	2.21	286	0	Cb1-6,4,10	100	255	6.35	255	0	Cb1-6,4,15	200	290	40.79	290	0
Cb2-2,3,5	100	125	2.26	125	0	Cb2-2,3,10	100	181	6.38	181	0	Cb2-2,3,15	200	325	121.71	325	0
Cb2-3,5,5	100	148	1.50	148	0	Cb2-3,5,10	100	246	22.77	246	0	Cb2-3,5,15	200	291	28.48	291	0
Cb2-6,4,5	100	133	1.23	133	0	Cb2-6,4,10	100	173	150.65	173	0	Cb2-6,4,15	200	327	93.40	327	0
Cb3-2,3,5	100	329	2.35	329	0	Cb3-2,3,10	100	315	5.43	315	0	Cb3-2,3,15	200	351	1597.96	351	0
Cb3-3,5,5	100	147	1.23	147	0	Cb3-3,5,10	100	179	156.89	179	0	Cb3-3,5,15	200	265	19.51	265	0
Cb3-6,4,5	100	196	0.94	196	0	Cb3-6,4,10	100	269	92.69	269	0	Cb3-6,4,15	200	286	17.59	286	0
Cb4-2,3,5	100	275	0.80	275	0	Cb4-2,3,10	100	249	87.33	249	0	Cb4-2,3,15	200	249	274.38	249	0
Cb4-3,5,5	100	128	0.96	128	0	Cb4-3,5,10	100	239	69.13	239	0	Cb4-3,5,15	200	330	237.18	330	0
Cb4-6,4,5	100	182	1.49	182	0	Cb4-6,4,10	100	288	7.18	288	0	Cb4-6,4,15	200	242	18.49	242	0
Cb5-2,3,5	100	114	1.03	114	0	Cb5-2,3,10	100	194	28.25	194	0	Cb5-2,3,15	200	235	7.53	235	0
Cb5-3,5,5	100	172	2.39	172	0	Cb5-3,5,10	100	223	85.51	223	0	Cb5-3,5,15	200	218	613.70	218	0
Cb5-6,4,5	100	181	1.15	181	0	Cb5-6,4,10	100	317	39.61	317	0	Cb5-6,4,15	200	263	43.94	263	0
Cc1-2,3,5	100	119	1.45	119	0	Cc1-2,3,10	100	173	49.19	173	0	Cc1-2,3,15	200	208	389.06	208	0
Cc1-3,5,5	100	120	1.41	120	0	Cc1-3,5,10	100	157	21.06	157	0	Cc1-3,5,15	200	257	108.09	257	0
Cc1-6,4,5	100	142	2.35	142	0	Cc1-6,4,10	100	216	28.42	216	0	Cc1-6,4,15	200	256	65.96	256	0
Cc2-2,3,5	100	108	2.11	108	0	Cc2-2,3,10	100	163	14.38	163	0	Cc2-2,3,15	200	294	9.23	294	0
Cc2-3,5,5	100	162	1.16	162	0	Cc2-3,5,10	100	218	25.27	218	0	Cc2-3,5,15	200	249	321.93	249	0
Cc2-6,4,5	100	121	2.09	121	0	Cc2-6,4,10	100	162	6.43	162	0	Cc2-6,4,15	200	265	220.34	265	0
Cc3-2,3,5	100	117	0.94	117	0	Cc3-2,3,10	100	168	2.21	168	0	Cc3-2,3,15	200	211	487.11	211	0
Cc3-3,5,5	100	128	1.94	128	0	Cc3-3,5,10	100	224	65.13	224	0	Cc3-3,5,15	200	247	568.54	247	0
Cc3-6,4,5	100	137	1.12	137	0	Cc3-6,4,10	100	179	15.44	179	0	Cc3-6,4,15	200	303	28.15	303	0
Cc4-2,3,5	100	171	1.99	171	0	Cc4-2,3,10	100	211	23.50	211	0	Cc4-2,3,15	200	247	108.98	247	0
Cc4-3,5,5	100	154	0.86	154	0	Cc4-3,5,10	100	209	14.14	209	0	Cc4-3,5,15	200	259	10.44	259	0
Cc4-6,4,5	100	119	1.89	119	0	Cc4-6,4,10	100	221	20.91	221	0	Cc4-6,4,15	200	223	60.17	223	0
Cc5-2,3,5	100	107	1.18	107	0	Cc5-2,3,10	100	167	8.13	167	0	Cc5-2,3,15	200	221	4.08	221	0
Cc5-3,5,5	100	113	0.85	113	0	Cc5-3,5,10	100	167	12.75	167	0	Cc5-3,5,15	200	197	0.71	197	0
Cc5-6,4,5	100	118	1.12	118	0	Cc5-6,4,10	100	162	33.13	162	0	Cc5-6,4,15	200	268	56.68	268	0
Cd1-2,3,5	100	144	2.35	144	0	Cd1-2,3,10	100	196	3.34	196	0	Cd1-2,3,15	200	279	21.83	279	0
Cd1-3,5,5	100	170	1.46	170	0	Cd1-3,5,10	100	195	7.26	195	0	Cd1-3,5,15	200	268	20.72	268	0
Cd1-6,4,5	100	183	0.46	183	0	Cd1-6,4,10	100	241	27.82	241	0	Cd1-6,4,15	200	222	48.20	222	0
Cd2-2,3,5	100	128	2.08	128	0	Cd2-2,3,10	100	190	1.37	190	0	Cd2-2,3,15	200	293	64.14	293	0
Cd2-3,5,5	100	145	0.85	145	0	Cd2-3,5,10	100	235	11.52	235	0	Cd2-3,5,15	200	256	27.75	256	0
Cd2-6,4,5	100	123	1.94	123	0	Cd2-6,4,10	100	202	24.08	202	0	Cd2-6,4,15	200	235	49.67	235	0
Cd3-2,3,5	100	122	1.92	122	0	Cd3-2,3,10	100	176	4.87	176	0	Cd3-2,3,15	200	197	34.29	197	0
Cd3-3,5,5	100	131	1.80	131	0	Cd3-3,5,10	100	218	13.95	218	0	Cd3-3,5,15	200	245	17.78	245	0
Cd3-6,4,5	100	143	1.16	143	0	Cd3-6,4,10	100	196	11.49	196	0	Cd3-6,4,15	200	318	56.08	318	0
Cd4-2,3,5	100	195	1.86	195	0	Cd4-2,3,10	100	256	4.18	256	0	Cd4-2,3,15	200	272	35.89	272	0
Cd4-3,5,5	100	144	1.52	144	0	Cd4-3,5,10	100	230	8.86	230	0	Cd4-3,5,15	200	245	56.72	245	0
Cd4-6,4,5	100	129	2.18	129	0	Cd4-6,4,10	100	181	26.43	181	0	Cd4-6,4,15	200	241	56.37	241	0
Cd5-2,3,5	100	162	2.02	162	0	Cd5-2,3,10	100	202	14.18	202	0	Cd5-2,3,15	200	221	31.18	221	0
Cd5-3,5,5	100	174	0.91	174	0	Cd5-3,5,10	100	212	14.30	212	0	Cd5-3,5,15	200	186	100.73	186	0
Cd5-6,4,5	100	142	2.13	142	0	Cd5-6,4,10	100	163	24.84	163	0	Cd5-6,4,15	200	248	666.90	248	0
Averages			1.51		0.00	***Averages***			28.37		0.00	***Averages***			314.12		0.00

Table 19: DDD results on instances with 5, 10 and 15 OD demands expressed as a single OD demand