CIRRELT-2021-11

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March 2021

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Abstract. We introduce the location-or-routing problem (LoRP), which integrates the facility location and the vehicle routing problems by uncovering a new connection from the customer coverage perspective. In the LoRP, open facilities cover the customers in their neighborhood and the uncovered customers are transported to open facilities by capacitated vehicles. Each facility has a maximum coverage range and each vehicle route is constrained by a maximum length. In this setting, a customer can be covered either by 'location' or by 'routing', hence the problem name. We discuss several application areas of LoRP and present its relation to the location and routing problems. We develop a set covering model and a branch-and-price algorithm as an exact solution methodology. The results show that the facility coverage range is an important determinant of the number and location of open facilities. We find that the vehicle routes play a decreasing role on the total cost as facility range increases. Furthermore, our trade-off analyses on random graphs show that the total cost decreases almost linearly by increasing facility coverage range. We investigate the reasons behind this observation using arguments from asymptotic analysis and find that it is a common property when the customers are uniformly distributed.

Keywords: Transportation, location, routing, location-routing, branch-and-price.

Acknowledgements. The author thanks the associate editor Prof. James Campbell and two referees whose comments helped improve the quality of this paper significantly. The author acknowledges that the computations reported in this paper were performed on Compute Canada.

[ǂ] Revised version of the CIRRELT-2020-24

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Dépôt légal – Bibliothèque et Archives nationales du Québec
Bibliothèque et Archives Canada, 2021

1. Introduction

This paper proposes a generalized customer coverage model by integrating the location and vehicle routing problems. To this end, we introduce the location-or-routing problem (LoRP), in which the customers can be covered by capacitated facilities if they are located within the coverage range of any open facility or by a vehicle departing from an open facility subject to maximum route length and vehicle capacity constraints. The objective of LoRP is to minimize the total weighted cost of opening facilities, vehicle routing and customer coverage by open facilities.

Consider locating general service stores such as retailers, supermarkets or shopping malls in an urban area. In covering location problems, a customer is assumed to be *covered* (or equivalently *served*) if the distance to the closest open facility is within a certain threshold. This threshold, which is referred to as coverage range, generally represents the tolerance of the customer to travel to the closest open facility. The customers that lie beyond the coverage range of any open facility are assumed to be uncovered in covering location problems. When customers cannot be served by open facilities, complimentary shuttle buses are offered in many large cities including Beijing (Kai-yan et al., 2013; Wang and Nie, 2020), Toronto (Tsawwassen Mills, 2020; Vaughan Mills, 2020; CrossIron Mills, 2020), Victoria (Victoria Transport Policy Institute, 2020) and Istanbul (Historia Shopping Mall, 2020; Starcity Outlet, 2020; Canpark Shopping Mall, 2020). Put differently, the open facilities cover a subset of customers, and the uncovered customers are served by vehicles. The LoRP arises in a number of application contexts.

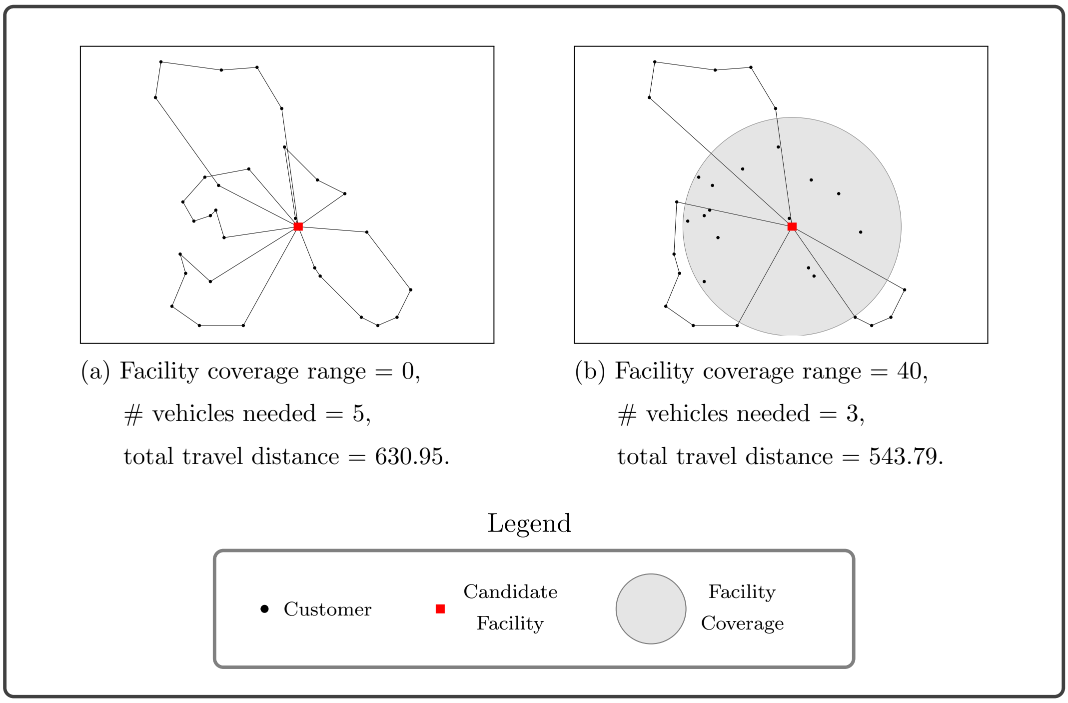
1. *Retail store, supermarket and shopping mall location,*

2. *School location and bus routing:* Generally, the school location (Antunes and Peeters, 2000) and school bus routing problems (Ellegood et al., 2020; Park and Kim, 2010) are solved hierarchically. When the two problems are considered simultaneously, the problem we deal with is LoRP, in which the students are assumed to be covered within a certain distance from the school and buses are used to transport students that lie beyond the coverage range,

3. *Urban delivery center location with drone operations:* When packages from urban delivery centers are transported by limited-range drones (Hong et al., 2018; Otto et al., 2018; Chauhan et al., 2019) and the uncovered customers are served by trucks located at these delivery centers, we encounter a LoRP application,

4. *Facility location in pandemics:* When there is a need for a large-scale testing or vaccination due to a pandemic such as COVID-19, the LoRP can also be applied to the location optimization of testing centers in urban areas. The aim is to cover the population by providing them with an opportunity either to visit a nearby center or to get tested at their home by a mobile medical clinic vehicle.

In general, the LoRP is applicable when a facility provides public or private service to a neighborhood, and the customers beyond the facility coverage range receive service by vehicles either at their location or by being transported to the facility. Though each different application has



(a) Facility coverage range = 0, # vehicles needed = 5, total travel distance = 630.95.

(b) Facility coverage range = 40, # vehicles needed = 3, total travel distance = 543.79.

Figure 1: Vehicle routes and total travel distance with and without facility coverage in parts (a) and (b), respectively.

its own peculiarities and side-constraints, the common understanding of service to a customer is similar: the location decisions induce the mode of service for different sets of customers. In LoRP, we capture this connection between the location and routing decisions.

Figure 1 demonstrates an example. There are a total of 30 customers. In Figure 1(a), when facilities cannot cover any customers, five vehicles are needed and the total travel distance is 630.95. When we assume that the facility covers the customers in its neighborhood as in Figure 1(b), the total travel distance reduces to 543.79, which includes only the three vehicle routes, and not the distance for the covered customers to access the facility.

1.1. Literature review

The LoRP is closely related to the location-routing problem (LRP), in which the goal is opening a set of facilities and visiting every customer by a capacitated vehicle starting at one of the open facilities. Similar to LoRP, the objective of LRP is to minimize the total cost of opening facilities and vehicle routing. The facility prototype considered in LRP is a depot or a warehouse (Perl and Daskin, 1985). Opening a facility does not directly cover any customer.

The idea of combining the location and routing problems is rooted in the study by Von Boventer (1961) and evolved into the LRP over time (Laporte, 1988; Laporte et al., 1988; Min et al., 1998). Different LRP extensions have been considered (Laporte et al., 1986; Alumur and Kara, 2007; Ahmadi-Javid and Seddighi, 2013; Hof et al., 2017; Arslan et al., 2021). Recent reviews on the

LRP include Nagy and Salhi (2007) and Prodhon and Prins (2014). The survey by Drexl and Schneider (2015) focuses on variants and extensions of LRP and the survey by Schneider and Drexl (2017) focuses on the standard LRP.

Combining two NP-hard problems, the LRP includes intricate relationships between the location and routing decisions and often necessitates advanced solution techniques. To this end, Berger et al. (2007) and Akca et al. (2009) develop exact branch-and-price (B&P) algorithms for solving the LRP. Belenguer et al. (2011) add capacity constraints on depots and vehicles and develop an exact branch-and-cut algorithm. Baldacci et al. (2011) observe that the LRP can be decomposed into a limited set of multi-capacitated depot vehicle routing problems and build an algorithm by various bounding procedures. Contardo et al. (2014) combine these ideas and introduce several new valid inequalities to accelerate the solution process. Escobar et al. (2014) propose a novel granular tabu search within a variable neighborhood search. Ponboon et al. (2016) and Farham et al. (2018) also include time windows in routing.

Considering a heterogeneous fleet of vehicles is an important facet of the vehicle routing problem (Koç et al., 2016c). The mix LRP (mLRP) relaxes the homogeneity assumption of the vehicles and considers a heterogeneous fleet in LRP (Koç et al., 2016a). This problem arises in the context of city logistics (Koç et al., 2016b). The mLRP is particularly important, because every vehicle type offers an alternative coverage option, similar to the location and routing decisions in LoRP. We further elaborate on this relationship in Section 2.

All of the studies described above consider the vehicle visits as the only way to serve a customer, and the location decisions are related to selection of the origin of the vehicle routes. Another closely related problem that does not have this assumption is the covering tour problem (CTP) (Gendreau et al., 1997). The CTP defines a coverage circle around a customer node, and all customers in this circle are assumed to be covered when the node is visited. CTP then finds a minimum cost vehicle route such that all customers are within a certain radius from any node on the route. Unlike the vehicle routing problem, serving a customer is possible without actually visiting the customer by a vehicle. Several variant have been developed such as bi-objective CTP (Jozefowiez et al., 2007), multi-vehicle CTP (Hachicha et al., 2000), multi-vehicle probabilistic CTP (Karaoğlan et al., 2018) and cumulative multi-vehicle CTP (Flores-Garza et al., 2017). Recently, Murakami introduced a generalized model to solve large-scale CTP. The coverage circle is the same in CTP for all the nodes in the network. In LoRP, on the other hand, a customer does not have a coverage radius, but a candidate facility has, which is a major difference between the two problems. In LoRP, facility location decisions are also made and a customer can be covered by a facility only, but not by any other customer.

1.2. Scientific contributions and organization of the paper

In our study, we take the location and routing decisions as alternative ways of serving the customers. As contributions of this study, we

- introduce the 'location-or-routing problem' and position it precisely in the vast body of vehicle routing and facility location literature,
- develop a B&P algorithm as an exact solution methodology and
- test the computational efficiency of the B&P algorithm on a wide range of instances and investigate the trade-off among several problem parameters,
- show that the facility range is an important determinant of the number and location of open facilities. We find that the vehicle routes play a decreasing role on the total cost as the facility range increases. Furthermore, the results show that the cost decreases linearly by increasing facility range.

We introduce the formal problem definition and a set covering formulation in Section 2, present location-or-routing as a problem class in Section 3, develop an exact B&P algorithm in Section 4, test its computational efficiency in Section 5 and discuss the results in Section 6. We conclude the study in Section 7.

2. Problem definition and formulation

In this section, we formally define the LoRP, investigate its relation with the LRP and develop a set covering formulation and valid inequalities.

2.1. Problem definition

Let I be the set of candidate facilities, J be the set of customers and $G = (N, A)$ be a directed network, where $N = I \cup J$ is the set of nodes and $A = \{(i, j) : i, j \in N, i \neq j\}$ is the set of arcs. The length of arc $(i, j) \in A$ is d_{ij}, which represents the travel cost of arc (i, j). The cost of covering customer j by facility i is proportional to d_{ij}. Without loss of generality, we assume that the distance matrix satisfies the triangular inequality. Each candidate facility $i \in I$ has a setup cost f_i, a coverage range r_i and a capacity C_i. Each customer $j \in J$ has a demand q_j, which must be covered either by a facility or by a vehicle. A facility i can cover a customer j if $d_{ij} \leq r_i$. Let I_j be the set of facilities covering customer $j \in J$ and J_i be the set of customers covered by facility $i \in I$. There is a homogeneous fleet of vehicles with capacity Q and route lengths are constrained to be maximum T.

Definition 1. *Location-or-routing problem (LoRP) is defined as selecting a set of facilities to open and determining a set of vehicle routes, each of which start at an open facility, visit a subset of customers and return to the same facility, respecting the vehicle capacity and maximum length constraints such that every customer is covered either by a facility or by a vehicle route and the total cost of opening facilities, routing vehicles and covering the customers by facilities is minimized.*

Remark 1. *If the coverage range $r_i = 0$ for all $i \in I$, LoRP then transforms into a LRP.*

Remark 2. *If the maximum route length of vehicles $T = 0$ and there is no cost for covering customers by facilities, LoRP then transforms into a set covering problem.*

Therefore, the problem introduced in this paper is a generalization of the LRP and the set covering problem.

Remark 3. *LoRP is a special case of mLRP.*

The correctness of Remark 3 is rooted in the observation that covering a customer j by facility i can be represented by a dummy vehicle route $p_{ij} = (i, j, i)$ with its length equal to the cost of covering customer j by facility i. This, however, requires introducing a different vehicle type for every customer $j \in J$ and facility $i \in I_j$ with capacity q_j. Therefore, using the formulations developed for mLRP for solving the LoRP is impractical.

2.2. A set covering formulation

We define a (closed) path p as an ordered set of directed arcs, starting and ending at the same candidate facility node $i \in I$. We use the terms route and path interchangeably. Set J_p represents the customers visited on path p, A_p represents the arcs in p, p_0 represents the starting (and ending) node of p and d_p represents the routing cost of path p. We refer to a path as feasible if it respects the maximum route length and vehicle capacity constraints, that is, $\sum_{(i,j)\in A_p} d_{ij} \leq T$ and $\sum_{(i,j)\in A_p: j\neq p_0} q_j \leq Q$. Let $\widetilde{\mathcal{P}}_j$ be the set of paths visiting customer $j \in J$, $\overline{\mathcal{P}}_i$ be the set of paths starting from facility $i \in I$, $\widehat{\mathcal{P}}_{ij} = \widetilde{\mathcal{P}}_j \cap \overline{\mathcal{P}}_i$, and $\mathcal{P} = \bigcup_{i\in I} \overline{\mathcal{P}}_i$. Let x_i equal 1 if facility $i \in I$ is selected and 0 otherwise, y_p equal 1 if path $p \in \mathcal{P}$ is selected and 0 otherwise and z_{ij} equal 1 if customer j is allocated to facility i and 0 otherwise. Note that allocating a customer to a facility specifies which facility covers that customer in a particular solution. A customer can still be covered by multiple facilities. We formulate the LoRP as follows.

$$\text{minimize} \quad w_F \sum_{i\in I} f_i x_i + w_R \sum_{p\in\mathcal{P}} d_p y_p + w_A \sum_{j\in J}\sum_{i\in I_j} d_{ij} z_{ij} \tag{1}$$

$$\text{subject to} \quad \sum_{i\in I_j} z_{ij} + \sum_{p\in\widetilde{\mathcal{P}}_j} y_p = 1 \qquad j \in J \tag{2}$$

$$\sum_{j\in J_i} q_j z_{ij} + \sum_{p\in\overline{\mathcal{P}}_i} q_p y_p \leq C_i x_i \qquad i \in I \tag{3}$$

$$z_{ij} \leq x_i \qquad i \in I, j \in J_i \tag{4}$$

$$x_i, y_p, z_{ij} \geq 0 \text{ and integer} \qquad i \in I, j \in J_i, p \in \mathcal{P}, \tag{5}$$

where w_F, w_R and w_A are nonnegative weights in the objective function associated with fixed, routing and allocation costs, respectively. The objective function minimizes the weighted sum of these three cost components. Constraints (2) ensure that all customers are covered. Even though multiple open facilities may be within the coverage range of a customer, constraints (2) ensure that all customers are covered only one time in a solution. Constraints (3) respect facility capacities and also ensures that a path starts from an open facility. Constraints (4) link location and allocation variables and constraints (5) are the domain restrictions.

2.3. Valid inequalities

Aside from enforcing the facility capacities to be respected, constraints (3) ensure that $x_i = 1$ when there exists a path in the optimal solution starting at facility i. The same implication can be achieved by the following stronger set of constraints.

$$\sum_{p \in \widehat{\mathcal{P}}_{ij}} y_p \leq x_i \qquad i \in I, j \in J \tag{6}$$

which states that if a customer j is served by a vehicle route starting from a facility i, then facility i should be open. Constraints (6) are known to be stronger (Akca et al., 2009) than constraints (3) and improve the linear programming (LP) relaxation of the model.

Let v_i be an integer variable representing the number of vehicles starting their routes from facility $i \in I$. This variable type is not necessary for problem description, but it is mainly used to implement the branching rules in a B&P algorithm. The following inequalities are then valid.

$$\sum_{p \in \overline{\mathcal{P}}_i} y_p = v_i \qquad i \in I \tag{7}$$

$$\sum_{i \in I} v_i \geq \left\lceil \frac{\sum_{j \in J \setminus \cup_{i \in I} J_i} q_j}{Q} \right\rceil \tag{8}$$

Constraint (7) assigns the number of vehicles used at facility i to variable v_i and Constraint (8) puts a lower bound on the number of vehicles to be used. This bound is generally weaker in LoRP than LRP because customers are not necessarily visited by vehicles and they may be covered by a facility.

Combining the initial model with these valid inequalities, we obtain the following formulation, which we refer to as the set covering (SC) model.

$$\text{(SC) minimize} \quad w_F \sum_{i \in I} f_i x_i + w_R \sum_{p \in \mathcal{P}} d_p y_p + w_A \sum_{j \in J} \sum_{i \in I_j} d_{ij} z_{ij}$$

$$\text{subject to } (2) - (4), (6) - (8)$$

$$x_i, y_p, z_{ij}, v_i \geq 0 \text{ and integer} \qquad i \in I, j \in J_i, p \in \mathcal{P} \tag{9}$$

Note that we relax binary variables x, y and z into integer variables without loss of generality due to the minimization type of the objective function. This eliminates the need to add an upper bound of 1 on the binary variables when solving the LP relaxation of the SC model.

3. Location-or-routing as a problem class

The problem we define in this paper integrates the location and routing problems. We now discuss several variants of LoRP, which we obtain by considering different problems in location science.

3.1. Uncapacitated location-or-routing problem (ULoRP)

The problem as we define above considers capacitated facilities and hence can be referred to as the *capacitated LoRP*. By relaxing the facility capacity constraints, we obtain the *uncapacitated LoRP*, which is formulated as follows.

$$\begin{aligned} &\text{minimize} \;\; w_F \sum_{i \in I} f_i x_i + w_R \sum_{p \in \mathcal{P}} d_p y_p + w_A \sum_{j \in J} \sum_{i \in I_j} d_{ij} z_{ij} \\ &\text{subject to } (2), (4), (6) - (9) \end{aligned}$$

Therefore, there is a capacitated and uncapacitated version of every problem considered below.

3.2. Set covering location-or-routing problem (SCLoRP)

In the set covering location problem, the coverage of customers does not contribute to the objective function. In a similar logic, the *set covering location-or-routing problem (SCLoRP)* minimizes the number of facilities to cover all the demand, which is modeled by setting $w_R = w_A = 0$ in the SC model.

3.3. Hard-cost minimizing location-or-routing problem (HMLoRP)

The first two cost components in the objective function of the SC model are hard costs that are directly related to setting up the infrastructure and carrying out the operations, whereas the third component is a soft cost and is not directly incurred. It models the inconvenience of the customers that directly transport to the facilities. When we consider only the hard costs in the objective function, we obtain the *hard-cost minimizing location-or-routing problem (HMLoRP)*, which is modeled by setting $w_A = 0$ in the SC model. This implies that a vehicle visit is not necessary for customers that are covered by at least one facility.

Proposition 1. *In HMLoRP, we set $\widehat{\mathcal{P}}_{ij} = \emptyset$ when $d_{ij} \leq r_i$, for all $i \in I$ and $j \in J$.*

PROOF. Assume that $d_{i_0 j_0} \leq r_{i_0}$, for $i_0 \in I$ and $j_0 \in J$ and that $y_{p_0} = 1$ in the optimal solution for path $p_0 = (i_0, \ldots, i_1, j_0, i_2 \ldots, i_0)$ starting at i_0 and visiting customer j_0. Since customer j_0 can be covered at no cost by facility i_0, there exists a feasible path $p_1 = p_0 \setminus \{j_0\} = (i_0, \ldots, i_1, i_2 \ldots, i_0)$

with $d_{p_0} = d_{p_1}$ due to triangular inequality. Thus, an alternative optimal solution with $y_{p_0} = 0$ and $y_{p_1} = 1$ exists. Applying the same argument for every i and j pair with $d_{ij} \leq r_i$ gives the desired result. □

Note that due to Proposition 1, increasing coverage range implies fewer feasible routes in HMLoRP. This proposition is also valid for the SCLoRP.

3.4. P-median location-or-routing problem (pMLoRP)

Transforming the first term in the objective function into a constraint and taking the parameters d_p and d_{ij} as time, the objective function represents the total time that customers spend to reach to facilities. Here, d_p represents the vehicle riding time and d_{ij} represents the direct transportation time of customers to their nearest facilities. This problem combines the p-median location and vehicle routing problems and is therefore referred to as the *p-median location-or-routing problem*, pMLoRP(p), where p is the number of facilities to be located. The model is as follows.

$$\text{minimize} \quad w_R \sum_{p \in \mathcal{P}} d_p y_p + w_A \sum_{j \in J} \sum_{i \in I_j} d_{ij} z_{ij} \tag{10}$$

$$\text{subject to } (2) - (4), (6) - (9)$$

$$\sum_{i \in I} x_i \leq p \tag{11}$$

where constraint (11) limits the number of open facilities by p.

3.5. P-center location-or-routing problem (pCLoRP)

If the objective is to minimize the maximum time a customer spends to reach to a facility, we then obtain the *p-center location-or-routing problem*, pCLoRP(p), where p is the number of facilities to be located. Let η be an auxiliary variable representing the maximum time that it takes for any customer to reach to a facility in the network and parameter $\hat{d}_p$ be the the ride time of the first customer on the route, which equals the total route time less the travel time from the facility to the first customer. We can then model pCLoRP(p) as follows.

$$\text{minimize} \quad \eta \tag{12}$$

$$\text{subject to } (2) - (4), (6) - (9), (11)$$

$$\eta \geq d_{ij} z_{ij} \qquad i \in I, j \in J_i \tag{13}$$

$$\eta \geq \sum_{p \in \widehat{\mathcal{P}}_{ij}} \hat{d}_p y_p \qquad i \in I, j \in J \tag{14}$$

where constraints (13) and (14) ensures that η is correctly assigned.

3.6. Maximum covering location-or-routing problem (MCLoRP)

Another extension is obtained by changing the objective function as the maximization of the customer coverage and by considering the cost as a constraint, which would lead to the *maximum covering location-or-routing problem (MCLoRP)*. Let t_j equal 1 if a customer is covered and 0 otherwise. We can then model the MCLoRP as follows.

$$\text{maximize} \quad \sum_{j \in J} q_j t_j \tag{15}$$

$$\text{subject to } (3)-(4),(6)-(9)$$

$$w_F \sum_{i \in I} f_i x_i + w_R \sum_{p \in \mathcal{P}} d_p y_p + w_A \sum_{j \in J} \sum_{i \in I_j} d_{ij} z_{ij} \leq B \tag{16}$$

$$\sum_{i \in I_j} z_{ij} + \sum_{p \in \widetilde{\mathcal{P}}_j} y_p = t_j \qquad j \in J \tag{17}$$

$$t_j \in \{0,1\} \qquad j \in J \tag{18}$$

The objective function maximizes the customer coverage. Constraint (16) forces a budget of B, constraints (17) assign $t_j = 1$ when customer j is covered (either by a facility or by a vehicle) and constraints (18) are the domain restrictions.

Therefore, the LoRP can be considered as a class of problems. In the next section, we develop an algorithm to solve the SC model. This directly allows solving the uncapacitated and capacitated versions of LoRP, SCLoRP, HMLoRP and pMLoRP. On the other hand, pCLoRP and MCLoRP requires calculating the reduced cost of y variables differently. Nevertheless, this does not alter the problem structure or the pricing problem and the methodology we develop below can be adapted in a straightforward manner to solve the pCLoRP and MCLoRP.

4. Solution methodology

We now develop an exact B&P algorithm to solve the SC model. In this section, we present a column generation algorithm, the pricing problem, branching rules, and other implementation details including generation of initial set of columns, variable fixing and upper bound heuristics.

4.1. Column generation

Solving the LP relaxation of SC model, which we refer to as SC-R, is an integral part of the B&P algorithm. We start the column generation algorithm by solving the LP relaxation of a restricted SC model with an initial set of columns only. This provides us with the dual variables, which in turn allows us to obtain the reduced cost of all path variables in the SC model. We then add at least one variable with a negative reduced cost, if one exists, and resolve the LP relaxation. This iterative procedure is continued until no such variable with a negative reduced cost exists after solving the restricted SC model, which gives us a certificate that the solution obtained is optimal for the SC-R.

Let α_j and γ_i be the dual variables unrestricted in sign, associated with constraints (2) and (7), respectively, and β_i and δ_{ij} be the nonnegative dual variables associated with constraints (3) and (6), respectively, The reduced cost of variable y_p is then given in the following expression.

$$c_p = w_R d_p + \sum_{j \in J_p} (\delta_{p_0 j} - \alpha_j + q_j \beta_{p_0}) - \gamma_{p_0}. \tag{19}$$

Having $\min_{p \in \mathcal{P}}\{c_p\} \geq 0$ ensures that all path variables with nonnegative reduced costs are in the problem and the SC-R is solved optimally. If there exists a path p with $c_p < 0$, we add the corresponding variable y_p to the formulation. Therefore, the goal after solving the restricted SC-R is to identify a path with negative reduced cost. This problem is referred to as the pricing problem, which is the topic of the next section.

4.2. Pricing problem

For candidate facility i, consider graph $\hat{G}_i = (\hat{N}_i, \hat{A}_i)$, where $\hat{i}$ is a duplicate node of facility i, $\hat{N}_i = J \cup \{i, \hat{i}\}$ and $\hat{A}_i = \big\{(m,n) \in A : m,n \in J\big\} \bigcup \big\{(i,m) : m \in J\big\} \bigcup \big\{(m,\hat{i}) : m \in J\big\}$. The length of arc $(m,n) \in \hat{A}_i$ is

$$\hat{d}_{mn} = \begin{cases} w_R d_{mi} - \gamma_i & \text{if } n = \hat{i} \\ w_R d_{mn} + \delta_{in} - \alpha_n + q_n \beta_i & \text{otherwise} \end{cases} \qquad (m,n) \in \hat{A}_i. \tag{20}$$

Traveling an arc $(m,n) \in \hat{A}_i$ consumes two types of resources, r^1_{mn} and r^2_{mn} from the vehicle capacity (Q) and the maximum route length (T), respectively.

$$r^1_{mn} = \begin{cases} q_n & \text{if } n \neq \hat{i} \\ 0 & \text{otherwise} \end{cases} \qquad (m,n) \in \hat{A}_i, \tag{21}$$

$$r^2_{mn} = \begin{cases} d_{mn} & \text{if } n \neq \hat{i} \\ d_{im} & \text{otherwise} \end{cases} \qquad (m,n) \in \hat{A}_i. \tag{22}$$

Note that the length of path $p = (p_0, p_1, \ldots, p_n, \hat{p}_0)$ in $\hat{G}_i$ is equal to the reduced cost of y_p in SC-R, that is, $\sum_{(m,n) \in p} \hat{d}_{mn} = \sum_{(m,n) \in A_p} w_R d_{mn} + \sum_{n \in J_p} (\delta_{p_0 n} - \alpha_n + q_n \beta_{p_0}) - \gamma_{p_0} = c_p$. Furthermore, $\sum_{(m,n) \in p} r^1_{mn}$ is equal to the customer demand on path p and $\sum_{(m,n) \in p} r^2_{mn}$ is equal to the distance of path p, which are restricted to be at most Q and T, respectively.

The pricing problem is then to find a shortest path in graph $\hat{G}_i$ for every candidate facility $i \in I$ subject to two side constraints associated with the vehicle capacity Q and the maximum route length T. Let u_{mn} equal 1 if and only if arc (m,n) is selected. The formulation presented below models the pricing problem for a given facility i in graph $\hat{G}_i$.

$$\text{minimize} \quad \sum_{(m,n)\in\hat{A}} \hat{d}_{mn} u_{mn} \tag{23}$$

$$\text{subject to} \quad \sum_{n:(m,n)\in\hat{A}} u_{mn} - \sum_{n:(n,m)\in\hat{A}} u_{nm} = \begin{cases} 1 & \text{if } m = i \\ -1 & \text{if } m = \hat{i} \\ 0 & \text{otherwise} \end{cases} \qquad m \in \hat{N}_i \tag{24}$$

$$\sum_{(m,n)\in\hat{A}} r^1_{mn} u_{mn} \le Q \tag{25}$$

$$\sum_{(m,n)\in\hat{A}} r^2_{mn} u_{mn} \le T \tag{26}$$

$$\sum_{(m,n)\in S} u_{mn} \le |S| - 1 \qquad S \subset \hat{A}_i \tag{27}$$

$$u_{mn} \in \{0,1\} \qquad (m,n) \in \hat{A}_i. \tag{28}$$

The objective function minimizes the reduced cost. Constraints (24) are the node balance equations. Constraints (25) and (26) ensure that the selected path respects the resource constraints. Constraints (27) ensure that the selected path is elementary and constraints (28) are the domain restrictions.

The pricing problem itself is NP-hard, however, several algorithms exist for solving a resource constrained shortest path problem (Pugliese and Guerriero, 2013). In this paper, we adopt a state-of-the-art algorithm developed by Lozano et al. (2016), referred to as the pulse algorithm. The algorithm solves an elementary resource constrained shortest path problem with time windows. In our implementation, we use step size = 25 and lower time limit = 25. We also set r^2_{mn} as the time consumption on arc (m,n) and set the time window of a candidate facility as $[0,T]$. Resource r^1_{mn} counts towards the capacity Q. This ensures that the path given by the algorithm is elementary resource constrained shortest path respecting the vehicle capacity and the maximum route length constraints.

4.3. Determining feasibility

Before starting the algorithm, we use the following observation to determine if an instance is feasible.

Proposition 2. *For $j \in J$, let ℓ_j be the distance to the closest candidate facility, that is, $\ell_j = \min_{i\in I} d_{ij}$. A given LoRP instance is feasible if and only if one of the following two conditions hold for every customer $j \in J$:*

- *there exists $i \in I$ such that $d_{ij} \le r_i$,*

- $q_j \leq Q$ *and* $\ell_j \leq T/2$.

Proof. (Necessity) Assume that a given LoRP instance is feasible. Then, for each customer $j \in J$, there exists either a facility that covers j or a feasible path visiting j. The former provides the first condition above. Let p_j be the path visiting customer j, N_j be the set of nodes on path p_j and A_j be the set of arcs on path p_j. Due to feasibility, we have $\sum_{k \in N_j} q_k \leq Q$, which implies that $q_j \leq Q$. Similarly, we have $\sum_{(m,n) \in A_j} d_{mn} \leq T$ and due to triangular inequality, we have $2\ell_j \leq \sum_{(m,n) \in A_j} d_{mn} \leq T$, which provides the second condition above.

(Sufficiency) Assume that at least one of the two conditions holds for every $j \in J$. First, assume that the former condition holds. Then, there exists $i \in I$ such that $d_{ij} \leq r_i$ and customer j is covered. Now, assume that the second condition holds. Let i_j be the closest facility to customer $j \in J$. Since $q_j \leq Q$ and $\ell_j \leq T/2$, opening candidate facility i_j and selecting path (i_j, j, i_j) covers customer j. This solution with all customers covered either by an open facility or by a path is feasible for the given problem instance. □

4.4. Initial set of columns

The initial set of columns is needed to ensure that the first restricted problem is feasible and that the dual variables can be obtained. Therefore, there must exist a path variable in the formulation corresponding to each customer. Similarly, there must exist a path variable starting at every candidate facility. Furthermore, these paths must be feasible. We build a feasible solution as follows. Let i_j be the closest candidate facility to customer $j \in J$ and p_j be a path that visits customer j from i_j. That is, $p_j = (i_j, j, i_j)$ for every $j \in J$. We add a variable for path p_j for every customer $j \in J$ if $d_{p_j} > T$, which ensure that the restricted SC-R is feasible when the instance is feasible due to Proposition 2.

4.5. Branching rules and variable fixing

If the optimal solution of SC-R is integer, it is also optimal for SC model. If it has fractional terms, we need to branch for an integer solution. We implement a four-stage hierarchical branching. In all levels, we select the most fractional variable to branch on. The first level branches on the facility variables x. At the master problem level, we only add a constraint to enforce the branching rule. We also do not need to solve the pricing problem associated with a facility i if $x_i = 0$. When all facility variables are binary, we branch on v variables. Similar to the facility location variables, we only add a single constraint to the master problem in order to enforce the branching rules. The pricing problem is not impacted by the branching rules on v variables. The third branching level is on flow variables. We branch on implicit variable r_{ij}^k which equals 1 if arc (i, j) is used in the solution by a vehicle starting at facility k, and 0 otherwise. To enforce $r_{ij}^k = 1$, we remove all the variables from the master problem corresponding to the paths starting at facility k and visiting nodes i or j without using arc (i, j). We also remove all variables corresponding to paths starting at a facility $\hat{k} \neq k$ and using arc (i,j). Note that, because of the problem structure, nodes i and j can still be covered by facilities with $r_{ij}^k = 0$. To avoid this, we also ensure that no facility covers nodes i and j by setting $\sum_{h \in J_i} z_{hi} = 0$ and $\sum_{h \in J_j} z_{hj} = 0$. At the pricing problem level, we remove all arcs leaving node i and entering node j except for arc (i,j) when solving the pricing problem corresponding to facility k. For other facilities $\hat{k} \neq k$, we remove nodes i and j when solving the pricing problem. To forbid using arc (i,j) is more straightforward, we remove all variables corresponding to paths using arc (i,j) from the master problem and forbid using arc (i,j) in the pricing problem. The fourth level branches on the z_{ij} variables. Similar to the x and v variables, we add a constraint to the master problem in order to enforce the branching rules and the pricing problem is not impacted by the branching rules on z_{ij} variables.

We also implement variable fixing by reduced cost for facility location variables (Savelsbergh, 1994). Let ψ_i be the reduced cost of variable x_i for all $i \in I$ and $\overline{z}$ and $\underline{z}$ be the upper and lower bounds on the optimal objective function value, respectively, at any given node of the branch-and-bound (B&B) tree. The variable x_i is set equal to zero if $\psi_i > \overline{z} - \underline{z}$. The implementation is similar to first level branching rule.

4.6. Upper bound heuristic

At any node in the B&B tree, we can build a mixed integer linear program (MILP) from the SC-R model by imposing integrality constraints on the continuous variables in order to obtain a feasible solution for the SC model. If the solution generated improves the incumbent solution, we keep the new solution as the incumbent and continue exploring the B&B tree. We build the first MILP after solving the SC-R at the root node, which provides an upper bound when starting to explore the B&B tree. Note that any feasible solution of such a restricted MILP model is also feasible for the SC model. Therefore, we do not necessarily run the restricted MILP to optimality. In our implementation, we run the restricted model for at most 120 seconds. In our computational experiments, we observed that the MILP model generally runs much faster, in a few seconds. Similarly, we build the restricted MILP model every time 1,000 or more new path variables are generated. This allows us to obtain good upper bounds as we build the B&B tree.

5. Computational study

We now present the experimental setting, present computational performance of algorithms and discuss the results. In this section, we solve uncapacitated HMLoRP (u-HMLoRP), capacitated HMLoRP (c-HMLoRP), uncapacitated pMLoRP (u-pMLoRP), and capacitated pMLoRP (c-pMLoRP). We implemented our algorithms using Java, and all the experiments were conducted on the Cedar cluster of Compute Canada using single thread and 10GB of RAM on a Linux environment. We used CPLEX 12.10.0 for solving the linear programs. The time limit for all experiments is set to three hours.

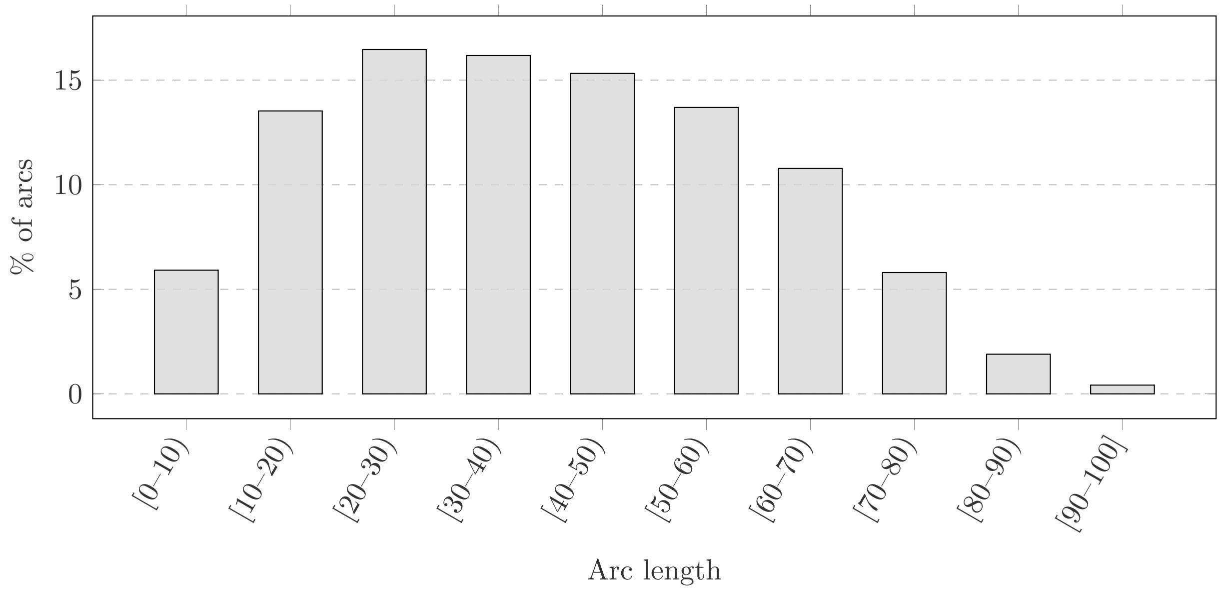


Figure 2: Arc length distribution in the 42 networks considered

5.1. Experimental setting

Table 1: Datasets

Set	Subset	# nodes	# facilities	# networks	# instances
Akca	Akca-30-5	30	5	6	1296
	Akca-40-5	40	5	6	1296
Prodhon	Prodhon-20-5	20	5	4	864
	Prodhon-50-5	50	5	8	1728
	Prodhon-100-5	100	5	6	1296
	Prodhon-100-10	100	10	6	2376
	Prodhon-200-10	200	10	6	2376
Total				42	11,232

We use the networks of Akca et al. (2009) and Prodhon (2006) (Table 1). We consider the large instances of the Akca set containing 6 networks with 30 customers (namely, r30x5a-1, r30x5a-2, r30x5a-3, r30x5b-1, r30x5b-2, r30x5b-3) and 6 networks with 40 customers (namely, r40x5a-1, r40x5a-2, r40x5a-3, r40x5b-1, r40x5b-2, r40x5b-3), all of which have 5 candidate facilities. In the Prodhon set, there are 4 networks with 20 nodes, 8 networks with 50 nodes, 12 networks with 100 nodes, and 6 networks with 200 nodes, which makes 30 networks in total with 5 to 10 capacitated facilities. The fixed setup costs of facilities are equal to 100 in all networks in the Akca set and they are between 5029 and 126,029 in the Prodhon set. The vehicle capacities vary between 275 and 390 in the Akca set and between 70 and 150 in the Prodhon set. The facility capacities are between 1000 and 1750 in the Akca set and between 70 and 1260 in the Prodhon set. The maximum distance between any pair of nodes in the network changes between 101 and 130 in the Akca set and between 4977 and 6930 in the Prodhon set. The coordinate system of the Akca networks is in order of tens and that of the Prodhon networks is in order of thousands. To simplify the design of experiments and to improve the presentation of the results, we scale the arc lengths so that the arc between the two farthest nodes in every network equals 100. Figure 2 plots the arc length distribution in all 42 networks considered. The horizontal axis is the arc length and the vertical axis is the percentage of the arcs in the graphs.

We have three sets of experiments.

- *Experiment set-1:* The first experiment set corresponds to the base case. We use a fixed coverage range for every facility, which we refer to as R. We test for $R \in \{0, 10, 20, 30, 40, 50\}$ and $T \in \{100, 125, 150\}$. We solve uncapacitated and capacitated HMLoRP and pMLoRP(p) for $p = 1, \ldots, |I|$. For the Akca set, this makes 216 u-HMLoRP instances, 216 c-HMLoRP instances, 1080 u-pMLoRP instances, and 1080 c-pMLoRP instances. In the Prodhon set, we have 18 networks with 5 candidate facilities and 12 networks with 10 candidate facilities. This makes 540 u-HMLoRP instances, 540 c-HMLoRP instances, 3780 u-pMLoRP instances, and 3780 c-pMLoRP instances for Prohon set. In total, we have 11,232 problem instances. Note that, by definition, we have $w_A = 0$ for HMLoRP and $w_F = 0$ for pMLoRP. Respecting these two conditions, in this first set of experiments, we fix $(w_F, w_R, w_A) = (1, 1, 0)$ for HMLoRP and $(w_F, w_R, w_A) = (0, 1, 0)$ for pMLoRP. We use this set for presenting computational results in Section 5.2. We also discuss the results of the Akca set in Section 6.2.

- *Experiment set-2:* We develop this second set to investigate how the objective function weights affect the results. We test the same settings for the Akca set as in 'Experiment set-1', and change the weights as $(w_F, w_R, w_A) = (4, 1, 0)$ by amplifying the fixed costs for HMLoRP and $(w_F, w_R, w_A) = (0, 1, 0.2)$ by increasing the allocation costs for pMLoRP, which makes a total of 2592 instances. We discuss their results in Section 6.3.

- *Experiment set-3:* In the third set, we solve large scale instances involving 100 customers and 10 to 60 facilities. To ensure reproducibility, we use the same 3 networks from the Prodhon-100-10 dataset (Table 1) and only add new facilities, which are obtained from the Prodhon-100-10 and the Prodhon-200-10 datasets. This allows us to obtain 60 different candidate facilities. We then build 6 copies of each of the 3 networks by adding 10, 20, 30, 40, 50, or 60 facilities. For these 18 different networks, we solve u-HMLoRP and c-HMLoRP for $R \in \{0, 10, 20, 30, 40\}$ and $T \in \{40, 50, 60\}$, which makes 540 instances. We again set $(w_F, w_R, w_A) = (1, 1, 0)$ for HMLoRP and $(w_F, w_R, w_A) = (0, 1, 0)$ for pMLoRP. We discuss their results in Section 6.4.

Therefore, there are a total of 14,364 instances. In the following, we first present the computational results and then study the impacts of R and T on the total cost and facility and vehicle utilizations.

5.2. Computational results

We carry out the computational tests on 'Experiment set-1' and report the solution status of instances in Table 2. The first column is the name of the dataset, the second column is the name of problem (including both uncapacitated and capacitated problem types) and the third column is the number of instances. The number of optimal, infeasible, feasible, and unknown solutions are shown in columns 3−6, respectively. The unknown instances are certified to be feasible using Proposition 2, but the root node relaxation could not be solved and therefore, no bound information is available. Among the 11,232 instances, a total of 7910 are solved optimally (5666) or proven to be infeasible (2244), which corresponds to more than 70% of the dataset. The remaining 3322 instances are 'Feasible' or 'Unknown'. There are no unknown solutions in the Akca set. Note the number of infeasible pMLoRP instances is large for $p \leq 2$.

Table 2: Solution status breakdown with respect to dataset and problem type in 'Experiment set-1'

Dataset	Problem	# instances	Solution status (# of instances)			
			Optimal	Infeasible	Feasible	Unknown
Prodhon	HMLoRP	1080	383	96	227	374
	pMLoRP(1)	1080	86	798	43	153
	pMLoRP(2)	1080	331	468	108	173
	pMLoRP(3)	1080	558	96	147	279
	pMLoRP(4)	1080	620	96	124	240
	pMLoRP(5)	1080	629	96	131	224
	pMLoRP(6)	432	219		63	150
	pMLoRP(7)	432	215		70	147
	pMLoRP(8)	432	216		72	144
	pMLoRP(9)	432	216		73	143
	pMLoRP(10)	432	217		72	143
Akca	HMLoRP	432	376	48	8	
	pMLoRP(1)	432	98	330	4	
	pMLoRP(2)	432	352	72	8	
	pMLoRP(3)	432	384	48		
	pMLoRP(4)	432	382	48	2	
	pMLoRP(5)	432	384	48		
Total		11,232	5666	2244	1152	2170

Table 3 reports the solution status breakdown as the percentage of the total number of instances in the same dataset. For example, there are 864 instances in the Prodhon-20-5 dataset and 75.7% are solved optimally and 24.3% are proven to be infeasible. Note that the total percentage of feasible and unknown instances increases as the number of nodes in the network increases.

In Table 4, the solution status breakdown is reported for different R, which mainly affects the number of feasible and unknown instances. When R is high, the number of vehicles routes to

Table 3: Solution status breakdown as percentages with respect to dataset in 'Experiment set-1'

Dataset	# nodes	# instances	Solution status (% of instances)			
			Optimal	Infeasible	Feasible	Unknown
Prodhon-20-5	20	864	75.7%	24.3%	0.0%	0.0%
Akca-30-5	30	1296	80.8%	19.0%	0.2%	0.0%
Akca-40-5	40	1296	71.7%	26.9%	1.5%	0.0%
Prodhon-50-5	50	1728	46.3%	27.8%	19.6%	6.4%
Prodhon-100-5	100	1296	29.6%	34.3%	12.0%	24.1%
Prodhon-100-10	100	2376	44.3%	10.6%	19.7%	25.5%
Prodhon-200-10	200	2376	33.7%	11.1%	7.1%	48.1%
Avearge		11,232	50.4	20.0	10.3	19.3

Table 4: Solution status breakdown with respect to R in 'Experiment set-1'

Facility range (R)	# instances	Solution status (# of instances)			
		Optimal	Infeasible	Feasible	Unknown
0	1872	499	374	232	767
10	1872	506	374	300	692
20	1872	732	374	372	394
30	1872	1156	374	128	214
40	1872	1344	374	84	70
50	1872	1429	374	36	33
Total	11,232	5666	2244	1152	2170

be generated is small and therefore, the solution process is more efficient and more instances are solved optimally. On the other hand, the number of infeasible instances is not impacted by R, which is always less than T in the 'Experiment set-1'. Therefore, if a customer can be covered by a facility, it can always be covered by the vehicle too, but the opposite is not true. The main factor affecting the number of infeasible instances is T. There are 2244 infeasible instances, which corresponds to 20.0% of all the instances and we report their breakdown in Table 5. The first column is the name of the problem and the second column is the number of infeasible instances. Columns 3−5 report the infeasible instance counts for the uncapacitated problem for $T = 100, 125$ and 150, respectively. Columns 6−8 report the same statistics for the capacitated problem. The majority of infeasible instances are in pMLoRP when both p and T are small. Since a customer can be covered by location or routing, feasibility requires at least one of the coverage types for every customer. Note that there are also 432 instances of pMLoRP with $6 \leq p \leq 10$, all of which are feasible and therefore not reported in this table.

Table 5: Breakdown of infeasible instances with respect to problem type and T in 'Experiment set-1'

Problem	# infeasible	Uncapacitated problem			Capacitated problem		
	instances	T=100	T=125	T=150	T=100	T=125	T=150
HMLoRP	144	54	18		54	18	
pMLoRP(1)	1128	252	102	18	252	252	252
pMLoRP(2)	540	96	18		180	126	120
pMLoRP(3)	144	54	18		54	18	
pMLoRP(4)	144	54	18		54	18	
pMLoRP(5)	144	54	18		54	18	
Total	2244	564	192	18	648	450	372

The optimality gaps are reported in Table 6. The averages are 0.7%, 1.2%, 2.4%, 0.6%, 0.3%, and 0.2% when R equals 0, 10, 20, 30, 40, and 50, respectively. The gaps are generally higher for capacitated instances.

The solution times are significantly affected by R (Table 7). Their averages are 5912.8, 4839.9, 4656.7, 2097.7, 982.1, and 489.3 seconds for $R = 0$, 10, 20, 30, 40, and 50, respectively. When R increases, several customers are covered by the open facilities. This results in smaller number of vehicle routes to be generated by the column generation algorithm and the solution times decrease accordingly. The parameter T also has an impact on the average solution times because the pricing problem is less constrained and takes more time when T is high.

6. Discussion

In this section, we discuss the impacts of several parameters on the total cost and on the facility and vehicle utilization. We first demonstrate the impacts of new problem type on the solutions

Table 6: Average optimality gaps for different R and T in 'Experiment set-1'

Problem type	Maximum route length (T)	Facility range (R) 0	10	20	30	40	50
Uncapacitated	100	0.8%	1.3%	1.6%	0.3%	0.1%	0.0%
	125	0.4%	0.9%	2.4%	0.7%	0.4%	0.0%
	150	0.5%	0.8%	1.9%	0.3%	0.4%	0.5%
Capacitated	100	0.9%	1.8%	2.8%	0.9%	0.1%	0.0%
	125	0.9%	1.2%	2.9%	0.9%	0.2%	0.2%
	150	1.0%	1.2%	2.6%	0.6%	0.5%	0.4%
Average		0.7%	1.2%	2.4%	0.6%	0.3%	0.2%

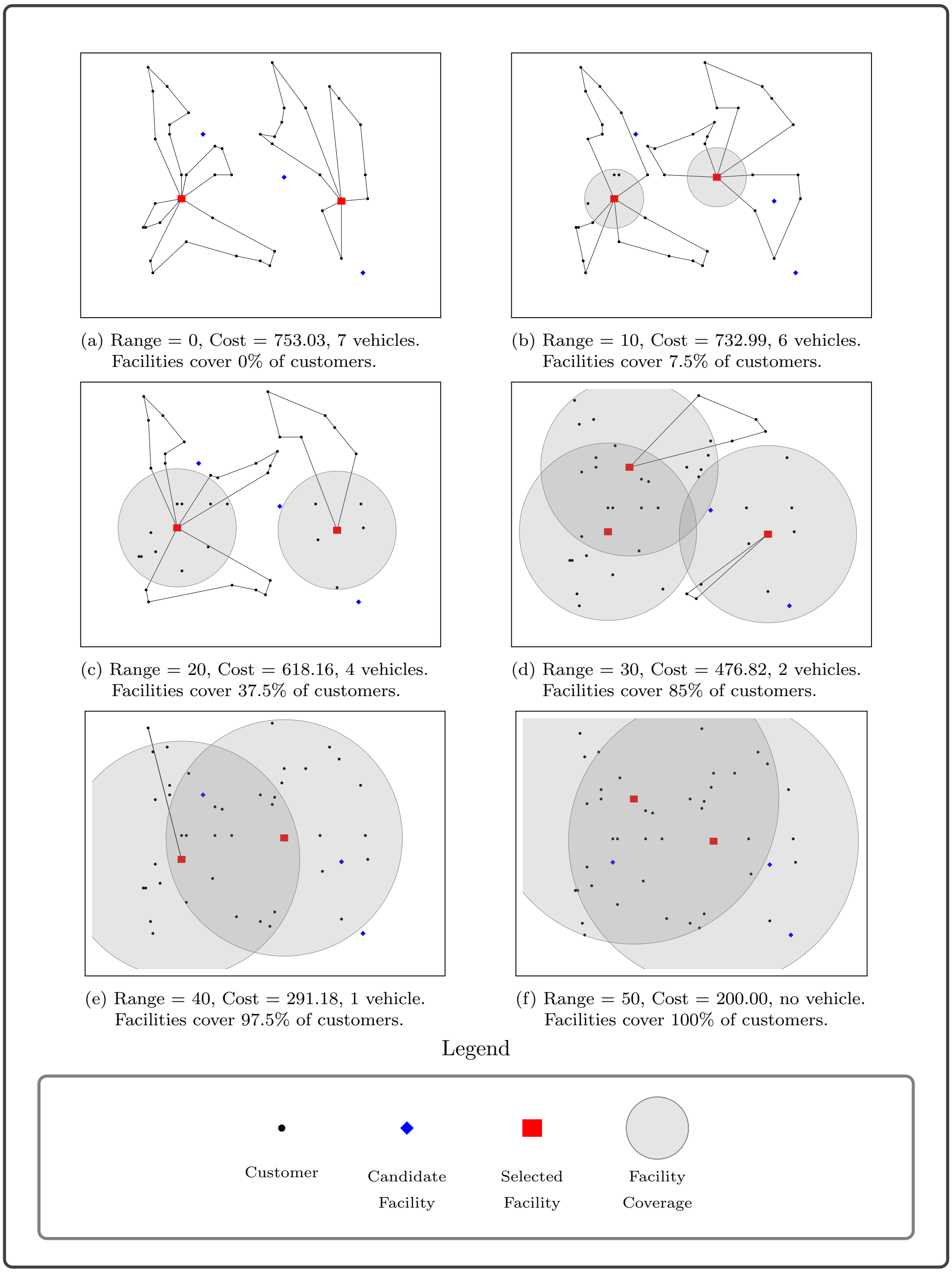
Table 7: Solution times (s) for different T and R in 'Experiment set-1'

Maximum route length (T)	Facility range (R) 0	10	20	30	40	50
100	4984.9	4976.5	3565.8	1237.1	219.0	19.6
125	5896.1	5855.2	4765.8	2153.0	983.9	400.8
150	6857.5	6688.1	5638.6	2903.1	1743.5	1047.5
Average	5912.8	5839.9	4656.7	2097.7	982.1	489.3

using an illustrative example in Section 6.1. We then discuss the results of 'Experiment set-1', 'Experiment set-2', and 'Experiment set-3' in Sections 6.2, 6.3, and 6.4, respectively.

6.1. An illustrative example

We first demonstrate the effect of facility coverage range on the optimal solutions using the example in Figure 3. The network we consider is 'r40x5a-2' of the Akca set with $T = 150$. There are 40 nodes and 5 candidate facilities. We solve the HMLoRP for $R = 0, 10, 20, 30, 40, 50$ and the optimal locations and vehicle routes are shown in Figures 3(a)−(f), respectively. Starting with the top-left figure, the optimal cost is 753.03 when $R = 0$ and 7 vehicles are used. When $R = 10$, the cost is not significantly affected, it only reduces 2.66% and 1 fewer vehicle is used (Figure 3(b)). The impact is more obvious when $R = 20$, in which case the cost reduction is more than 17.9% and 4 vehicles are used (Figure 3(c)). When $R = 30$, the cost reduces by 36.7% and the number of facilities increases to 3, which collectively cover 35 customers (Figure 3(d)). In this setting, only 5 customers are covered by 2 vehicles. Increasing R to 40 (Figure 3(e)) leaves only 1 uncovered customer by the 2 open facilities, which is also covered by the open facilities when $R = 50$ (Figure 3(f)). The problem effectively transforms into a set covering problem.



(a) Range = 0, Cost = 753.03, 7 vehicles. Facilities cover 0% of customers.

(b) Range = 10, Cost = 732.99, 6 vehicles. Facilities cover 7.5% of customers.

(c) Range = 20, Cost = 618.16, 4 vehicles. Facilities cover 37.5% of customers.

(d) Range = 30, Cost = 476.82, 2 vehicles. Facilities cover 85% of customers.

(e) Range = 40, Cost = 291.18, 1 vehicle. Facilities cover 97.5% of customers.

(f) Range = 50, Cost = 200.00, no vehicle. Facilities cover 100% of customers.

Figure 3: Selected facilities and vehicle routes in optimal solutions for network ‘r40x5a-2’ of the Akca set, maximum route length $T = 150$ and facility coverage range $R \in \{0, 10, 20, 30, 40, 50\}$ in parts (a)−(f), respectively.

6.2. Experiment set-1: base case

In this section, we present the results of the Akca set in 'Experiment set-1'. We omit the results of the Prodhon set because there are many non-optimal solutions, which prevents drawing meaningful insights. For the same reason, we do not report the results corresponding to pMLoRP(1) and we remove the instances in the Akca set corresponding to networks 'r30x5b-3', 'r40x5b-1' and 'r40x5b-2' because they are infeasible when $T = 100$ but feasible when $T = 125$ or $T = 150$.

Table 8: Average costs for different problem types and R for the Akca set instances in 'Experiment set-1'

Problem	Facility range (R)					
name	0	10	20	30	40	50
HMLoRP	781.7	771.4	694.0	553.7	396.2	222.2
pMLoRP(2)	582.1	570.7	502.5	397.9	235.6	27.6
pMLoRP(3)	535.9	517.8	419.8	274.0	122.1	0.0
pMLoRP(4)	523.0	495.8	377.7	233.5	116.4	0.0
pMLoRP(5)	515.6	484.7	355.9	230.0	116.4	0.0

Table 8 reports the costs of HMLoRP for different problem types and R. The averages are 781.7, 771.4, 694.0, 553.7, 396.2, and 222.2 for $R = 0, 10, 20, 30, 40$, and 50, respectively. There is almost a linear relationship between the cost and R when range is at least 20 (Figure 4). The slope of the linear part of the blue curve is 15.7. That is, every unit increase in R decreases the cost by 15.7. The linear relationship between the cost and the range is rooted in the random distribution of customers in the plane and we elaborate on this topic in Section 6.2.1. The average costs of pMLoRP for $2 \leq p \leq 5$ are also reported in Table 8 and are shown in Figure 4. We observe a similar trend in pMLoRP to the HMLoRP. The average costs of HMLoRP are higher because they also include the opening cost of facilities. Note that the costs of pMLoRP(p) for $3 \leq p \leq 5$ are zero when $R = 50$ because opening facilities does not contribute to the objective function in pMLoRP, the allocation weight $w_A = 0$ and no vehicle is necessary to cover the customers because they are covered by facilities.

Table 9 reports the average number of open facilities in HMLoRP solutions. There are generally more open facilities in c-HMLoRP solutions than in u-HMLoRP solutions. The average number of open facilities is decreasing when T increases, because more customers can be covered by vehicles. In the R dimension, the average number of facilities in solutions are 2.04, 2.11, 2.39, 2.81, 2.46 and 2.22 for $R = 0, 10, 20, 30, 40$, and 50, respectively. Figure 5 plots this relationship. There are two parts of this function. The average number of open facilities is increasing until $R = 30$ and then decreasing. The reason is the tradeoff between opening a new facility to cover customers (in which case fixed cost is incurred), and sending vehicles from the other open facilities (in which case routing cost is incurred). In the first part of the function, the cost of opening a new facility to cover customers is less than the cost of covering them by vehicles. The marginal benefit of

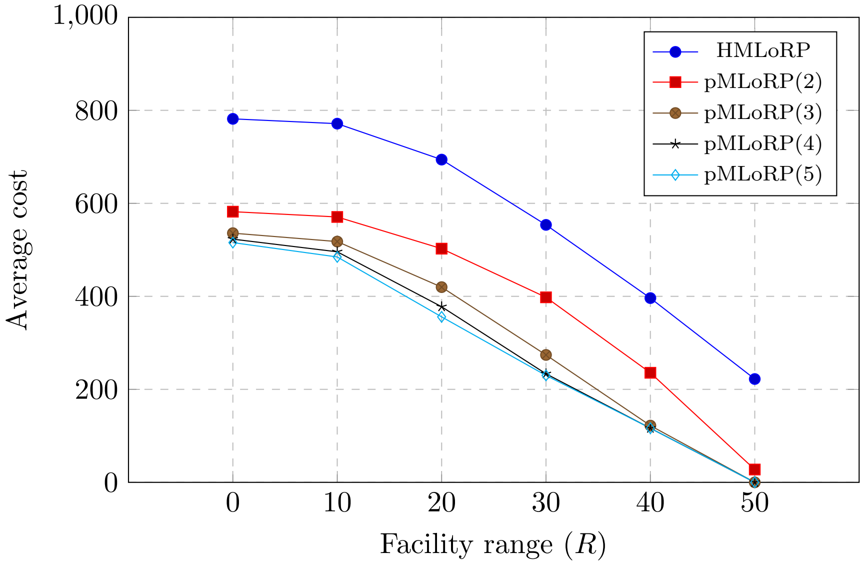


Figure 4: Average costs for different problem types and R for the Akca set instances in 'Experiment set-1'

Table 9: Average number of open facilities used in HMLoRP solutions for different T and R for the Akca set instances in 'Experiment set-1'

Problem	Max. route	Facility range (R)					
type	length (T)	0	10	20	30	40	50
Uncapacitated	100	2.33	2.44	2.67	3.00	2.67	2.22
	125	1.89	2.00	2.33	2.78	2.33	2.22
	150	1.56	1.67	2.11	2.56	2.22	2.22
Capacitated	100	2.33	2.44	2.67	3.11	2.67	2.22
	125	2.11	2.11	2.33	2.78	2.44	2.22
	150	2.00	2.00	2.22	2.67	2.44	2.22
Average		2.04	2.11	2.39	2.81	2.46	2.22

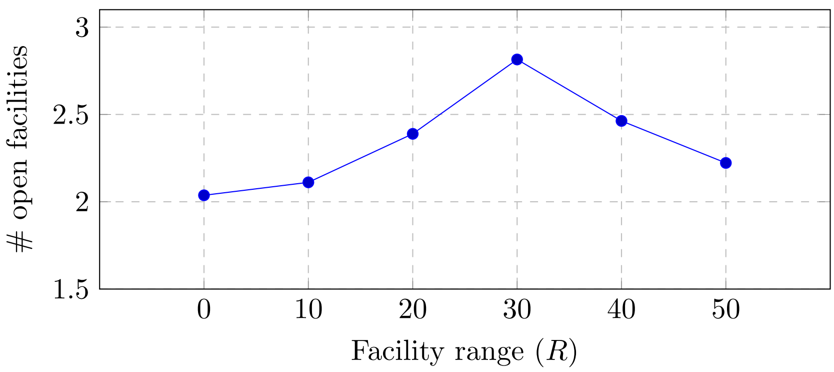


Figure 5: Average number of open facilities used in HMLoRP solutions for different R for the Akca set instances in 'Experiment set-1'

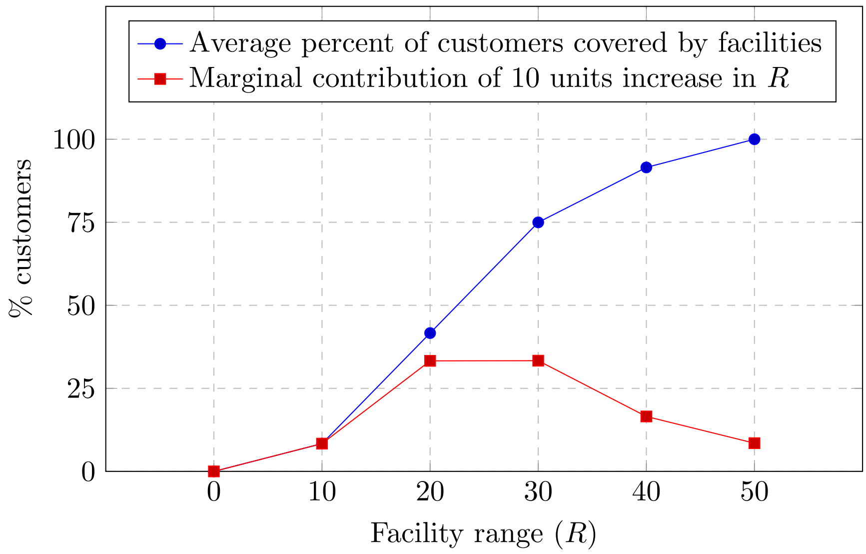


Figure 6: Average percent of customers covered by facilities in HMLoRP solutions for different R for the Akca set instances in ‘Experiment set-1’

opening a new facility is increasing as R approaches from 0 to 30 because each new facility covers more customers. In the second part of the function, the facility coverages start to overlap. In other words, the marginal contribution of each new facility is not good enough, and covering the customers by vehicles from open facilities is better in terms of the objective function. To this end, the average percent of customers covered by facilities and the marginal contribution of 10 units increase in R are shown in Figure 6. Note that the marginal contribution line in red is decreasing when $R \geq 30$.

Table 10: Average number of vehicles used in HMLoRP solutions for different T and R for the Akca set instances in ‘Experiment set-1’

Problem	Max. route	Facility range (R)					
type	length (T)	0	10	20	30	40	50
Uncapacitated	100	7.33	6.56	5.78	3.56	1.78	0.00
	125	6.33	5.89	4.44	2.67	1.44	0.00
	150	6.11	5.67	3.89	2.44	1.44	0.00
Capacitated	100	7.44	6.78	5.89	3.56	1.89	0.00
	125	6.22	5.67	4.44	2.67	1.33	0.00
	150	6.11	5.67	3.89	2.33	1.33	0.00
Average		6.59	6.04	4.72	2.87	1.54	0.00

Table 10 shows the number of vehicles used in the HMLoRP solutions. The average number of vehicles used is 6.59, 6.04, 4.72, 2.87, 1.54 and 0.00 when facility range is 0, 10, 20, 30, 40, and 50,

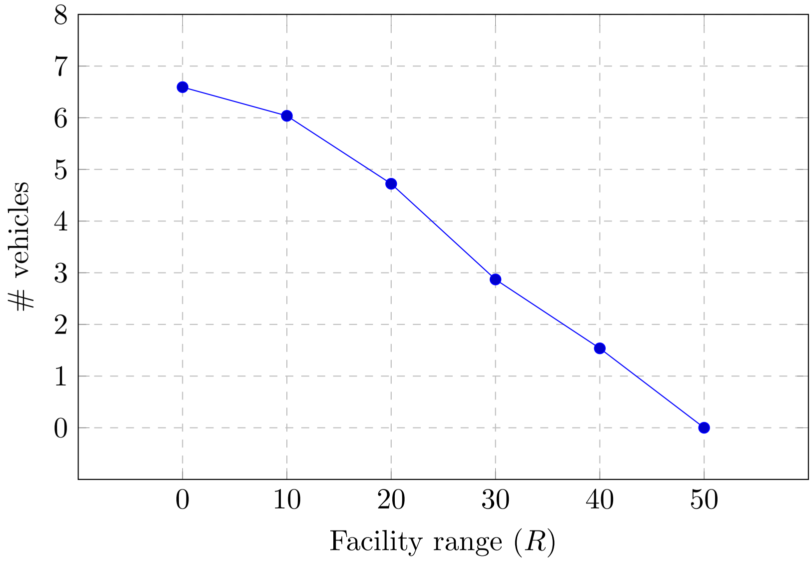


Figure 7: Average number of vehicles used in HMLoRP solutions for different R for the Akca set instances in 'Experiment set-1'

respectively. The clear relationship between these two factors is depicted in Figure 7. On average, the number of vehicles reduces by 1.3 for every 10 units of increase in R. This rate of change depends on the instance and may be as high as 1.8. Note that, for some networks and specific range values, increasing the range does not necessarily reduce the number of vehicles.

6.2.1. An approximation for the cost

It is a well-known result in continuous approximation models (Beardwood et al., 1959; Daganzo, 1984a,b) that, given n points that are uniformly and independently distributed in an (fairly compact) area of size A, we have $\frac{L^*}{\sqrt{n}} \to \beta\sqrt{A}$ as $n \to \infty$, where L^* is the optimal length in traveling salesman problem and β is a constant term. For empirical β values, we refer the reader to Franceschetti et al. (2017). For VRP, Daganzo (1984a) approximated the optimal length as $L^* = 2rm + 0.57\sqrt{nA}$, where r is the so called *stem distance* and m is the number of vehicles. Inspired by this line of thinking, we investigate the relationship that we empirically observe in Figure 4 between the facility range R and the cost. To this end, we consider the routing cost in the problem, expressed by the second term in the objective function (1) of SC model. For a facility coverage range of R, let L_R be the optimal routing cost and η_R be the *square root of the fraction of the uncovered customers by the facilities* ($0 \leq \eta_R \leq 1$). Figure 8 plots the average L_R and η_R for different R values. In HMLoRP instances in Figure 8(a), the routing cost is shown in red with values on the left axis and the η_R is shown in blue with values on the right axis. The L_R and η_R for pMLoRP are shown in Figure 8(b). The strong relationship is easily observed on the average values. Furthermore, $\eta_R L_0$ can be used to approximate L_R for any problem instance. In particular, we find that this approximation is within 5%, 10%, 20% and 30% of the true optimal routing cost in 65.7%, 75.3%, 88.9% and 95.7% of the instances. As an example, consider the instance in Figure 3. In Table 11, we report the percent of uncovered customers by any facility, η_R, total cost, fixed cost, routing cost, approximation of the routing cost and the absolute error in columns 2−8, respectively. The absolute errors are below 5% except when $R = 30$. The error would be only 1.1% if the number of uncovered customers was 4 instead of 6.

Table 11: Routing cost continuous approximation in the example in Figure 3

Facility Range (R)	Uncovered customers by any facility (%)	η_R	Total cost	Fixed cost	Routing cost	Approximation ($L_R = \eta_R L_0$)	Absolute error (%)
0	100.0	1.00	753.0	200.0	553.0	553.0	0.0
10	92.5	0.96	733.0	200.0	533.0	531.9	0.2
20	62.5	0.79	618.2	200.0	418.2	437.2	4.6
30	15.0	0.39	476.8	300.0	176.8	214.2	21.1†
40	2.5	0.16	291.2	200.0	91.2	87.4	4.1
50	0.0	0.00	200.0	200.0	0.0	0.0	0.0

† This error would be 1.1% if the number of uncovered customers was 4 instead of 6.

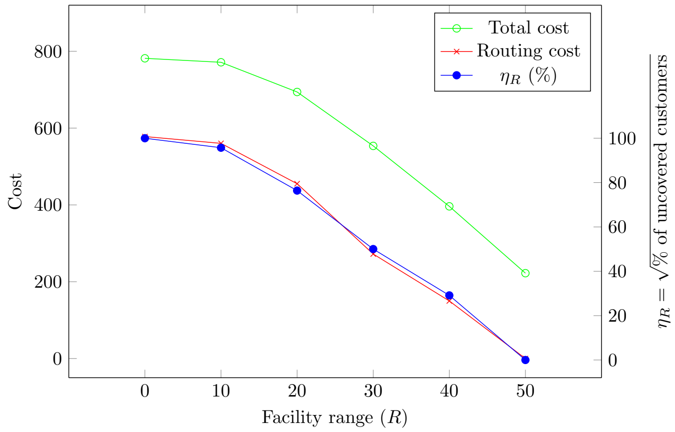
It is noteworthy that, unlike its counterpart in the vehicle routing problem, the continuous approximation for the LoRP cost is not impacted by the decreasing size of the area as facility range increases. Even though the coverage of customers by the open facilities induces a nonconvex shape of uncovered customers, the relationship is strong and it closely approximates the routing costs. This is mainly because the vehicles still need to travel this distance and all the uncovered customers are beyond the coverage range.

The routing cost changes proportional to η_R. We now take one step further and observe that the average number of customers covered by a facility is directly proportional to the area of the facility coverage, which equals πR^2. Therefore η_R, which is defined as the square root of the fraction of the uncovered customers by the facilities, changes linearly in R. This explains the linear relationship of the routing cost by increasing facility range.

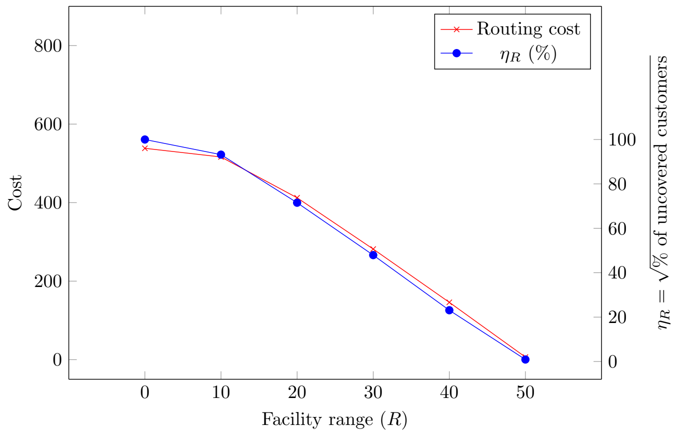
6.3. Experiment set-2: changing the objective function coefficients

In this section, we report the results of instances in 'Experiment set-2'. The main purpose is to investigate the changes in average costs, number of open facilities and number of vehicles used with respect to the base case in 'Experiment set-1'. To this end, the objective function weights are set as $(w_F, w_R, w_A) = (4, 1, 0)$ for HMLoRP and as $(w_F, w_R, w_A) = (0, 1, 0.2)$ for pMLoRP.

The average costs for different problem types and R are reported in Table 12 and plotted in Figure 9. For the pMLoRP, the solutions are not impacted significantly by the change in the objective function weights. When $R = 0$, the same objective function values are obtained as in the base case (c.f. Table 8), because the customers are not covered by the facilities and $w_R = 0$ for pMLoRP. For higher R values, the costs are always greater than those in the base case, but the increase is insignificant. For the HMLoRP, on the other hand, the average costs increase at least



(a) Average total cost, routing cost and the square root of the uncovered customer percentage in HMLoRP



(b) Average routing cost and the square root of the uncovered customer percentage in pMLoRP for $p = 2, \ldots, 5$

Figure 8: The change in routing cost is aligned with the square root of the fraction of uncovered customers by the facilities

by 70% with respect to the base case mainly due to the increase in w_F. Furthermore, the average numbers of open facilities decrease in HMLoRP solutions and change between 1.8 and 1.84 for all R values (Table 13). Because of the decrease in the number of open facilities, the customers are mainly covered by vehicles, which justifies the increase in the average numbers of vehicles in Table 14 with respect to those in Table 10.

Table 12: Average costs for different problem types and R in 'Experiment set-2'

Problem	Facility range (R)					
name	0	10	20	30	40	50
HMLoRP	1338.2	1331.5	1274.6	1175.0	1022.6	813.2
pMLoRP(2)	582.1	572.1	515.5	432.7	293.5	113.9
pMLoRP(3)	535.9	519.3	438.1	315.7	183.7	77.3
pMLoRP(4)	523.0	498.6	398.4	274.7	173.7	72.3
pMLoRP(5)	515.6	488.7	377.8	269.0	170.7	69.4

Table 13: Average number of open facilities used in HMLoRP solutions for different T and R in 'Experiment set-2'

Problem	Max. route	Facility range (R)					
type	length (T)	0	10	20	30	40	50
Uncapacitated	100	2.22	2.22	2.22	2.22	2.22	2.22
	125	1.45	1.45	1.45	1.55	1.55	1.55
	150	1.08	1.08	1.08	1.08	1.25	1.25
Capacitated	100	2.22	2.22	2.22	2.22	2.22	2.22
	125	2.00	2.00	2.00	2.00	2.00	2.00
	150	2.00	2.00	2.00	2.00	2.00	2.00
Average		1.80	1.80	1.80	1.81	1.84	1.84

The results show that the location decisions highly depend on the tradeoff between the location and routing multipliers in the objective function of the SC model, w_F and w_R, respectively. We find that a greater number of facilities are opened when the w_F is small; whereas, for large w_F values, the number of facilities is small, and the customers are primarily served using vehicles. In any case, the open facilities are spread and not clustered. The routing multiplier w_R, on the other hand, has an opposite effect, for smaller w_R values, the number of open facilities is smaller. We observe that the vehicle routes play a decreasing role in cost as R increases.

6.4. Experiment set-3: greater number of facilities

In this section, we present the results for 'Experiment set-3'. We consider 18 networks with 100 customers and 10, 20, 30, 40, 50, or 60 candidate facilities. We set the step size parameter of the pulse algorithm equal to 5 to solve the pricing problem. We solve u-HMLoRP and c-HMLoRP

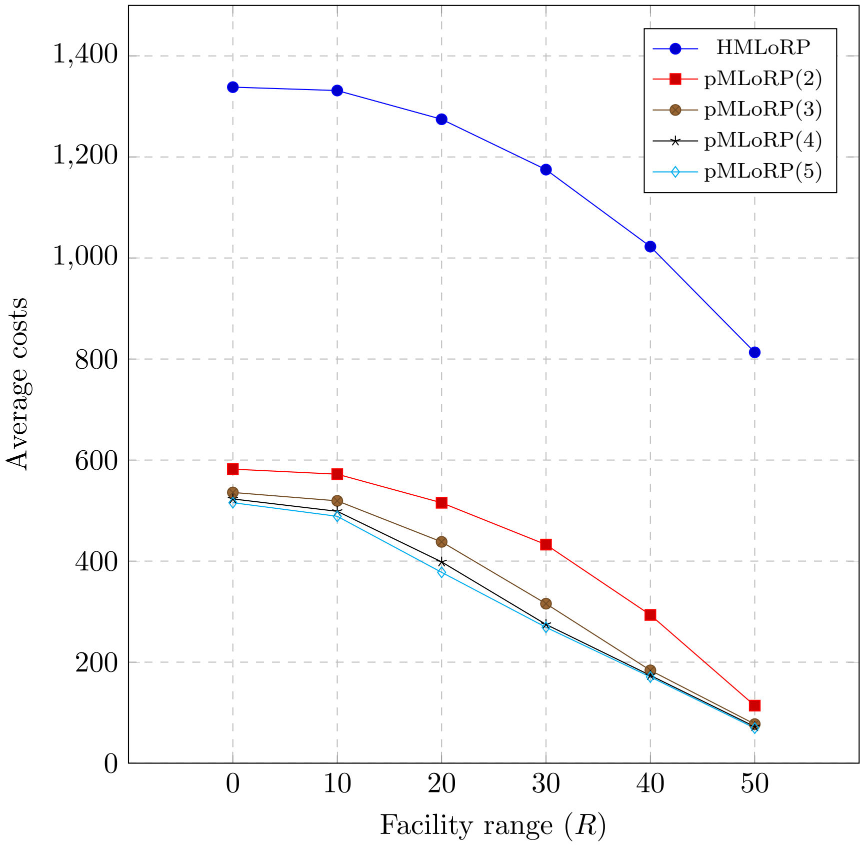


Figure 9: Average costs for different problem types and R in ‘Experiment set-2’

Table 14: Average number of vehicles used in HMLoRP solutions for different T and R in ‘Experiment set-2’

Problem type	Max. route length (T)	Facility range (R) 0	10	20	30	40	50
Uncapacitated	100	7.33	6.89	6.33	5.00	2.89	0.00
	125	6.78	6.78	6.00	4.89	3.44	1.56
	150	6.00	5.89	5.11	4.22	3.44	1.78
Capacitated	100	7.44	7.22	6.44	5.22	3.00	0.00
	125	6.33	5.78	4.89	3.67	2.22	0.44
	150	6.44	5.67	4.11	3.22	1.78	0.22
Average		6.72	6.37	5.48	4.37	2.80	0.67

for $R \in \{0, 10, 20, 30, 40\}$ and $T \in \{40, 50, 60\}$. There are infeasible instances when T and the number of candidate facilities are both small (Table 15), because some customers are not within the coverage range of any facility and they cannot be reached by any vehicles from these facilities because of the maximum route length. We therefore do not consider the instances in which the number of candidate facilities is 10, 20 or 30.

Table 15: Infeasible instances in 'Experiment set-3'

Max. route length (T)	# candidate facilities	Range (R) 0	10	20	Total
40	10	6	6	6	18
	20	6	6	6	18
	30	4	4	4	12
50	10	6	6	6	18
Total		22	22	22	66

Table 16 reports the costs for different T and R. The cost is the highest when $T = 40$, number of candidate facilities is 40 and $R = 0$. This corresponds to the most restricted setting considered in the table. When $T = 40$, the costs do not change significantly for increasing R from 0 to 20. However, there is a significant reduction when $R = 30$. This is when the facility range exceeds a certain threshold beyond which vehicle visits are unnecessary to cover customers because they are covered by facilities. We can also observe this result in Table 17, where we observe that the average number of open facilities reduce significantly when R increases to 30.

Table 16: Average costs of HMLoRP solutions (in thousands) for different T and R in 'Experiment set-3'

Max. route length (T)	# candidate facilities	Facility range (R) 0	10	20	30	40
40	40	392.09	391.98	391.01	178.61	138.51
	50	365.08	364.97	364.00	178.61	138.51
	60	365.08	364.97	364.00	178.61	138.51
50	40	230.88	230.76	230.23	178.61	138.51
	50	230.87	230.76	230.23	178.61	138.51
	60	230.87	230.76	230.24	178.61	138.51
60	40	179.79	179.70	179.30	178.61	138.51
	50	179.79	179.70	179.30	178.61	138.51
	60	179.80	179.70	179.30	178.61	138.51
Average		261.58	261.48	260.84	178.61	138.51

In Table 17, the average number of open facilities change between 2.83 and 7. The number of open facilities is significantly affected by R, but the number of candidate facilities in the network

Table 17: Average number of open facilities used in HMLoRP solutions for different T and R in 'Experiment set-3'

Max. route length (T)	# candidate facilities	Facility range (R) 0	10	20	30	40
40	40	7.00	7.00	7.00	3.67	2.83
	50	6.67	6.67	6.67	3.67	2.83
	60	6.67	6.67	6.67	3.67	2.83
50	40	4.67	4.67	4.67	3.67	2.83
	50	4.67	4.67	4.67	3.67	2.83
	60	4.67	4.67	4.67	3.67	2.83
60	40	3.67	3.67	3.67	3.67	2.83
	50	3.67	3.67	3.67	3.67	2.83
	60	3.67	3.67	3.67	3.67	2.83
Average		5.04	5.04	5.04	3.67	2.83

affects the average number of open facilities only marginally. Put differently, a high number of potential locations does not significantly impact the results. The facility coverage range on the other hand is the primary determinant on the number and location of open facilities.

7. Conclusion and future research

The facility location and vehicle routing are closely related problems in transportation. We have uncovered a new connection between the location and routing decisions by introducing the location-or-routing problem (LoRP), in which a customer can be covered either by a facility or by a vehicle visit. Each selected facility covers the customers in the neighborhood around itself defined by a coverage range, similar to the set covering problem. If a customer lies beyond the coverage range of any open facility, then a vehicle visit is required to cover the customer. This new problem has applications in location optimization of retail stores, supermarkets, shopping malls, schools, urban delivery centers and medical testing centers. We have presented a set covering model with an exponential number of variables for solving the LoRP. As a solution method, we have developed an exact branch-and-price algorithm and provided insights on the total costs. In particular, experiments on random graphs have shown that the total cost decreases almost linearly as the facility coverage range increases. We have also demonstrated that the facility range is an important determinant of the number and location of open facilities. We have found that the vehicle routes play a decreasing role on the total cost as the facility range increases. Several extensions of LoRP are possible. Basic extensions include considering stochastic nature of the demand. Additionally, the customer demand and the facility coverage range are assumed to be independent in this study. However, in retail store, supermarket or shopping mall location applications, the coverage of the demand may decay as the distance between a facility and a customer increases. Lastly, since multiple facilities can potentially cover a customer, backup coverage may be considered when making decisions.

Acknowledgment

The author thanks the associate editor Prof. James Campbell and two referees whose comments helped improve the quality of this paper significantly. The author acknowledges that the computations reported in this paper were performed on Compute Canada.

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