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# The Impact of Product Recall on Advertising Decisions and Firm Profit While Envisioning Crisis or Being Hazard Myopic

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**Abstract.** Occurrence of a product recall can have a disastrous effect on the firm responsible for the recall. Any major recall by a firm can negatively affect the goodwill of the firm. Consequently, the firm incurs a substantial indirect cost due to decline in sales and loss in profit. Moreover, a competitor's opportunistic reaction can intensify the recalling firm's damages. Strategic use of advertising recovers lost goodwill and mitigates the damages made by a product recall. In this paper, using a goodwill based model under a differential game framework, we analyze the equilibrium strategies of two competing manufacturers when either one firm or both can issue a product recall at a random time, and investigate (i) the firms' equilibrium advertising strategies (ii) analyze the impact of the recall on a firm's profit (iii) introduce and investigate the effect of "hazard myopia" (a firm's inability to foresee the crisis likelihood) on a firm's advertising decisions and profit. Our study finds that the equilibrium advertising strategies for both the firms vary depending on the impact and likelihood of the recall. Especially, we find that when both the firms are focal firms without the prior knowledge of who will recall first in a planning horizon, adjusting optimal advertising at an appropriate time is utterly important. Surprisingly, a recall with a minor impact can increase the focal firm's long-term expected profit. On the other hand, hazard myopia can be profitable if the long-term effect of the recall is small. Our findings suggest that advertising levels of firms should differ in pre-recall and post-recall regimes depending on the impact and likelihood of the recall.

**Keywords:** Product recall, dynamic game, equilibrium advertising effort, brand image, market share, crisis likelihood.

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## 1. Introduction

Product recalls are common occurrences in industries like consumer products, vehicles, food products and health products. The Transport Canada database shows that there has been 10402 recalls issued only in the automobile industry starting from 2010 till January 2018. In a contemporary competitive market, rival firms strive for quality excellence. However, the data from Transport Canada shows that despite firms' best efforts to ensure quality, product-recalls occur often and therefore should be anticipated during a management's decision making. The impact of product recalls vary depending on the harm caused by the products under recall. While small recalls, causing little media attention and customer awareness may go unnoticed, the major recalls often result in huge losses for the firms at fault as consumers lose confidence in the brand, supplier relationships get affected negatively or competitors take advantage of the situation (Craig and Thomas, 1996; Eilert, 2013). The anticipation of product harm crisis is important from a managerial point of view because it can affect decisions like advertising, pricing or quality investments and largely impact the firm's reputation and financial performance (Rubel, 2018; Rubel et al., 2011; Chao et al., 2009; Cleeren et al., 2008; Gao et al., 2015; Chen et al., 2009; Eilert et al., 2017). Examples of strategic decision making to alleviate recall impacts are abundant. Salmonella contamination led to the recall of some Cadbury chocolate flavours from the UK market. After the recall, Cadbury's marketing efforts increased substantially resulting in an increase in advertising and the launch of new products (Walsh, 2006). The increased efforts of marketing or advertising were in the expectation of regaining lost market. In a more recent incident, Samsung globally recalled Note 7. The impact was huge and it wiped off \$16 billion of the company's market value initially (Reuters, 2016). Following the recall, Samsung made extensive advertising campaigns to regain customers' trust. The advertising went to the extent of showing the new eight-point battery check that exceeded the industry quality standards (Fenech, 2017). This shows how Samsung signalled quality improvement via advertising.

The words product recall and product harm crisis are synonymous in most of the marketing literature though the strict meaning of the two words is not same(Cleeren et al., 2017). While product harm crisis is the antecedent, product recall is the consequence. However, for the sake of this paper we use the words "product recall", "crisis" or product harm crisis" to mean the same thing - product recall.

The motivation of our paper stems out from the dilemma that the previous literature and the various industry reports yield about a firm's advertising efforts after a recall. Furthermore, there is a relative lack of research in the area of decision making in marketing channels to mitigate recall impacts (Wowak and Boone, 2015). We investigate the equilibrium advertising policies of two competing firms when one firm anticipates a recall (one focal firm) or both firms (two focal firms) predict recalls. Additionally, we analyze the situation where a firm ignores the crisis likelihood. Moreover, we examine the effects of crisis likelihood and damage on the firm profit for both the cases: competition with one focal firm and competition with two focal firms.

High pre-crisis brand equity can protect a firm from the recall crisis and therefore a firm, in anticipation of a recall, can advertise more during pre-crisis regime to build its brand image (Cleeren et al., 2008). Alternatively, management might have an urge to make more advertising efforts in the post-crisis regime in order to gain back its image and lost market share. An increase in the advertising can be a lost effort if the consumers are unforgiving towards the firm at fault. Consequently, the competitor might also exhibit opportunistic behaviour thereby increasing its advertising (Cleeren et al., 2013; Craig and Thomas, 1996). Both the arguments, favouring increased ad spending before or after a recall are valid. Every firm has a certain likelihood and damage rate for a particular product recall. Managers of a firm should envision the risk of recall in their advertising decisions throughout the planning horizon because crisis anticipation increases managers' time preference (Rubel et al., 2011).

Uncertain occurrence, substantial damage which incurs additional costs like costs for managing reverse logistics, customer compensations, litigation costs, etc. and above all an erosion of brand value or goodwill are the characteristics of a product recall. Our paper focuses mainly on analyzing optimal advertising decision when firms compete with respect to brand image or goodwill. Product recall affects the baseline sales of the recalling firm (Van Heerde et al., 2007). Very severe damage due to quality failure and subsequent recall can jeopardize the image of the firm at fault. The car airbags recall by Takata Corporation of Japan cost the company in billions. Consequently, Takata filed bankruptcy in 2016 (Tajitsu, 2017). In another incident, the Westland/Hallmark Meat packing company, accused of improper cattle handling, recalled more than 143 million pounds of beef in 2008 due to an intervention by the FDA. The company, under a $ 500 million settlement with various plaintiffs had to file bankruptcy in 2012. A highly publicised and costly recall caused swift goodwill erosion thereby accelerating the firm's jeopardy.

High goodwill results in better market share and hence better performance. A firm's goodwill is often linked with advertising. As a matter of fact, advertising is a major factor in maximizing brand equity (Meenaghan, 1995; Achenbaum, 1989; Lindsay, 1990). Meenaghan (1995) observes - "at all levels of marketing imagery advertising is identified as one of the principal components of image creation". For example, Nike's famous "Just do it" campaign not only boosted the company's brand image but also increased the sales from $877 million to $9.2 billion within a decade. A high brand image has a positive impact on the market share or sales of a firm. Numerous goodwill based demand models assume the positive effect of goodwill on demand (Dockner et al., 2000; Karray and Zaccour, 2005). If consumers' perceived brand image is congruent with a consumer's social, actual, ideal images, the brand is purchased and the sales of the brand are impacted positively (Ataman and Ülengin, 2003).

We wish to validate the findings related to optimal advertising during recall (Rubel et al., 2011; Cleeren et al., 2013, 2008). Firms fear the goodwill loss as a result of a recall. To the best of our knowledge, the extant literature, related to the effect of product recalls, did not shed light on the optimal advertising policies when goodwill of the competing firms affect their individual demands. We develop a dynamic game theoretic model where demand is a function of the firms' goodwills. In our model, a product recall affects goodwill, and consequently demand. Thus our model investigates optimal advertising decisions while envisioning three significant aspects of a product recall – uncertain recall time, the effect of advertising on goodwill and impact of the recall risk and damage on the optimal policies.

The previous literature, based on mathematical modelling framework, has stressed the importance of envisioning the impending product recall while making decisions (Rubel, 2018; Rubel et al., 2011). We believe that the assumption that firms have perfect information about the hazard rate, might not always be applicable. For example, a firm can be a new entrant in the market. In such cases, the entrant or the incumbent might not predict or consider the crisis possibility of the entrant. If a firm has no recall history and it maintains a high quality or brand value, the firm itself or the competitor can also possibly ignore the possibility of a recall hazard. In this context, we propose the term "hazard myopic" for a firm if it ignores the chances of a recall. In the marketing literature, "managerial myopia" is used in a different sense with respect to advertising. Marketing myopia with respect to advertising refers to the strategy where the focus is promoting a product rather than building the brand or paying attention towards the customers needs (Levitt, 2008; Friedman and Friedman, 1976; Sharma, 2015). The models under the dynamic game framework in the marketing literature have used the term myopia to signify that a player ignores the state evolution of the system, e.g. (Taboubi and Zaccour, 2002; Benchekroun et al., 2009; Zu and Chen, 2017). We consider another form of myopia - overlooking hazard possibility while determining the long-term profit (hazard myopic). In a situation where we have this type of myopia, a viable case of interest is to examine the firms' equilibrium advertising policies and profits. When firms are not "hazard myopic" we call them farsighted in this paper. Thus, farsighted firms envision their respective hazard rates.

By analyzing our model we answer the following research questions:

1. What are the equilibrium advertising policies of the individual players before product recall and after product recall?
2. How are the equilibrium advertisings of the competing firms affected by the intensity of the recall and the hazard rate of the recall?
3. Under the model assumptions, is post recall advertising always higher than the pre-recall advertising?
4. What is the impact of the impending recall on the performance of the competing firms?
5. What is the impact of hazard myopia on advertising and performance of the competing firms?

In this paper, we have developed a brand image based demand function and a dynamic model which captures advertising competition and uncertain recall time. The recall is also "precise" i.e the firms know the exact items to be recalled and can observe the "drop" in goodwill as a result of the damage caused by the recall (Ketchen Jr et al., 2014). In addition, the recall is partial and non-defective products remain in the market after the recall. We have analyzed the advertising decisions and the impact of a recall in two scenarios -
(i) A focal firm and a non-focal firm compete
(ii) Two focal firms compete.
Our results augment or support those of the previous literature (Rubel et al., 2011; Rubel, 2018; Cleeren et al., 2013; Gao et al., 2015). We find that the a firm's profit margin and sensitivity to competition affect its equilibrium advertising policies. Whether a firm should increase or decrease advertising in the post-recall regime depends on what profit margin the firm wants to attain, how sensitive the market is towards the firms' goodwills and, most importantly on the $(\chi, \eta)$ pair, where $\chi$ in the recalling firm's hazard rate and $\eta \in (0,1)$ is the damage to goodwill caused by the recall. Crisis intensity and hazard rate are major determining factors for firm profits. In fact, a low impact low recall hazard rate can positively affect the performance of a firm. High competition can increase profit for a firm. High recall probability or intensity negatively affects a firm's profit. Though we have not considered price as a decision variable, some of our results about the firm

| Authors | Regime of Analysis | Focus | Theory/Approach | Dependent variables | Main effects | Interaction Effects |
|---|---|---|---|---|---|---|
| Cleeren et al. (2008) | Post-crisis | Consumers | Hazard model | Timing of first purchase after the crisis | Advertising, Loyalty, Familiarity, Usage | Loyalty*time |
| Rubel et al. (2011) | Pre-crisis, Post-crisis | Firms, Consumers | Stochastic control problem | Sales of affected brands, competitors and number of recalls | Advertising | Crisis likelihood*Damage<br>Crisis likelihood*Advertising |
| Zhao et al. (2011) | Pre-crisis, Crisis, Post-cisis | Consumers | Consumer learning model | Brand choice of affected brands and non-affected brands | Advertising, Price, Risk Aversion | |
| Liu and Shankar (2015) | Pre-crisis, Crisis, Post-cisis | Firms, Consumers | State-space model with random coefficients | Market share of affected and non-affected brands | | Advertising*publicity<br>Advertising*Severity of Crisis |
| Gao et al. (2015) | Event window of [0, +1] days | Investors | Event study | Future product reliability, injuries and recall frequency | Newness of recalled product, severity of hazard | Pre-recall advertising adjustment*newness of recalled product; Pre-recall advertising adjustment*severity of hazard |
| Our paper | Pre-crisis, Post-crisis | Firms | Differential game | Demand/Sales | Advertising, Profit | **For one focal firm**:<br>a) Crisis likelihood*Damage to goodwill*Advertising;<br>b) Crisis likelihood*Damage to Goodwill* Firm profit;<br>c) Hazard myopia*Damage to Goodwill *Advertising;<br>d) Hazard myopia*Damage*Firm Profit;<br><br>**For Two Focal Firms**:<br>a) Crises likelihoods*Damage to Goodwills*Advertisings;<br>b) Crises likelihoods*Damage to Goodwills* Firm profits; |

Table 0: Contribution of our Paper

performance points towards the importance of pricing before and after a product recall. The findings from the model also highlight that a brand having high initial goodwill suffers from less profit loss by a recall. However, we found that a high-intensity recall can substantially erode firm profit irrespective of its initial goodwill. We summarize our contribution to the existing literature in the following paragraph.

First, we investigated the equilibrium advertising policies and long-term expected profits of the rival firms under a dynamic goodwill based game theoretic duopoly model when product recall can strike at an uncertain time. While the previous studies (Rubel et al., 2011; Cleeren et al., 2008; Gao et al., 2015) considered advertising decisions and the impact of the impending recall on the same, our study articulates the effect of a recall on a focal as well a non-focal firms' advertising strategies and the resulting long-term expected profits.

Second, we have considered another form of myopia - overlooking hazard possibility while determining the long-term profit (hazard myopic). In a situation where we have this type of myopia, a viable case of interest is to examine the firms' equilibrium advertising policies and profits. When firms are not "hazard myopic" we call them farsighted in this paper. Thus, farsighted firms envision their respective hazard rates.

Third, we believe that this paper is the first to have developed the feedback strategies in the case when two focal firms with different crisis likelihoods exist and the duopoly competition continues even after both the crises occur. This is in contrast with the previous studies (Rubel, 2018) and provides us with some useful managerial implications. The details of our contribution with respect to the related papers are given in the Table 0. The table is partly adopted from a literature review in the area of marketing and product harm crisis (Cleeren et al., 2017), but is modified to highlight our contribution.

The rest of the paper is arranged in the following manner. In section 2, a detailed description of the model is given. In this section, we show how uncertain recall time, damaging effect of recall can be incorporated in the model and propose the solution procedure. Section 3 gives a comprehensive analysis of the model and illustrates how the advertising policies differ for a single focal firm, myopic firms or two focal firms. Section 4 supports the analysis done in section 3 by numerical experiments and elucidate the significance of our analytical findings. Finally, in section 5 we conclude with managerial implications of our research and future research directions.

## 2. Model Formulation

In this section, we define the model comprising of the demand functions, the process of recall occurrence, the effect of recall and the manufacturers' profit-maximizing problems in the different scenarios. We consider a market structure involving two manufacturers, competing for goodwill. We will refer the manufacturers as $M_1$ and $M_2$. Consumers are sensitive towards the difference in the goodwill of the manufacturers. While a manufacturer's own goodwill positively affects its demand, the competitor's goodwill negatively affects its demand. The manufacturers try to maintain high brand goodwill by continuous advertising efforts. Product-recall can occur at an uncertain time within the planning horizon, $[0, \infty)$. A recall dampens the goodwill of the focal firm and consequently decreases the firm's demand thereby affecting the firm's profit. Therefore, advertising, a commonly used weapon for maintaining or increasing goodwill, can be a viable strategy to counter the negative effects of the product recall. The main objective of our model is to find the equilibrium advertising in pre-crisis and post-crisis regimes and analyze the effect of recall probability and intensity on the equilibrium strategies and firms' profits. We assume that the players take the advertising decisions simultaneously and the equilibrium strategies are therefore solutions of the Nash game described later.

We consider two different scenarios depending on the number of focal firms. First, we investigate the case when there is one focal firm, i.e., one of the manufacturers is prone to a product recall. Second, we extend our model to the situation where there are two focal firms, i.e., both the manufacturers are prone to recall. We analyze these two cases separately. For each of the above two scenarios, we consider that the firms are farsighted, i.e., they perfectly predict the hazard rate at the beginning of the planning horizon. We further extend the one focal firm model to the case when the firms are "hazard myopic". Myopic behaviour of a firm in the context of this paper refers to the characteristic that the firm does not foresee the recall and makes a decision without accounting for the impending hazard. A firm, when myopic, will become aware of a recall when it is issued. This is because of the widespread media attention on adverse events like a recall. It is plausible that the firms reconsider their advertising decisions at the time when the firm itself or the rival issues a recall. Table 1 summarizes the different scenarios and decisions while Table 2 summarizes the model notations.

| Case | Players | Behaviour | Regime 1 | Regime 2 | Regime 3 |
|---|---|---|---|---|---|
| One Focal firm $M_1$ | $M_1$ | Farsighted | $A_{11}^*$ | $A_{12}^*$ | NA |
| | $M_2$ | | $A_{21}^*$ | $A_{22}^*$ | NA |
| | $M_1$ | Myopic | $A_{11}^*$ | $A_{12}^*$ | $A_{13}^*$ |
| | $M_2$ | | $A_{21}^*$ | $A_{22}^*$ | $A_{23}^*$ |
| Two Focal firms | $M_1$ | Farsighted | $A_{11}^*$ | $A_{12}^*$ | $A_{13}^*$ |
| | $M_2$ | | $A_{21}^*$ | $A_{22}^*$ | $A_{13}^*$ |

Table 1: My caption

| Model Parameter | Description |
|---|---|
| $\theta_i(t)$ | Goodwill of manufacturer $i$ at time $t$ |
| $A_{ij}(t)$ | Advertising effort by the manufacturer $i$ in regime $j$ |
| $A^*_{ij}(t)$ | equilibrium advertising effort of the manufacturer $i$ in regime $j$ |
| $\alpha$ | Initial total market size over which the manufacturers compete |
| $\beta_i$ | Consumer sensitivity of brand image difference |
| $\gamma$ | Marginal market share sensitivity to difference in brand image |
| $\delta_i$ | Absorption of goodwill for manufacturer $i$ |
| $r$ | discounting factor |
| $\rho(t)$ | Market share of manufacturer 1 |
| $\eta_i$ | Damage to goodwill caused by the recall |
| $\chi$ | Hazard rate |
| $D_i(\theta_1, \theta_2, \rho)$ | Demand for manufacturer $i$ in regime $j$ |
| $m_{ij}$ | Unit Profit Margin for manufacturer $i$ in regime $j$ |
| $V_{ij}$ | Value function for manufacturer $i$ in regime $j$ |

Table 2: Model Parameters

### 2.1. The Recall Occurrence:

The planning horizon in our study is $[0, \infty)$. We also assume that during the planning horizon, recall occurs only once by any manufacturer. When we consider only one focal firm, let $t_r$ be the random time of the recall. Consider $\chi \in (0, 1)$ to be the hazard rate. We define the probabilistic switching of the pre-crisis and the post-crisis regime by means of the stochastic process $[R(t) : t \geq 0]$ defined below:

$$\begin{aligned} \lim_{dt \to 0} \frac{P[R(t+dt) = 2 | R(t) = 1]}{dt} &= \chi, \\ \lim_{dt \to 0} \frac{P[R(t+dt) = 1 | R(t) = 2]}{dt} &= 0. \end{aligned} \tag{1}$$

When both the firms are prone to recall, $t_{r1}$ and $t_{r2}$ are the random times of recall. The firms do not know ex ante if $t_{r1} > t_{r2}$ or $t_{r1} < t_{r2}$. We exclude the case of $t_{r1} = t_{r2}$. Each firm $i$ has its own hazard rate $\chi_i$ following a stochastic process of occurrence, $[R_i(t) : t \geq 0]$. The regime switches are given by the following equations.

$$\begin{aligned} \lim_{dt \to 0} \frac{P[R_i(t+dt) = 2 | R_i(t) = 1]}{dt} &= \chi_i, \\ \lim_{dt \to 0} \frac{P[R_i(t+dt) = 1 | R_i(t) = 2]}{dt} &= 0. \end{aligned} \tag{2}$$

Applications of such regime switching and piecewise deterministic games can be found in the closely related studies (Boukas et al., 1990; Haurie and Moresino, 2006; Rubel et al., 2011).

The random timings of a recall essentially split the planning horizon into different decision epochs or regimes. When there is one focal firm, there are two possible decision epochs. When there are two focal firms, there are three possible decision epochs. Figure 1 and Figure 2 depicts the recall occurrence and the regimes for decision making.

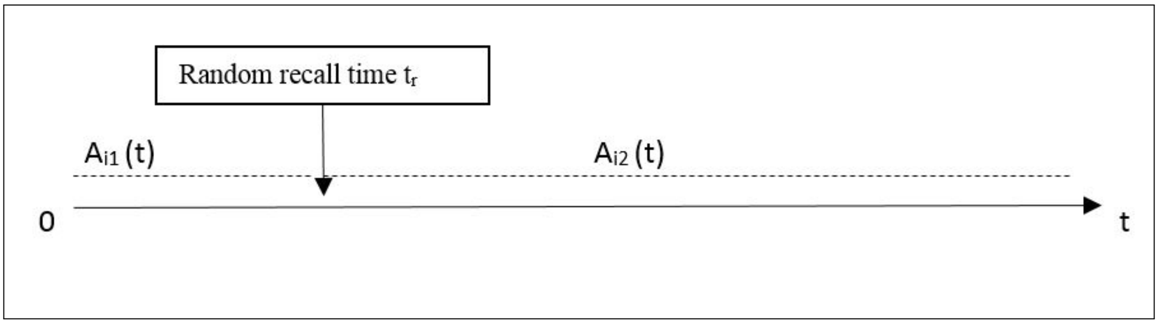



Figure 1: Advertising decisions - one focal firm

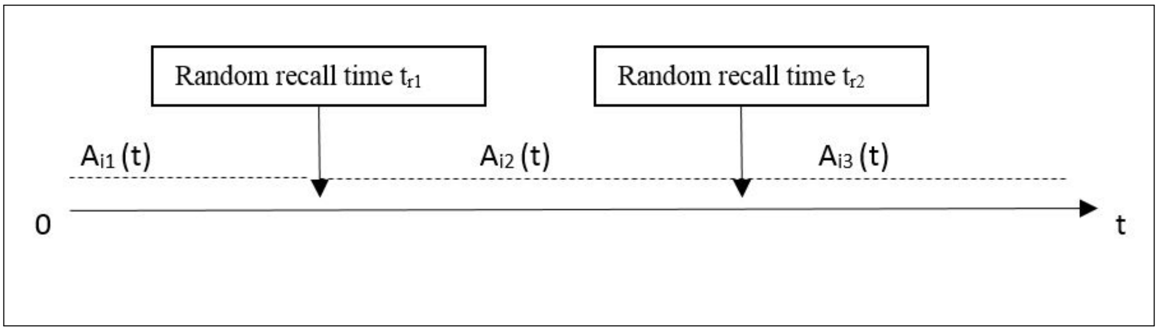



Figure 2: Advertising decisions -two focal firms

## 2.2. Demand Functions

We consider dynamic linear demand functions for the manufacturers $M_1$ and $M_2$. [1]. One underlying assumption is that the manufacturers have no restrictions or capacity issues and are able to meet demand if it is high. Since decisions are taken in pre-crisis and post-crisis regimes, and decisions affect the state trajectory, ideally we need a time index j for the demand functions, state variables and the decision variables. However, to avoid notational complexity and confusion *we drop the time index from the notation of demand and state variables.* We use the indices $i, j$ for the decision variables ($A_{ij}$), profit margins, $m_{ij}$ and value functions $V_{ij}$. If $D_i(\theta_1(t), \theta_2(t), \rho(t))$ is the demand for manufacturer $M_i$ , $i \in \{1,2\}$, the demands for the two manufacturers $M_1$ and $M_2$ are respectively:

$$\begin{aligned} D_1(t) &= \rho(t)\alpha + \beta_1(\theta_1(t) - \theta_2(t)), \\ D_2(t) &= (1 - \rho(t))\alpha + \beta_2(\theta_2(t) - \theta_1(t)). \end{aligned} \tag{3}$$

[1]Henceforth, the terms goodwill and brand image will be used interchangeably

The relatively simple nature of the demand function assures tractability of our model. Morover, some studies in the previous literature adopt a structurally similar price-dependent demand model (Balasubramanian and Bhardwaj, 2004). In the above demand functions, $\theta_i(t)$ is the goodwill of the manufacturer $i$, $\alpha$ is the initial market size , $\beta_i$ is the consumer sensitivity towards difference of the goodwills of $M_i$ and $M_j$ and $\rho(t)$ is the market share of manufacturer 1. In our model analysis, we do not impose any state constraint for the solution to our problem. However, to ensure that our analyses and interpretations are correct, we choose the parametric values such that the market share remains positive and less than 1. Similar solution approaches can be found in some of the previous literature (Prasad and Sethi, 2004). The state variables of our dynamic system are the firms' goodwills, $\theta_i(t)$ and the market share $\rho(t)$. The states evolve according to the following differential equations:

$$\begin{aligned} \dot{\theta}_1(t) &= kA_{1j}(t) - \delta_1\theta_1(t), & \forall t \in [0, \infty), \theta_1(0) = \tilde{\theta}_1, \\ \dot{\theta}_2(t) &= kA_{2j}(t)) - \delta_2\theta_2(t), & \forall t \in [0, \infty), \theta_2(0) = \tilde{\theta}_2, \\ \dot{\rho}(t) &= \gamma(\theta_1(t) - \theta_2(t)), & \rho(0) = \tilde{\rho}. \end{aligned} \tag{4}$$

$A_{ij}$ is the advertising effort of $M_i$ in regime $j$ and $\delta_i$ is the decay parameter (Nerlove and Arrow, 1962). $\gamma$ is the sensitivity of the marginal market share to the difference in the brand image. $\tilde{\theta}_1 \geq 0, \tilde{\theta}_2 \geq 0$ and $0 \leq \tilde{\rho} \leq 1$ are the initial values of the brand images and market shares at the beginning of the planning horizon.

The demand functions (3) and the state variables (4) of our model are based on the following assumptions:

1. The demand for a firm is positively affected by its own goodwill and negatively by the goodwill of its rival
2. A firm's marginal goodwill is positively affected by its own advertising efforts
3. The marginal market share of a firm is positively affected by its own goodwill and negatively by the goodwill of its rival

While our assumption 1 is captured by the demand functions, the assumptions 2 and 3 are reflected in the evolution of the states of our dynamic system, the states being - the brand images $\theta_1(t)$, $\theta_2(t)$ and the market share $\rho(t)$. Assumption 1 and 2 comport with some of the models discussed in the previous literature (Dockner et al., 2000; Karray and Zaccour, 2005; Nair and Narasimhan, 2006) . While (Dockner et al., 2000) proposed a multiplicative dynamic demand model with linear state equations, (Karray and Zaccour, 2005) considers a linear demand function which is affected positively by the brand image and the efforts(investments) a firm makes to maintain its brand image. (Kim and Chung, 1997) showed that brand image is positively related to market share which supports our assumption 3. Using the Lanchester model, one study found that the closed-loop strategies of firms depend on the realized market share (Chintagunta and Vilcassim, 1992). This underscores the importance of introducing the third state variable - market share, $\rho(t)$ in our study. We also investigate the closed loop strategies but, as we will see later, in our case, the advertising strategies will be degenerate stationary Markov strategies. Hence, these stationary strategies will not be influenced by market share, $\rho(t)$, directly. However, the consumer sensitivity towards the marginal market share, $\gamma$ will affect the advertising strategies.

### 2.3. Capturing the effect of recall as a jump state:

In the case of a product recall, there is a damaging effect of the recall on the recalling firm's baseline sales (Van Heerde et al., 2007; Rubel et al., 2011). The damaging effect, however, depends on the firm's existing goodwill and the magnitude of the recall. Product recall causes erosion of brand goodwill (Eilert, 2013; Fenech, 2017). In our model the damage due to product recall is captured by the diminishing goodwill of the focal firm after the recall. If, in general, the manufacturer $M_i$ recalls, We express the jump state by $\theta_{i2}(t_r^+) = (1 - \eta_i)\theta_{i1}(t_r^-))$, where $t_r$ is the time of the recall for $M_i$. Similar applications of jump states have been considered in the related literature (Kamien and Schwartz, 2012). The focal firm's demand function increases with its own goodwill. So a loss in goodwill lowers its demand. The solution procedure of our model is given in section 2.4. In the following section, we illustrate how to incorporate the jump state using state equations (8).

After a product recall advertising effectiveness can reduce(Liu and Shankar, 2015). The absorption parameter of the Nerlove arrow model can be used to capture the ineffectiveness of advertisement and the brand size. The marginal brand image is negatively affected by the absorption factor, $\delta$. We propose that an increase in $\delta$ in the post-crisis regime signifies the advertising ineffectiveness. Therefore, for a firm who issues a recall, if in the second regime the $\delta$ changes to $\delta + \Delta$, where $0 \leq \Delta$, we can say that the advertising effectiveness has decreased. Here $\Delta$ measures the intensity of the advertising effectiveness. Thus the damage to the recalling firm can be twofold - damage by goodwill loss and loss of advertising effectiveness.

### 2.4. The Manufacturers' Decision Problems in presence of one focal firm

#### 2.4.1. Farsighted Firms

Without the loss of generality, we assume that $M_1$ is the focal firm. Both $M_1$ and $M_2$ anticipate the hazard rate, $\chi$ at the beginning of the planning horizon. The manufacturers face their individual profit maximizing problems. We have considered the quadratic costs of advertising efforts. Such a cost function, given by $C(A_{ij}) = \frac{\mu_i}{2}A_{ij}^2$ has been widely used in the literature (e.g. (Karray and Zaccour, 2005)). Since a product-recall splits the decision horizon into two regimes - pre-crisis and post-crisis, in general the decisions made by the firms will differ in the two horizons. The time of recall is stochastic. Therefore the long-term profit is an expected sum of the profits in the post-crisis and pre-crisis regimes. Let $A_{ij}$ be the advertising effort and $\pi_{ij}$ be the profit for manufacturer $M_i$ in regime $j$.

For Manufacturer $i$, the profits in each regime are given by:

$$\begin{aligned} \pi_{i1} = \pi_{i1}(A_{i1}) &= \int_0^{t_r} e^{-rt}[m_{i1}D_{i1}(\theta_1, \theta_2, \rho) - \frac{\mu_1}{2}A_{i1}^2]dt \\ \pi_{i2} = \pi_{i2}(A_{i2}) &= \int_{t_r}^{\infty} e^{-rt}[m_{i2}D_{i2}(\theta_1, \theta_2, \rho) - \frac{\mu_1}{2}A_{i2}^2]dt \end{aligned} \tag{5}$$

Now, $t_r$, the time of recall, is a random time which is not known in advance. This implies that the profits in the two periods are random variables. So the long-term expected profit is given by, $\Pi_i(A_{i1}, A_{i2}) = E[\pi_{i1} + e^{-rt_r}\pi_{i2}]$ where the expectation E[.] is taken with respect to the crisis occurrence process. The discount factor $e^{-rt_r}$ appears because $\pi_{11}$ accrues at $t = 0$ and $\pi_{12}$ at $t = t_r$. Thus, it is required to discount $\pi_{12}$ back to $t = 0$ to add the two long-term profits. The value of the long-term profit will therefore depend on the strategies $(A_{i1}, A_{i2})$ chosen in the two regimes for $i \in \{1,2\}$. The problem above is a random stopping problem. If $f(t)$ and $F(t)$ are the probability density and cumulative density functions of the stochastic occurrence process, then the hazard rate is $h(t) = \chi, F(t) = 1 - e^{-\int_0^t h(s)ds}$. Therefore, $f(t) = \chi e^{-\int_0^t h(s)ds}$. The long-term expected profit for the manufacturer $i$ can be written as:

$$\Pi_i(A_{i1}, A_{i2}) \;=\; E[\int_0^{t_r} e^{-rs}\pi_{i1}ds + e^{-rt_r}\pi_{i2}] \text{ where } i \in \{1,2\}, \tag{6}$$

where the first term under the expectation gives the profit of the pre-crisis period and the second term gives the profit of the post-crisis regime. Therefore, the sum of the two profits gives the long-term profit over the planning horizon. Integrating by parts and making algebraic manipulations (Haurie and Moresino, 2006; Rubel et al., 2011) the above expression can be transformed into the following equation (see appendix):

$$\Pi_i(A_{i1}, A_{i2}, \chi) = \int_0^\infty e^{-(r+\chi)t}\{\pi_{i1} + \chi\pi_{i2}\}dt. \tag{7}$$

The management of the both the firms would like to optimize their long-term profit. Thus, given the demand dynamics and the state evolution, they would like to choose the advertising efforts which would maximize the expected profit over the planning horizon. In order to solve the problems for the manufacturers, we have to start by solving for the value functions of the post-crisis regime first (Rubel et al., 2011; Haurie and Moresino, 2006) . The value functions for the $M_1$ and $M_2$ in the second regime is given by:

$$\begin{aligned}
V_{12}(\theta_1,\theta_2,\rho) &= \underset{A_{12}}{\text{Max}} \int_{t_r}^\infty e^{-rt}[m_{12}D_{12}(t) - \frac{\mu_1}{2}A_{12}^2]dt, && (8)\\
V_{22}(\theta_1,\theta_2,\rho) &= \underset{A_{22}}{\text{Max}} \int_{t_r}^\infty e^{-rt}[m_{22}D_{22}(t) - \frac{\mu_2}{2}A_{22}^2]dt, &&\\
\text{Subject to} & &&\\
\dot{\theta}_{12}(t) &= kA_{12}(t) - \delta_1\theta_1(t), \quad \theta_1(t_r+) = (1-\eta)\theta_1(t_r-), &&\\
\dot{\theta}_{22}(t) &= kA_{22}(t) - \delta_2\theta_2(t), \quad \theta_2(t_r) = \theta_{2t_r}, &&\\
\dot{\rho}(t) &= \gamma(\theta_1(t) - \theta_2(t)), \qquad \rho(t_r) = \rho_{t_r}, &&
\end{aligned}$$

where $\theta_{2t_r}$ and $\rho_{t_r}$ are the goodwill of firm 2 and market share of firm 1 at time $t_r$ respectively. We want to find the feedback strategies for the firms. Therefore, as a standard solution procedure we start by writing the HJB equations (Dockner et al., 2000). Subsequently, the first order conditions on the decision variables will help us in finding the equilibrium strategies from the HJB equations. We note that the value functions $V_{ij}(\theta_1,\theta_2,\rho)$ are concave in $A_{ij}$ since $\frac{\partial^2 V_{ij}}{\partial A_{ij}^2} = -\mu_i < 0$. The HJB equations for the manufacturers 1 and 2 in the second regime are respectively given by:

$$
\begin{aligned}
rV_{12}(\theta_1,\theta_2,\rho) &= \underset{A_{12}}{\text{Max}}[(\rho\alpha + \beta_1(\theta_1 - \theta_2)m_{12} - \frac{\mu_1}{2}A_{12}^2 + \frac{\partial V_{12}}{\partial \theta_1}\dot{\theta}_1(t) + \\
&\quad \frac{\partial V_{12}}{\partial \theta_2}\dot{\theta}_2(t) + \frac{\partial V_{12}}{\partial \rho}\dot{\rho}(t)] \qquad (9)\\
rV_{22}(\theta_1,\theta_2,\rho) &= \underset{A_{22}}{\text{Max}}[((1-\rho)\alpha + \beta_1(\theta_1 - \theta_2))m_{22} - \frac{\mu_1}{2}A_{12}^2 + \frac{\partial V_{22}}{\partial \theta_1}\dot{\theta}_1(t) + \\
&\quad \frac{\partial V_{22}}{\partial \theta_2}\dot{\theta}_2(t) + \frac{\partial V_{22}}{\partial \rho}\dot{\rho}(t)]
\end{aligned}
$$

The problems for the firms in the first regime are thus given below:

$$
\begin{aligned}
V_{11}(\theta_1,\theta_2,\rho) &= \underset{A_{11}}{\text{Max}} \int_0^\infty e^{-(r+\chi)t}[m_{11}D_1(t) - \frac{\mu_i}{2}A_{11}^2 + \chi V_{12}((1-\eta)\theta_1,\theta_2,\rho)]dt \qquad (10)\\
V_{21}(\theta_1,\theta_2,\rho) &= \underset{A_{21}}{\text{Max}} \int_0^\infty e^{-(r+\chi)t}[m_{21}D_2(t) - \frac{\mu_i}{2}A_{21}^2 + \chi V_{22}((1-\eta)\theta_1,\theta_2,\rho)]dt \\
&\text{Subject to} \\
\dot{\theta_1}(t) &= kA_{11}(t) - \delta_1\theta_1(t), \quad \forall t \in [0,\infty), \theta_1(0) = \theta_{10} \\
\dot{\theta_2}(t) &= kA_{21}(t)) - \delta_2\theta_2(t), \quad \forall t \in [0,\infty), \theta_2(0) = \theta_{20} \\
\dot{\rho}(t) &= \gamma(\theta_1(t) - \theta_2(t)), \qquad \rho(0) = \rho_0
\end{aligned}
$$

where $\theta_{10}, \theta_{20}$ are the initial goodwills of firm 1 and firm2 and $\rho_0$ is the market share of firm 1 at time 0.

From equation (9),the pre-crisis HJB equations for the manufacturers 1 and 2 are given by :

$$
\begin{aligned}
(r+\chi)V_{11}(\theta_1,\theta_2,\rho) &= \underset{A_{11}}{\text{Max}}[(\rho\alpha + \beta_1(\theta_1 - \theta_2))m_{11} - \frac{\mu_1}{2}A_{11}^2 + \frac{\partial V_{11}}{\partial \theta_1}\dot{\theta}_1(t) + \\
&\quad \frac{\partial V_{11}}{\partial \theta_2}\dot{\theta}_2(t) + \frac{\partial V_{11}}{\partial \rho}\dot{\rho}(t) + \chi V_{12}((1-\eta)\theta_1,\theta_2,\rho)] \qquad (11)\\
(r+\chi)V_{21}(\theta_1,\theta_2,\rho) &= \underset{A_{21}}{\text{Max}}[((1-\rho)\alpha + \beta_2(\theta_2 - \theta_1))m_{21} - \frac{\mu_2}{2}A_{21}^2 + \frac{\partial V_{21}}{\partial \theta_1}\dot{\theta}_1(t) + \\
&\quad \frac{\partial V_{21}}{\partial \theta_2}\dot{\theta}_2(t) + \frac{\partial V_{21}}{\partial \rho}\dot{\rho}(t) + \chi V_{22}((1-\eta)\theta_1,\theta_2,\rho)]
\end{aligned}
$$

### 2.4.2. Hazard Myopic Firms

The marketing literature has stressed the importance of envisioning the impending product recall while making decisions. However, the assumption that the rival firms would always know and consider each others hazard rate can be questionable. Then some viable questions can be -

- What is the effect of ignoring crisis probability on the decision and performance of the firms?
- Is there a condition under which a myopic firm has a better performance?

Though the firms overlook the hazard probability at the beginning of the planning horizon, it is understandable that they will become aware of a recall when it is announced.i.e. they will notice the change in goodwill as the damage $\eta$ occurs. Hence, the firms can potentially change the equilibrium advertising policies to ensure better profits. Assuming one focal firm, we consider that Both $M_1$ and $M_2$ ignore $\chi$, the hazard rate.

When both $M_1$ and $M_2$ are myopic, the solution procedure starts with solving for the equilibrium decisions and value functions for the first regime and then doing the same for the second regime. With crisis not in sight, the decision problems for the manufacturers in the first regime are:

$$\begin{aligned}
V_{11}(\theta_1,\theta_2,\rho) &= \operatorname*{Max}_{A_{11}} \int_0^\infty e^{-rt}[m_{11}D_1(t) - \frac{\mu_i}{2}A_{11}^2]dt \\
V_{21}(\theta_1,\theta_2,\rho) &= \operatorname*{Max}_{A_{21}} \int_0^\infty e^{-rt}[m_{21}D_2(t) - \frac{\mu_i}{2}A_{21}^2]dt \\
\text{Subject to}& \\
\dot{\theta_1}(t) &= kA_{11}(t) - \delta_1\theta_1(t), \quad \forall t \in [0,\infty), \theta_1(0) = \theta_{10} \\
\dot{\theta_2}(t) &= kA_{21}(t)) - \delta_2\theta_2(t), \quad \forall t \in [0,\infty), \theta_2(0) = \theta_{20} \\
\dot{\rho}(t) &= \gamma(\theta_1(t) - \theta_2(t)), \qquad \rho(0) = \rho_0
\end{aligned} \tag{12}$$

The second regime decision problems are same as (11) with the exception that the margins change to $m_{12}$ and $m_{22}$ respectively for the focal and the non focal firms. The state equations also change as the damage occurs. Thus the second regime problems are given by:

$$\begin{aligned}
V_{12}(\theta_1,\theta_2,\rho) &= \operatorname*{Max}_{A_{12}} \int_{t_r}^\infty e^{-rt}[m_{12}D_1(t) - \frac{\mu_1}{2}A_{12}^2]dt \\
V_{22}(\theta_1,\theta_2,\rho) &= \operatorname*{Max}_{A_{22}} \int_{t_r}^\infty e^{-rt}[m_{22}D_2(t) - \frac{\mu_2}{2}A_{22}^2]dt \\
\text{Subject to}& \\
\dot{\theta_1}(t) &= kA_{11}(t) - \delta_1\theta_1(t), \quad \forall t \in (t_r,\infty), \theta_1(t_r^+) = (1-\eta)\theta_1(t_r^-) \\
\dot{\theta_2}(t) &= kA_{21}(t)) - \delta_2\theta_2(t), \quad \forall t \in (t_r,\infty), \theta_2(t_r) = \theta_{t_r} \\
\dot{\rho}(t) &= \gamma(\theta_1(t) - \theta_2(t)), \qquad \rho(t_r) = \rho_{t_r}
\end{aligned} \tag{13}$$

Proceeding similarly as in the far sighted firms' cases, we can similarly write derive the HJB equations for the myopic firms and solve for the equilibrium feedback strategies.

### 2.5. The Manufacturers' Decision Problems in presence of two focal firms

In this case, the market consists of two focal manufacturers $M_i$. Both manufacturers are prone to recall and each has a specific hazard rate of $\chi_i$ where $i \in \{1,2\}$. We are assuming that after the recall both the manufacturers still sell their products in the market or in other words the recalls are partial. For example, in Feb 2018, Fujitsu Canada, recalled only certain computers with defective batteries. Other units or models of the same (E-series) were still available in the market after the recall.

When the firms are farsighted, both the hazard rates are common knowledge. However, at the beginning of the horizon neither firm knows who will recall first. Consequently, the value function of firm $i$ at the beginning of the horizon is given by:

$$\begin{aligned}
V_{i1}(\theta_1,\theta_2,\rho_i) = \operatorname*{Max}_{A_{i1}} \mathop{\mathbb{E}}_{R_i,R_j} \Big( \int_0^{\min\{t_i,t_j\}} e^{-rt}\Pi_{i1}(t)dt + e^{-rt_i}V_{i2}(\theta_i,\theta_j,\rho_i).\boldsymbol{\Phi[t_i < t_j]} \\
+e^{-rt_j}\hat{V}_{i2}(\theta_i,\theta_j,\rho_i).\boldsymbol{\Phi[t_j < t_i]}\Big)
\end{aligned} \tag{14}$$

where the operators $\boldsymbol{\Phi[t_i < t_j]}$ and $\boldsymbol{\Phi[t_j < t_i]}$ are defined in the following manner:

$$\begin{aligned}
\boldsymbol{\Phi[t_i < t_j]} &= 1 \quad \text{if} \quad t_i < t_j \\
&= 0 \quad \text{otherwise} \\
\boldsymbol{\Phi[t_j < t_i]} &= 1 \quad \text{if} \quad t_j < t_i \\
&= 0 \quad \text{otherwise}
\end{aligned} \tag{15}$$

and the second regime value functions $V_{i2}$ are given by

$$\begin{aligned}
V_{i2}(\theta_i, \theta_j, \rho_i) &= \operatorname*{Max}_{A_{i2}} \int_0^\infty e^{-(r+\chi_j)t}\{\Pi_{i2}(t)dt + \chi_j V_{i3}(\theta_i, (1-\eta_j)\theta_j, \rho_i)\} \\
\hat{V}_{i2}(\theta_i, \theta_j, \rho_i) &= \operatorname*{Max}_{A_{i2}} \int_0^\infty e^{-(r+\chi_i)t}\{\Pi_{i2}(t)dt + \chi_i V_{i3}((1-\eta_i)\theta_i, \theta_j, \rho_i)\}
\end{aligned} \tag{16}$$

Here $\Pi_{i1}(t)$ is the profit of the manufacturer $i$ st the pre recall regime. Similar formulation of value function can be found in (Rubel, 2018) where one of the firms becomes a monopolist after the recall as products are taken off the market. We have relaxed this assumption in our model because recalls can be partial as well. Hence both the firms continue to compete after recall. In other words, the market remains a duopoly market in all the three decision regimes - the pre-recall regime, the regime between the first recall and second recall and the post-recall regime for both the firms. Hence, the horizon now has three decision epochs - *the pre-recall epoch*, *the epoch between the recalls made by $M_1$ and $M_2$* and *the post-recall epoch.*

The derivation of the value function for the firm $i$ in regime 1 are based on the premises that the occurrences of the two recalls are independent of one another. The probability of the random variable $\min\{t_i, t_j\}$ can be defined as - $\Pr(\min\{t_i, t_j\} > t)$=$\Pr\{(t_i > t) \cap (t_j > t)\}$=$\Pr(t_i > t).\Pr(t_j > t)$=$e^{-t(\chi_i+\chi_j)}$. Thus proceeding in a similar manner as in equations (10) and making some algebraic manipulations, we get:

$$\begin{aligned}
V_{i1}(\theta_i, \theta_j, \rho_i) = \operatorname*{Max}_{A_{i1}} \int_0^\infty e^{-(r+\chi_i+\chi_j)t}\{&\Pi_{i1}(t)dt + \chi_i V_{i2}(\theta_i, \theta_j, \rho_i) \\
&+ \chi_j \hat{V}_{i2}(\theta_i, \theta_j, \rho_i)\}
\end{aligned} \tag{17}$$

Similarly, for the $j^{th}$ firm, the value function in the first regime is given by:

$$\begin{aligned}
V_{j1}(\theta_i, \theta_j, \rho_i) = \operatorname*{Max}_{A_{j1}} \int_0^\infty e^{-(r+\chi_i+\chi_j)t}\{&\Pi_{j1}(t)dt + \chi_i V_{j2}(\theta_i, \theta_j, \rho_i) \\
&+ \chi_j \hat{V}_{j2}(\theta_i, \theta_j, \rho_i)\}
\end{aligned} \tag{18}$$

The details of the derivation of the first regime value function is given in the appendix.

**Remark:** The recall damages are still captured by the jump states in this case when two focal firms compete. However, it is only after the first recall , i.e., at $t_{r1}$ that we know who has recalled first. If $M_i$ recalls first at $t_{r1}$ and $M_j$ recalls second at $t_{r2}$, then

$$\begin{aligned}
\theta_i(t_{r1}^+) &= (1-\eta_i)\theta_i(t_{r1}^-), \\
\theta_j(t_{r2}^+) &= (1-\eta_j)\theta_j(t_{r2}^-)
\end{aligned}$$

capture the jump states and the impacts of the recalls.

## 3. Analytical Results and Discussion

First, we want to reiterate that what our goals are and how the model elements are articulated to help us reach our goals. Our goals are to find the equilibrium advertising of the manufacturers in the pre-crisis and post-crisis regimes in the two cases - one focal firm issuing a recall, two focal firms susceptible to recall. First, we solve the manufacturers' problems to find equilibrium advertising strategies. We start with finding the value function for the post-crisis regime using HJB equations (8). Then using equations (10) we find the equilibrium advertising for the first regime. Proofs of all the propositions, unless trivial, are given in the appendix.

### 3.1. Equilibrium Advertising and Profits - One Focal Firm

We investigate the scenario when one of the competing firms has a probability of recall and both the firms make their decisions taking into account the hazard rate of the focal firm. In other words, the hazard rate here is common knowledge for both the players. $A^*_{ij}$ is the equilibrium advertising effort of manufacturer $i$ in regime $j$, where $i, j \in \{1, 2\}$.

#### 3.1.1. Decisions of the Farsighted Firms

**Proposition 1:(a)** *The post-crisis equilibrium advertising efforts of the manufacturers $M_1$ and $M_2$ are respectively given by*:

$$
\begin{aligned}
A^*_{12} &= \frac{km_{12}(\beta_1 + \gamma\alpha/r)}{\mu_1(r + \delta_1)}, \\
A^*_{22} &= \frac{km_{22}(\beta_2 + \gamma\alpha/r)}{\mu_2(r + \delta_2)}.
\end{aligned} \tag{19}
$$

**(b)** *For the pre-crisis advertising efforts of the manufacturers $M_1$ and $M_2$ are respectively given by*:

$$
\begin{aligned}
A^*_{11} &= \frac{k}{\mu_1(r + \chi + \delta_1)}[(\beta_1 + \frac{\gamma\alpha}{r + \chi})m_{11} + (\frac{\gamma\alpha}{r(r + \chi)} + \frac{1 - \eta}{r + \delta_1}(\beta_1 + \frac{\gamma\alpha}{r}))\chi m_{12}], \\
A^*_{21} &= \frac{k}{\mu_2(r + \chi + \delta_2)}[(\beta_2 + \frac{\gamma\alpha}{(r + \chi)})m_{21} + \frac{\gamma\alpha\chi}{(r + \chi)r}m_{22}].
\end{aligned} \tag{20}
$$

The above are the feedback advertising strategies of the two manufacturers in the two regimes. The strategies, in this case, are stationary and hence Markov perfect. In the absence of a crisis, i.e when the hazard rate, $\chi = 0$ and hence the damage, $\eta = 0$, the strategies of both the regimes are identical. Once a recall has occurred, the firm $M_1$ does not anticipate any more recall during the planning horizon. This is a primary assumption of our model. Hence, the equilibrium advertisements are free from the hazard rate in the second regime. The equilibrium advertising efforts in the pre-crisis regime incorporate the hazard rate $\chi$ . This emphasizes the foresightedness of the management of both the focal and non-focal firms as they take the probability of recall into account when they make their advertising decisions even before the recall occurs. This is consistent with the previous findings that risk begets impatience. (Rubel et al., 2011).

**<u>Remark</u>**: *We find that the pre-crisis advertising decisions incorporate the profit margins of the post-crisis regime. While we have assumed perfect information for the model parameters, in reality, knowing the second period margin in advance might not be feasible. None the less, our findings underscore the importance of an advance estimation of expected damage $\eta$ and the profit margin in case a recall occurs. If a firm can estimate these parameters with accuracy, it will remain in a better position with respect to the profit.*

In the following lemmas, we give some properties of the equilibrium advertising policies with respect to the parameters. Then we perform a comparative analysis of the equilibrium advertising expressions and infer on its variance in the pre and post-crisis regimes.

**Lemma 1:** *The equilibrium advertisements efforts of both the firms in the second regime is:*

*(a) increasing with the market size $\alpha$,*

*(b) increasing with respective margins of the firms $m_{12}$ or $m_{22}$,*

*(c) increasing with the sensitivity of demand towards brand difference $\beta_i$,*

*(d) increasing with the sensitivity of marginal market share towards brand difference $\gamma$,*

*(e) decreasing with the absorption $\delta_i$ and discounting factor $r$.*

Lemma 1 has some significant implications. Higher margins result in higher equilibrium advertising in the second regime for both the manufacturers. If a manufacturer wants a higher margin, it can either reduce manufacturing costs or increase the prices. We argue that the $M_1$ produces more thereby benefiting from the economies of scale and lowering the cost. In the post-crisis regime, this production cost savings can help the manufacturer to nullify the effects of the profit loss by the recall. Moreover, the resulting excess advertising will help him in gaining more market and positively impact his own revenue. The argument that the $M_1$ will increase the price after the recall is quite flimsy on the grounds of its recent goodwill loss. $M_2$ can take advantage of the situation by increasing its price (thereby increasing margin, advertisement and hence demand) or produce more at a lower cost as it expects to sale more in the wake of its rival's crisis.

For a firm with higher brand equity, in general, it can be assumed that brand absorption $\delta$ will be lesser than its rival with lower brand equity. We found that the advertising efforts of the firms are decreasing with $\delta_i$.This is apparently contradictory as brand equity can be built with more advertising. However, $\delta$ is only one factor that reflects brand equity and is not a measure of brand equity. Because of the structure of the game model, the contradiction arrives.

Now let us suppose the two firms in question are symmetric in all parameters with the exception of $\delta_1$ and $\delta_2$.This gives rise to two cases - $\delta_1 > \delta_2$ or $\delta_1 < \delta_2$. If $\delta_1 > \delta_2$, the recalling firm knows that the rival's marginal brand image is subject to heavier erosion than her own. Therefore, she tries to gain her lost sales by advertising more. On the other hand, if the rival firm has low absorption, $\delta$, it behaves opportunistically by advertising significantly more than the recalling firm to capitalize on the situation.

**Lemma 2:(a)** *In the pre-crisis regime, for the focal firm $M_1$, the equilibrium advertising is decreasing with the hazard rate:*

- *always if* $m_{11} = m_{12}$
- *always if* $m_{11} > m_{12}$ *and* $\gamma\alpha < \beta_1\delta_1$

**(b)** *In the pre-crisis regime, for the non-focal firm $M_2$, the equilibrium advertising is decreasing with the hazard rate:*

- *always if* $\chi > \sqrt{r(r+\delta_2)}$ *(weaker condition)*
- *if* $\chi > \Phi$ *where expression of* $\Phi$ *is given in the Appendix*

From the above lemma, for a focal firm, the pre-crisis advertising decreases with the likelihood of crisis if the margin is same in the two periods. This explains that if a high recall probability is anticipated by the recalling firm, the firm may not put a lot of efforts in advertising as high advertising may result is high sales and consequently high recall costs. When chances of recall for the rival is less, $M_2$ puts more advertising efforts to capture a larger share of the market from the beginning of the product life in the market. On the other hand, high hazard rate gives $M_2$ complacency and he expects to gain advantage from the crisis opportunity.

**Lemma 3:** *The equilibrium advertising effort for the focal firm in pre-crisis regime is decreasing with the damaging effect, $\eta$.*

Lemma 3 is consistent with the previous literature, mainly the model of (Rubel et al., 2011) based on (Sethi, 1983). The damage effect $\eta$ cannot be estimated ex-ante unless the recall root causes and affected units are known. A high anticipated damage in goodwill means lesser profit in the post-crisis period. Thus the significance of Lemma 3 is that, if a high damage rate is anticipated, the management might want to cut off advertisement costs in regime 1 in order to advertise more in the post-crisis regime thereby maximizing goodwill and sales. In our model, the damaging effect does not directly impact demand. But the damage rate accounts for the jump in the state variable $\theta_1$ which is the brand image of manufacturer 1. This, in turn, affects the demand negatively. Therefore, if sales or demand is affected by the damage, either directly or indirectly, and the management anticipates the recall, it is likely that a high damage expectation will reduce advertisement spending in the first regime.

#### 3.1.1.1. Is the post-crisis advertising higher than the pre-crisis advertising?

The previous literature found empirical evidence that ad spending increases after the recall. In our study, we find the conditions under which the post-crisis advertising efforts are more than the pre-crisis advertising. The mathematical structures of the advertising efforts that we derived, equations (19) and (20), reflect that there is no direct answer to the question - should advertising before recall be more than that after the recall i.e. is

$A^*_{11} < (>) A^*_{12}$ or $A^*_{21} < (>) A^*_{22}$? The obvious answer is that it depends on the parameter values of our model, most important of which are the $(\chi, \eta)$ pair.

**Proposition 2:(a)** *For the focal firm($M_1$),the post-crisis advertising effort is more than the pre-crisis advertising effort if,*

$$\frac{(\beta_1 + \frac{\gamma\alpha}{r})}{(\beta_1 + \frac{\gamma\alpha}{r+\chi})} \frac{(r + \eta\chi + \delta_1)}{(r + \delta_1)} - \frac{\gamma\alpha\chi}{(\beta_1(r+\chi) + \gamma\alpha)r} > \frac{m_{11}}{m_{12}}.$$

**(b)** *For the non-focal firm ($M_2$), the post-crisis advertising effort is more than the pre-crisis advertising effort if,*

$$\frac{(\beta_2 + \frac{\gamma\alpha}{r})(r + \chi + \delta_2)}{(\beta_2 + \frac{\gamma\alpha}{r+\chi})(r + \delta_2)} - \frac{\gamma\alpha\chi}{r\Big(\beta_2(r + \chi) + \gamma\alpha\Big)} > \frac{m_{21}}{m_{22}}.$$

*If $m_{21} \le m_{22}$, post-crisis advertising of the non focal firm is always greater than pre-crisis advertising.*

The above Proposition 2 shows that the focal firm's equilibrium advertising will be more or less in the post-crisis period depending on the crisis likelihood $\chi$. However, the threshold for this $\chi$ also depends on the margins. Assuming that the focal firm's second regime's margin is affected by the recall damage $\eta$, the advertising efforts in the post and pre-crisis period depends on $\chi$ (directly) and $\eta$ (indirectly). More about this is discussed in Section 4 where we find the threshold of $\chi$ for given parameter values and the damage $\eta$.

The previous literature (Rubel et al., 2011) showed that in a competitive environment non-focal firms should increase post-crisis ad spending in order to exploit the situation. However, our analysis suggests that equilibrium advertising for the non-focal firm should be definitely more in the second regime if, the firm's Second regime's margin is higher or equal to the margin in the first regime. Otherwise, the crisis likelihood will determine the level of advertising efforts.

From a strategic point of view, the profit margins can be influenced by pricing decisions. However, pricing strategies are not in the scope of our paper, albeit our analysis accentuates the importance of dual decision making with respect to pricing and advertising during a recall as suggested by some of the existing literature (Cleeren, 2015).

#### 3.1.1.2. The effect of recall on the expected profits of $M_1$and $M_2$

The value functions were explicitly derived by the method of comparison of coefficients. These are the instantaneous profits of the firms under consideration. It is of interest to have a look into the structure of the value functions as we can possibly find out the influence of the different parameters on the value functions. This in turn will give us an estimate of how the long-term profit can be influenced by the parameters.

**Proposition 3:** **(a)** *The value function for manufacturer $M_1$ in regime 2 is given by: $V_{12} = a_{12}\theta_1(t) + b_{12}\theta_2(t) + c_{12}\rho(t) + d_{12}$ where $a_{12}, b_{12}, c_{12}$ and $d_{12}$ are given by:*

$$\begin{aligned} a_{12} &= \frac{m_{12}}{(r + \delta_1)}(\beta_1 + \frac{\gamma\alpha}{r}), \\ b_{12} &= \frac{-m_{12}}{(r + \delta_2)}(\beta_1 + \frac{\gamma\alpha}{r}), \\ c_{12} &= \frac{m_{12}\alpha}{r}, \\ d_{12} &= \frac{k^2 m_{12}(\beta_1 + \frac{\gamma\alpha}{r})}{r}[(\frac{m_{12}(\beta_1 + \frac{\gamma\alpha}{r})}{2\mu_1(r + \delta_1)^2} - \frac{m_{22}(\beta_2 + \frac{\gamma\alpha}{r})}{\mu_2(r + \delta_2)^2})]. \end{aligned} \tag{21}$$

**(b))** *The value function for manufacturer* $M_1$ *in regime 1 is given by* $V_{11} = a_{11}\theta_1(t) + b_{11}\theta_2(t) + c_{11}\rho(t) + d_{11}$ *where* $a_{11}, b_{11}, c_{11}$ *and* $d_{11}$ *are defined as:*

$$\begin{aligned} a_{11} &= \frac{1}{(r+\chi+\delta_1)}[(\beta_1 + \frac{\gamma\alpha}{r+\chi})m_{11} + (\frac{\gamma\alpha}{r(r+\chi)} + \frac{1-\eta}{r+\delta_1}(\beta_1 + \frac{\gamma\alpha}{r}))\chi m_{12}], \qquad (22)\\ b_{11} &= \frac{-1}{(r+\chi+\delta_2)}[\beta_1 m_{11} + c_{11}\gamma + \frac{\chi m_{12}}{(r+\delta_2)}(\beta_1 + \frac{\gamma\alpha}{r})],\\ c_{11} &= \frac{\alpha}{(r+\chi)}[m_{11} + \frac{\chi m_{12}}{r}],\\ d_{11} &= \frac{k^2 a_{11}}{r+\chi}[\frac{a_{11}}{2\mu_1} + \frac{b_{21}}{\mu_2}]. \end{aligned}$$

**Proposition 4:(a)** *The value function for manufacturer* $M_2$ *in regime 2 is given by:* $V_{22} = a_{22}\theta_1(t) + b_{22}\theta_2(t) + c_{22}\rho(t) + d_{22}$ *where* $a_{22}, b_{22}, c_{22}$ *and* $d_{22}$ *are given by:*

$$\begin{aligned} a_{22} &= \frac{-m_{22}}{(r+\delta_1)}(\beta_2 + \frac{\gamma\alpha}{r}), \qquad (23)\\ b_{22} &= \frac{m_{22}}{(r+\delta_2)}(\beta_2 + \frac{\gamma\alpha}{r}),\\ c_{22} &= \frac{-m_{22}\alpha}{r},\\ d_{22} &= \frac{m_{22}}{r}[\alpha + k^2(\beta_2 + \frac{\gamma\alpha}{r})(\frac{m_{22}(\beta_2 + \frac{\gamma\alpha}{r})}{2\mu_2(r+\delta_2)^2} - \frac{m_{12}(\beta_1 + \frac{\gamma\alpha}{r})}{\mu_1(r+\delta_1)^2})]. \end{aligned}$$

**(b)** *The value function for manufacturer 2 in regime 1 is given by* $V_{21} = a_{21}\theta_1(t) + b_{21}\theta_2(t) + c_{21}\rho(t) + d_{21}$ *where* $a_{21}, b_{21}, c_{21}$ *and* $d_{21}$ *are given by:*

$$\begin{aligned} a_{21} &= \frac{-1}{(r+\chi+\delta_1)}[(\beta_2 + \frac{\gamma\alpha}{r+\chi})m_{21} + (\frac{\gamma\alpha}{r(r+\chi)} + \frac{1-\eta}{r+\delta_1}(\beta_2 + \frac{\gamma\alpha}{r}))\chi m_{22}], \qquad (24)\\ b_{21} &= \frac{1}{(r+\chi+\delta_2)}[(\beta_2 + \frac{\gamma\alpha}{r+\chi})m_{21} + \frac{\gamma\alpha\chi}{r(r+\chi)}m_{22}],\\ c_{21} &= \frac{-\alpha}{(r+\chi)}[m_{21} + \frac{\chi}{r}m_{22}],\\ d_{21} &= \frac{1}{(r+\chi)}(\alpha m_{21} + \frac{k^2 b_{21}^2}{2\mu_2^2} + a_{21}a_{11}\frac{k^2}{\mu_1} + \chi d_{22}). \end{aligned}$$

Table 3 summarizes the relationship amongst the state variables and the Value functions of the two firms in the two regimes.

| | $\theta_1$ | $\theta_2$ | $\rho$ |
|---|---|---|---|
| $V_{11}$ | ↑ | ↓ | ↑ |
| $V_{12}$ | ↑ | ↓ | ↑ |
| $V_{21}$ | ↓ | ↑ | ↓ |
| $V_{22}$ | ↓ | ↑ | ↓ |

Table 3: My caption

From Table 3, in both the pre-crisis and post-crisis regimes, the value function of $M_1$ is increasing with $\theta_1$, decreasing with $\theta_2$ and increasing with $\rho$. The value function of

$M_2$ is decreasing with $\theta_1$(brand image of the rival), increasing in $\theta_2$ (own goodwill) and deceasing in $\rho$(market share of the rival). This is not surprising because a high brand image or market share for a firm would mean better profits and the higher brand image of the competitor would also affect the profit of the firm. This supports that the each firm wants to maintain a high level of goodwill and thereby invest in advertising which in turn affects the goodwill.

### 3.1.2. Decisions of Hazard Myopic Firms

The hazard myopia leads to policies where the firms do not anticipate the hazard rate $\chi$ or damage $\eta$. As a result, unlike the far sighted case, these factors do not appear in the mathematical expressions of the pre crisis policies. However, the damage $\eta$ appears in the expression of the value functions of the firms. This is because the firms observe the damage caused after the recall. Thus myopia might lead to identical policies in the two regimes but affects the firms' performances as the firms observe the goodwill erosion.

**Proposition 5:** *In the presence of one focal firm($M_1$), when both the firms are hazard myopic, the equilibrium advertising policies are given by:*

$A^*_{1j} = \frac{km_j}{\mu_1(r+\delta_1)}(\beta_1 + \frac{\gamma\alpha}{r}),$
$A^*_{2j} = \frac{kn_j}{\mu_2(r+\delta_2)}(\beta_2 + \frac{\gamma\alpha}{r}),$ for $j \in \{1,2\}$.

Since we considered the firms to be myopic, the hazard rate $\chi$, does not affect the value function of the firms (Proposition 5). However, to understand the effect of $\eta$, on firm's profits, we need to elaborate the procedure of the calculation of value functions for the firms under this scenario.

The procedure to find the value function (expected long-term profit) of the myopic firm is different from case when the firms are far sighted. The procedure is given below:

1. Using equations (11) find the equilibrium policies of the focal firm.
2. Using the equilibrium policy, evaluate
$V_{11}(\theta_1,\theta_2,\rho) = \int_0^{t_r} e^{-rt}[m_{12}D_1(t) - \frac{\mu_1}{2}{A^*_{11}}^2]dt + V_{12}(\theta_1,\theta_2,\rho)$ where
$V_{12}(\theta_1,\theta_2,\rho) = a_{12}(1-\eta)\theta_1(t_r) + b_{12}\theta_2(t_r) + c_{12}\rho(t_r) + d_{12}$

Similarly, the non focal firm's value function can also be evaluated. A myopic firm does not anticipate the recall occurrence or the damage intensity. Thus his problem is deterministic and the profit will change for different values of $t_r$. In the numerical experiments section we compare the myopic and non myopic value functions and discuss more about this.

## 3.2. Equilibrium advertising in presence of two focal firms

When two firms anticipate recall, there are three decision epochs - pre crisis epoch(1), the epoch between two recalls(2), post crisis epoch(3). It is not known in advance who will recall first. The two recalls are characterized by two different stochastic processes with hazard rates $\chi_1$ and $\chi_2$ for $M_1$ and $M_2$. We first find the equilibrium advertising decisions in the three regimes. While the decisions at the beginning of epoch 1 and epoch 3 are unambiguously determined, the decision at the beginning of the second epoch depends on who recalls first.

**Proposition 6:** *The equilibrium advertising of the $i^{th}$ firm in the three epochs are given by:*

$$\begin{aligned}
A^*_{i1} &= \frac{k[(\beta_i m_{i1} + \chi_j \hat{a}_{i2} + \chi_i a_{i2} + \gamma c_{i1}]}{\mu_i(r + \chi_i + \chi_j + \delta_i)}, \qquad (25)\\
A^*_{i2} &= \frac{k}{\mu_i(r + \chi_j + \delta_i)}[\beta_i m_{i2} + \chi_j a_{i3} + \gamma c_{i2}] \text{ if manufacturer } i \text{ recalls first },\\
\hat{A}^*_{i2} &= \frac{k}{\mu_i(r + \chi_i + \delta_i)}[\beta_i \hat{m}_{i2} + \chi_i(1 - \eta_i)a_{i3} + \gamma \hat{c}_{i2}] \text{ if manufacturer } i \text{ recalls second },\\
A^*_{i3} &= \frac{k m_{i3}(\beta_i + \gamma\alpha/r)}{\mu_i(r + \delta_i)}.
\end{aligned}$$

The $a_{ij}$, $c_{ij}$, $\hat{a}_{ij}$ and $\hat{c}_{ij}$ are the coefficients of the state variables of the value functions in regimes j. If $M_i$ recalls first, the second regime value function is given by $V(\theta_i, \theta_j, \rho_i) = a_{i2}\theta_i + b_{i2}\theta_j + c_{i2}\rho_i + d_{i2}$ . If $M_i$ recalls second,the second regime value function is given by $V(\theta_i, \theta_j, \rho_i) = \hat{a}_{i2}\theta_i + \hat{b}_{i2}\theta_j + \hat{c}_{i2}\rho_i + \hat{d}_{i2}$. The expressions of these coefficients are given in the appendix. We also denote the second regime margins differently depending on who recalls first. $M_i$ recalls first, the margin is denoted by $m_{i2}$ , otherwise it is denoted by $\hat{m}_{i2}$. The logic behind using different notations for margin is that, $m_{i2}$ and $\hat{m}_{i2}$ are not necessarily same. For example, the profit margin may drop after a recall, and remain same when the rival recalls. In that case, $m_{i2} = m_{i1} > \hat{m}_{i2}$.

The equilibrium decisions are complex expressions of the model parameters. It is clear that for the $i^th$ firm the first regime advertising is decreasing with $\eta_i$. The second regime advertising is decreasing with $\eta_i$ if the $i^th$ firm recalls second. The variance of the advertising with respect to $\chi_i$ and $\chi_j$ depends on certain conditions of our parameters. We present one important proposition.

**Proposition 7 (a)**:*The first regime equilibrium advertising efforts $A^*_{i1}$ of $M_i$ is decreasing with the damage $\eta_i$.*
**(b)** *When $M_i$ recalls first, the second regime advertising $A^*_{i2}$ of $M_i$ is increasing in $\chi_j$ (the competitor's crisis likelihood) if $m_{i3} > m_{i2}$, i.e. the third period unit profit margin of $M_i$ is greater than its second period unit profit margin. Otherwise, $A^*_{i2}$ is decreasing in $\chi_j$.*
**(c)** *Is $M_i$ recalls second, it's second regime equilibrium advertising efforts is decreasing with the damage $\eta_i$.*
Due to the presence of many parameters and the multiplicative association of the parameters, it is analytically not tractable to use the first order derivative of $A^*_{i1}$ with respect to find
$A^*_{i1}$ *increases with* $\chi_i$ *if* $\chi_i$ *is greater than a threshold value which depends on* $\chi_j$.
The above result shows that whether pre-crisis advertising should increase with a firm's hazard rate depends on whether the competitor's hazard rate is sufficiently high or not. Of course, if the right-hand side of the above expressions in proposition 7 is negative, then the first regime advertising efforts are always increasing with $\chi_i$ or $\chi_j$.

**Proposition 8**:*The long term profit of the firm $M_i$ can be determined from the value function given by* $V_{i1}(\theta_i(t), \theta_j(t), \rho_i(t)) = a_{i1}\theta_i(t) + b_{i1}\theta_j(t) + c_{i1}\rho_i(t) + d_{i1}$, at $t = 0$, *where*

$$\begin{aligned}
a_{i1} &= \frac{\beta_i m_{i1} + \chi_i a_{i2} + \gamma c_{i1} + \chi_j \hat{a}_{i2})}{(r + \chi_i + \chi_j + \delta_i)}, \\
b_{i1} &= \frac{(-\beta_i m_{i1} + \chi_i b_{i2} - \gamma c_{i1} + \chi_j \hat{b}_{i2})}{(r + \chi_i + \chi_j + \delta_j)}, \\
c_{i1} &= \frac{(\alpha m_{i1} + \chi_i c_{i2} + \chi_j \hat{c}_{i2})}{(r + \chi_i + \chi_j)}, \\
d_{i1} &= \frac{(-\mu_i {A^*_{i1}}^2/2 + k a_{i1} A^*_{i1} + k b_{i1} A^*_{j1} + \chi_i d_{i2} + \chi_j \hat{d_{i2}})}{(r + \chi_i + \chi_j)},
\end{aligned}$$

*and the initial values* $\theta_i(0), \theta_j(0)$ *and* $\rho_i(0)$ *are known.*

## 4. Numerical Experiments

We have obtained the equilibrium strategies and value functions in closed forms for all scenarios under consideration. We found some straight forward relationships among the equilibrium advertisement efforts, the value functions and the system parameters. However, our system has many parameters and we present some numerical experiments to emphasize the efficiencies of our analysis. The numerical results help us draw some insights about the influence of our model parameters on firm performance. The model parameters are -

$$\alpha, \beta_1, \beta_2, \gamma, k, \chi, \mu_1, \mu_2, \eta, \delta_1, \delta_2, r, m_{11}, m_{21}, m_{12}, m_{22}$$

We study the impact of hazard probability and damage intensity on

- State Variables $\Big(\theta_i(t),\ \rho(t)\Big)$
- The equilibrium advertising $(A^*_{ij}(t))$
- Firms' profits $\Big(V_{ij}(0)\Big)$

We also show how the recall impacts the demand of the firm. Since our model is goodwill based, we investigate the effect of recall on a firm with high initial goodwill as compared to a firm with lower initial goodwill. We have chosen the parameter values in such a manner that equilibrium advertising efforts, profits, the goodwill, the margins, all remain within a small range of values and this helps us in scaling our figures properly. The parameter values also ensure the constraint that $0 \leq \rho(t) \leq 1$, $\theta_1(t) \geq 0$ and $\theta_2(t) \geq 0$. Some parameters are fixed for all the models. These are

$$\begin{aligned}
\alpha = 1, \gamma = .03, k = 1, \\
\mu_1 = \mu_2 = 250, \\
r = .07, \delta_1 = .03, \delta_2 = .03, \\
\beta_1 = \beta_2 = .5
\end{aligned} \tag{26}$$

In our analyses, we realized that the damage intensity $\eta$ and hazard rate $\chi$ have significant impacts on the firms' decisions, goodwills, market shares and profits. We, therefore, numerically study the effect of these two parameters on advertising, state variables and the profits. We classify the recalls based on the $(\chi, \eta)$ pair as -

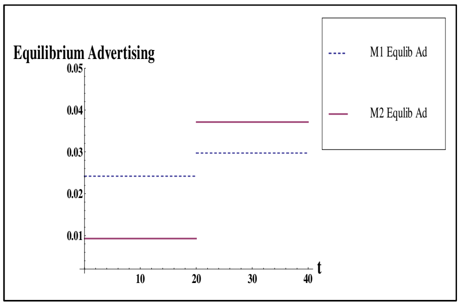
1. (Low probability, Low impact)=(.05,.05)
2. (High probability,High impact)=(.7,.7)
3. (Low probability, High impact)=(.05,.7)
4. (High probability, Low impact)=(.7,.05)
5. (Benchmark,Benchmark)=(.3,.3)

Previous empirical analysis found that in the automobile industry, both the benchmark recall probability and damage intensity are approximately 30% (Rubel et al., 2011). Therefore, our benchmark case considers$(\eta, \chi) = (.3, .3)$. We have not considered any recall costs in our models. Therefore, we believe our results will provide more accurate insights if we consider the decrease in margin of the focal firm in the second period as an implicit indicator of potential recall costs. We have assumed that the first period margin is always 1, i.e. $m_{i1} = 1,\ i = 1, 2$. When recall impact or damage is low ,i.e. $\eta = .05$, second period margin $m_{i2} = 1$ implying there is no loss in margin for the focal firm $i$. If $\eta = .3, m_{i2} = .8$ signifying a 20% loss in margin for the focal firm $i$. If the recall intensity or damage is high, .7, then $m_{i2} = .5$ for the focal firm $i$.

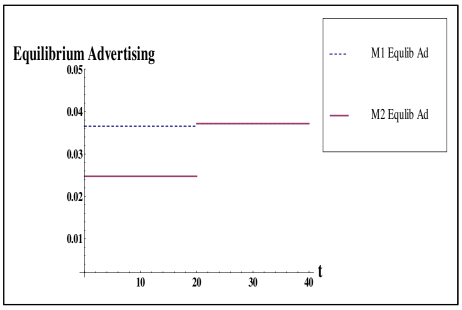
### 4.1. Equilibrium Advertising, Firm Performance, State Trajectories: One Focal Firm

#### 4.1.1. Effect of Product Recall on the Pre-crisis and Post-crisis Advertising

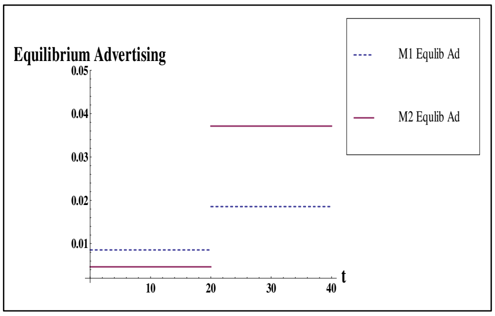
We compare the equilibrium advertising for both the firms in the two regimes using the parameter values as given in (26). Previous literature found that the approximate time to crisis about 2 years from product launch (Rubel et al., 2011; Hora et al., 2011). We, therefore, scaling $t$ in months, consider the time of recall as $t = 20$ in each case. Thus, $t \leq 20$ is the pre-crisis regime and $t > 20$ is the post-crisis regime. $M_1$, the focal firm, has both recall likelihood and damage effect as parameters in its equilibrium pre-crisis advertising policy, while $M_2$ has only recall probability as a parameter. The post crisis advertising decisions of both the firms are free from $\chi$ or $\eta$. Figure 3 (a),(b),(c) and (e) shows that the equlibrium advertising for both the firms is higher in the post-crisis period. The non focal firm always advertises more than or equal to the focal firm in the post-crisis period except when there is a low likelihood high impact recall (case 3(d)). Due to the low likelihood of the recall the focal firm advertises more in the pre-crisis period than in the post-crisis period. However, if the high impact recall really occurs, the firm lowers the advertising efforts. This can be due to budget cut for quality investments or other related costs of recall. From Figure 3(e) , we see that the focal firms maximum advertising is maintained in the first period if low impact recall is very likely.



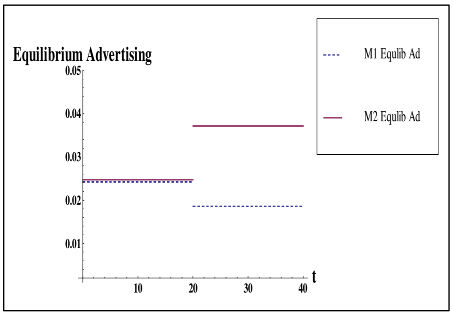

(a) χ=.3; η=.3; m2=.8;



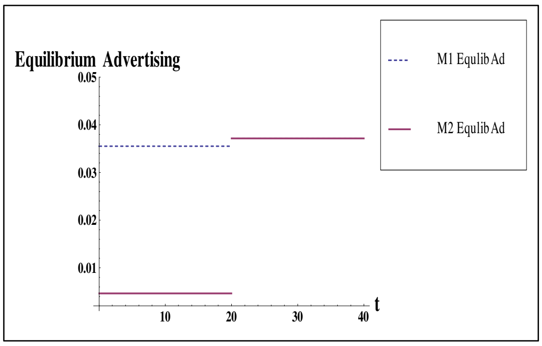

(b) χ=.05; η=.05; m2=1;




(c) χ=.7; η=.7; m2=.5;




(d) χ=.05; η=.7; m2=.5;




(e) χ=.7; η=.05; m2=1;

Figure 3: Comparison of Equilibrium Advertising

From Proposition 2 (a), the focal firm's ($M_1$) post-crisis advertising efforts are more than the pre-crisis advertising efforts if:

$$\frac{(\beta_1 + \frac{\gamma\alpha}{r})}{(\beta_1 + \frac{\gamma\alpha}{r+\chi})}\frac{(r + \eta\chi + \delta_1)}{(r + \delta_1)} - \frac{\gamma\alpha\chi}{(\beta_1(r + \chi) + \gamma\alpha)r} > \frac{m_{11}}{m_{12}}$$

and the non focal firm's post-crisis advertising efforts are more than the pre-crisis advertising efforts for manufacturer 2 if:

$$\frac{(\beta_2 + \frac{\gamma\alpha}{r})(r + \chi + \delta_2)}{(\beta_2 + \frac{\gamma\alpha}{r+\chi})(r + \delta_2)} - \frac{\gamma\alpha\chi}{r\Big(\beta_2(r + \chi) + \gamma\alpha\Big)} > \frac{m_{21}}{m_{22}}.$$ If $m_{21} \le m_{22}$, post-crisis advertising of the non focal firm is always greater than pre-crisis advertising.

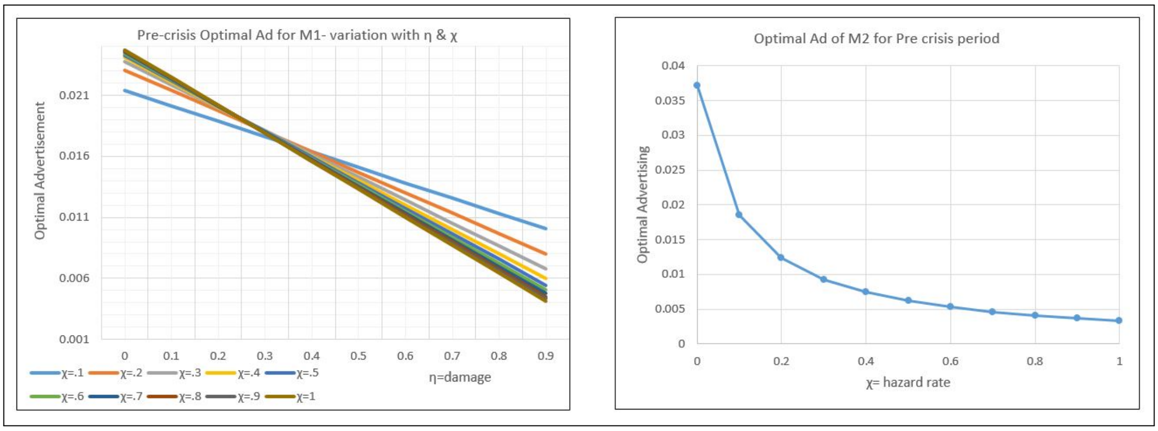



Figure 4: Equilibrium advertising for $M_1$ & $M_2$ in the pre-crisis regime- Variation with $\chi$ and $\eta$

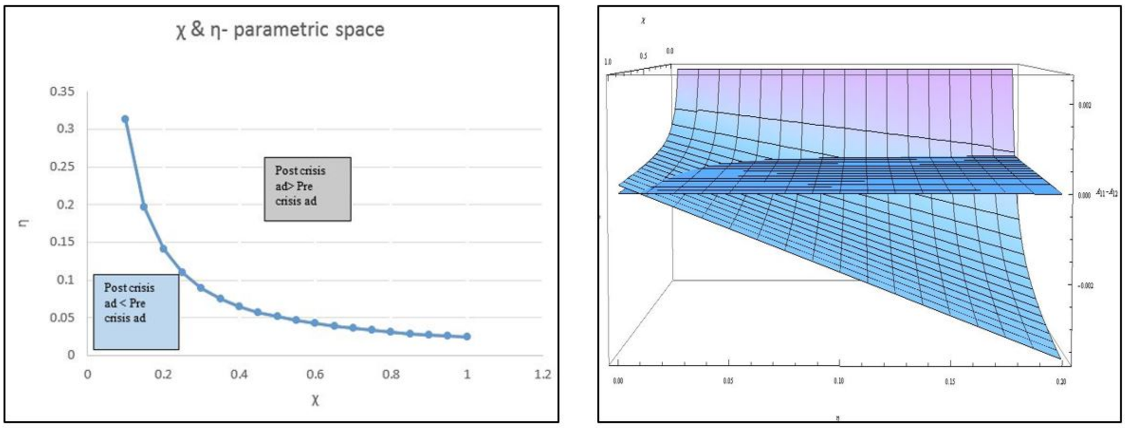



Figure 5: M1 - $\chi$ and $\eta$ space for Post-crisis ad to be more than pre-crisis ad

| **Category** | $\chi$ | $\eta$ | $m_{12}$ | $A_{11} - A_{12}$ | **Pre crisis ad >post-crisis Ad ?** |
|---|---|---|---|---|---|
| *Low impact low probability* | .05 | .05 | .9 | 0.0015 | Yes |
| *Low impact high probability* | .8 | .05 | .9 | -0.0016 | No |
| *Benchmark Case* | .3 | .3 | .75 | -0.0048 | No |
| *High impact low probability* | .05 | .7 | .5 | 0.0055 | Yes |
| *High impact high probability* | .8 | .7 | .5 | -0.0127 | No |

Table 4: Comparison of pre-crisis and post-crisis advertising for different cases

**Observation 1**:*The post-crisis advertising is higher than the pre-crisis advertising when there is a high probability of a recall capable of causing damage.*

This finding comports with the previous literature and explains the fact that the firms would engage in aggressive advertising after a recall of moderate intensity has been issued. We believe that the classification of the impact of recall as "low" or "high" will depend on the firm size or the recall costs incurred by the firm. However, in our theoretical results, we consider $\eta \leq .05$ to be fairly low impact. Even if we consider .05 to be low intensity if there is a high probability of such recall, firms post-crisis advertising increases in order to protect its image. Bad news spread fast and firm's image may get tarnished even due to a small recall because of brand switching or possible scepticism about its products that may arise. Sufficient advertising after the recall can prevent the public perception of potential recall damage to be more than the actual severity of the recall.

We also find that even a very low probability of high-intensity recall results in higher pre-crisis advertising. The focal firm will fear a slight risk of high-intensity recall because if the recall occurs it can be devastating for the firm. Thus, there might be a motivation for the firm to have high pre-crisis advertising in order to make the most profit from the pre-crisis market. Our finding also supports the empirical analysis of (Gao et al., 2015) who conclude that for a firm, if recall involves minor hazard, the pre-crisis advertising should be more. This is because more advertising sends a positive signal to the market about the firm's confidence in its product. In our case, an example of a minor hazard may be represented by a $\chi, \eta$ value of (.9,.02).

*The Role of Profit Margin:*

We found that margin plays an important role in determining whether the pre-crisis advertising is higher. Our numerical experiments show that if the margin in the post-crisis regime is at least the same as that of the pre-crisis regime, then the post-crisis advertising is higher for almost all practical cases for the focal firm. Numerically, there are instances when the post-crisis advertising may be lower at $m_{11} = m_{12}$. However, the value of $\eta$ is practically negligible for such cases and hence such cases are excluded from our analysis. From a managerial point of view, our findings highlight the importance of estimating three things- *the recall probability, the recall impact* and *the possible loss of margin after the recall.*

For the non-focal firm $M_2$, we find that as long as $m_{22} \geq m_{21}$, for all $\chi > 0$, the post-crisis advertising is always higher than the pre-crisis advertising. Under the model assumptions $m_{22} \geq m_{21}$. Thus, we can posit that the non-focal firm will always advertise more in the second post-crisis regime. This can be interpreted as the reaction of the non-focal firm as it adjusts equilibrium advertising to seize more market share when crisis damages its competitor's goodwill. It is worth noting that in the previous empirical estimations of recall probability (Rubel et al., 2011), in the automobile industry, the expected crisis time is 2 to 3 years and the baseline sales are affected by 35%. This is in agreement with the maximum value of $\eta$ (around .32) that we derived using numerical analysis and the analytical condition in proposition 2-(a) (Figure 6.). Thus, we can confidently assert that our parameter values are not contrived.

#### 4.1.2. Effect of Product Recall on Goodwill

We examine the behavior of the state variables, $\theta_1, \theta_2, \rho$ for different values of $\chi$ (crisis probability) and $\eta$ (damage intensity). We find that the interpretations of the variance of state variables largely depend on the two regime margins of the firms. We obtained the solutions of the state variables in the two regimes.

**Pre crisis solutions**:

$$\theta_1(t) = \frac{e^{-t\delta_1}\left(-A_{11}^* k + e^{t\delta 1} A_{11}^* k + \theta_{10}\delta 1\right)}{\delta 1} \qquad \text{with initial condition } \theta_1(0) = \theta_{10}$$

$$\theta_2(t) = \frac{e^{-t\delta_2}\left(-A_{21}^* k + e^{t\delta 2} A_{21}^* k + \theta_{20}\delta 2\right)}{\delta 2} \qquad \text{with initial condition } \theta_2(0) = \theta_{20}$$

$A_{i1}^*$ are the equilibrium advertising of the manufacturers in the pre crisis regimes.

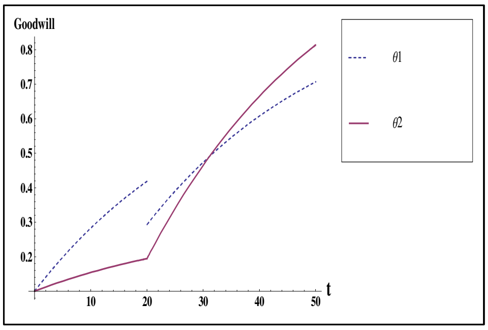
The post crisis solutions are:

$$\begin{aligned}
\theta_1(t) &= \frac{e^{-t\delta_1}(e^{t\delta_1}A^*_{12}k - A^*_{12}ke^{t_r\delta_1} + ae^{tr\delta_1}\delta_1)}{\delta_1}, \theta_1(t_r+) = a = (1-\eta)\theta_1(t_r-) \\
\theta_2(t) &= \frac{e^{-t\delta_2}(e^{t\delta_2}A^*_{22}k - A^*_{22}ke^{t_r\delta_2} + be^{tr\delta_2}\delta_1)}{\delta_2}, \theta_2(t_r+) = b = \theta_2(t_r-)
\end{aligned}$$

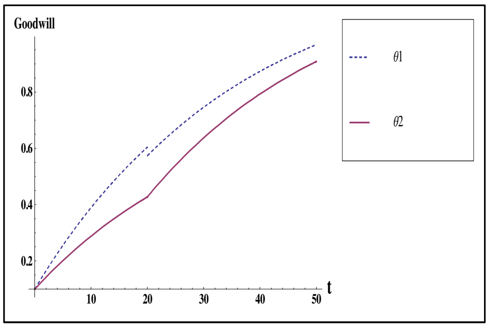
The different scenarios of recall impact and probabilities are considered. The scenarios are given in table below.

| | Low impact,<br>Low likelihood | Low impact,<br>High likelihood | High impact,<br>Low likelihood | High impact,<br>High likelihood |
|---|---|---|---|---|
| $\chi$ | 0.05 | 0.7 | 0.05 | 0.7 |
| $\eta$ | 0.05 | 0.05 | 0.7 | 0.7 |

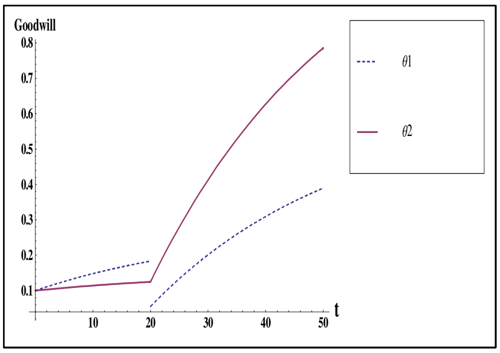
Table 5: Recall Likelihood and Impact



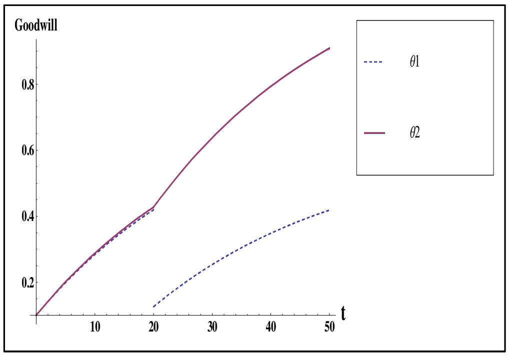

(a) χ=.3; η=.3



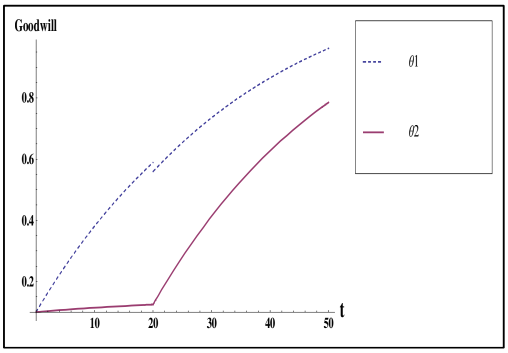

(b) χ=.05; η=.05




(c) χ=.7; η=.7




(d) χ=.05; η=.7




Similar Firms: $\theta_1(0) = \theta_2(0) = 0.1$;

(e) χ=.7; η=.05

Figure 6: State Trajectories(goodwill) in pre and post-crisis regimes

Figure 6 shows how the brand goodwills vary before and after recall for similar firms(same initial goodwill; same sensitivity to brand difference and same absorption capacity). The margins in pre and post-crisis regimes are assumed to be same($m_{11}$ = $m_{12}$ = 1)when recall impact is low ($\eta \leq .2$) and if recall impact is high,(e.g $\eta = .7$) there is an erosion in margin due to recall costs and $m_{12}$ = .5. It is impossible to predict an exact relationship between recall impact and margin erosion. However, we believe that our numbers are sufficiently indicative to illustrate the true effect of recalls on the state variables.

Whenever there is a recall, invariably in each case, the goodwill of $M_2$ rises in the second regime. This is intuitive because $M_2$ advertises more in the second period as long as $m_{22} \geq m_{21}$. For the focal firm, a low impact recall increases the goodwill of the firm. This might be surprising but is consistent with the findings of the optimal advertising strategies given in Figure 3 which shows that advertising is higher for a low recall likelihood. A product recall is often followed by market overreaction (Govindaraj et al., 2004). Therefore, in case of small impact recalls, the focal firm advertises aggressively so that the stock of goodwill is high enough to nullify the effects of overreaction. Moreover, from a consumer's perspective, high goodwill and high quality are often associated. Consequently, if a brand's goodwill is high, then despite a recall, customers can be still loyal to the brand diminishing the potential damage (Kalaignanam et al., 2013).

### 4.1.3. Market Shares $\rho$(t) Trajectories

Analysis of the market shares of the firms for different values of crisis likelihood and impact reveals some interesting insights. Congruous with the advertising policies and resulting stock of goodwill of the firms, the market share of the focal firm plummets if the recall impact, $\eta$ is high. However, we find that a low likelihood high impact $(\chi = 0.05, = 0.7)$ recall reduces the market share of the focal firm in the pre-crisis period from 0.5 at t=0 to 0.495705 at t=20, the time of recall (Figure 7(b)). On the other hand, a high likelihood high impact recall $((\chi = 0.7, = 0.7)$ increases the market share from 0.5 to 0.523519 in the pre-crisis period. This apparent intriguing conflict is theoretically sound. Recall that $\dot{\rho}(t) = \gamma(\theta_1(t) - \theta_2(t))$. Figure 6(c) and 6(d) shows that the difference $\theta_1(t) - \theta_2(t)$, on which the evolution of the market share $\rho(t)$ depends, is much more in the case of high likelihood high impact recall than in the instance of low likelihood low impact recall. This explains the above conflict. In practice, a low possibility raises scepticism about the occurrence of the recall. Consequently, the non-focal firm advertises more in a reaction to the focal firm's high advertising, thereby capturing a high market share.

The steady state for $\rho$ is attained if $\dot{\rho}(t) = \gamma(\theta_1(t) - \theta_2(t)) = 0$. Since $\gamma > 0, (\theta_1(t) - \theta_2(t)) = 0 \Rightarrow \theta_1(t) = \theta_2(t)$ when the market share reaches a steady state. However, $0 \leq \rho \leq 1$ may not always be satisfied and $\rho(t)$ may be greater than 1 as t increases. The time t, at which the market share becomes greater than 1, can be increased by scaling down (decreasing the value of) $\gamma$. We believe that the time frame we have considered is practical enough to draw useful insights. $\rho(t) = 1$ can also be interpreted as the time when the $M_2$ goes out of business and the problem under study becomes irrelevant as we are considering competition only and not a monopoly.

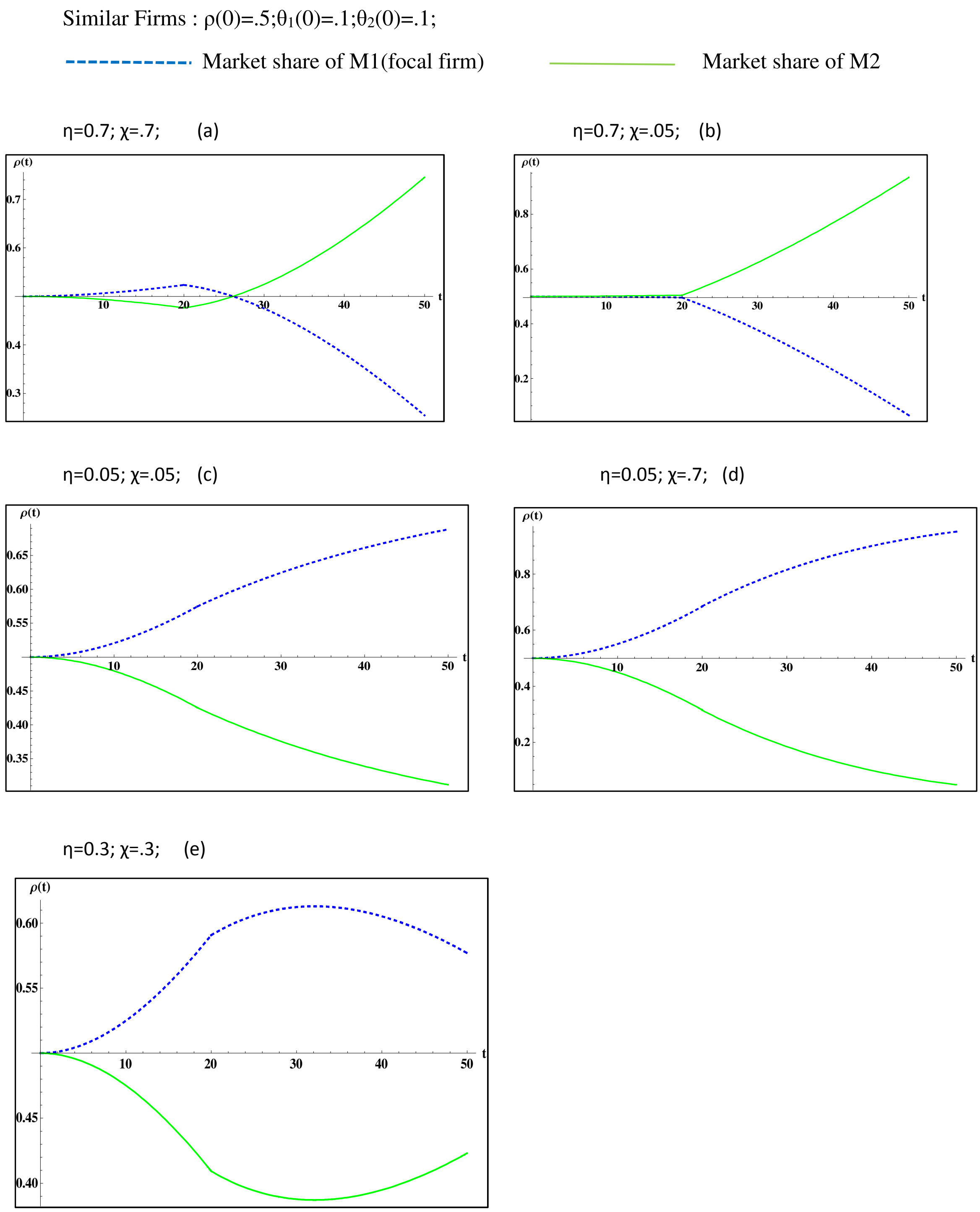



Figure 7: Market share of similar firms

#### 4.1.4. Effect of Product Recall on Firm Performance

Since, the expressions of the value function are complex, we numerically examine the effect of the recall likelihood and the intensity of the damage on the profit. The analytic investigation leaves us with a complex parametric space and it is hard to infer anything insightful from the analysis. We evaluate the value functions i.e the long-term expected profits, $V_{11}(t)$ and $V_{21}(t)$ at time t=0. This gives us the long-term expected profit of the manufacturers. An existing high brand equity can insure a firm against potential damage during crisis (Dawar and Pillutla, 2000; Cleeren et al., 2008). Hence, we consider following three scenarios:

i) $M_1$, the recalling firm and $M_2$ are symmetric implying that they have a similar market share and brand image initially before at t=0

ii) $M_1$ is a bigger brand than $M_2$ before the recall

iii) $M_2$ is bigger brand than $M_1$ before the recall

Using Mathematica software we depict the long-term profits corresponding to the above scenarios using the values of table 6 :

| | $\theta_1(0)$ | $\theta_2(0)$ | $\eta$ | | $\rho(0)$ |
|---|---|---|---|---|---|
| Similar Firms | 0.1 | 0.1 | 0.05(m2=1) | 0.7(m2=.5) | 0.5 |
| Focal firm is bigger | 0.13 | 0.1 | 0.05(m2=1) | 0.7(m2=.5) | 0.65 |
| Non focal firm is bigger | 0.1 | 0.13 | 0.05(m2=1) | 0.7(m2=.5) | 0.35 |

Table 6: Scenarios - similar firms and dissimilar firms

We compare the long-term profits of the firms for the two extreme cases:

1. Damage intensity $\eta$ is low at 5% and the effect on profit margin is negligible ($m_{11} = m_{12} = 1$),

2. Damage intensity $\eta$ is high at 70% and there is a high margin erosion ($m_{11} = 1, m_{12} = .5$).

The long-term expected profits vary with the hazard rate (others parameters remaining fixed). Therefore we express the value function as a function of $\chi$ and find the local maximum by using the first order condition on the value function. The first order conditions yield a polynomial of degree 5 in $\chi$. We use mathematica software to solve the first order condition and find the real positive roots of the polynomial. This gives us the values of $\chi$ for which $V_{11}(t=0)$ or $V_{21}(t=0)$ are maximum.

**Observation 2** :

(a) *When recall impact is low, the focal firm's profit increases in the hazard rate up to a threshold (approximately $\chi = 0.08$ in our case) and then decreases. However, a high impact recall is always detrimental to the firm's profit.*

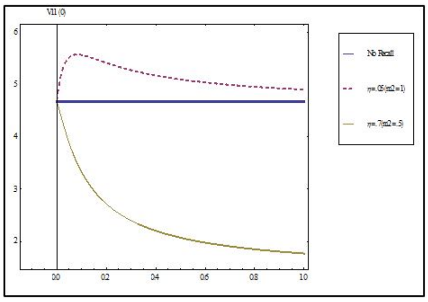
(b) *A "no-recall" scenario is more profitable for the focal firm than a high impact, highly likely recall scenario.*

(c) *The non-focal firm has a significantly higher profit when the recall impact is high. The non-focal firm's profit is monotonic increasing with the hazard rate for a high impact recall.*

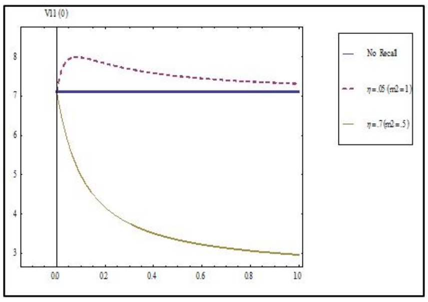
(d) *A low impact recall having low likelihood decreases the profit of the non-focal firm.*

Our analysis shows that when there is a low risk of recall, the long-term profit for both the focal firm actually increases. A cursory glance into the result might be surprising. However, a detailed introspection reveals that this is consistent with the findings of another study (Rubel, 2018) which suggests that the crisis anticipation softens price competition and thus generates more profit. Price is not a decision variable in our model, but the equilibrium advertising efforts in the two regimes are functions of the margins which may vary.

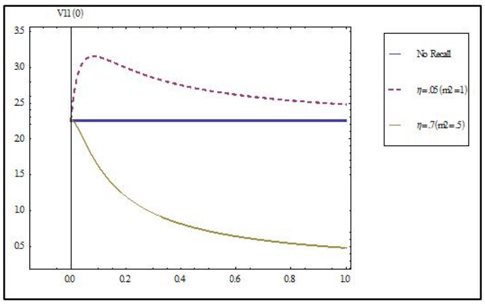
A product recall can be categorized as an unsystematic risk and low likelihood of such risks can positively affect firm performance (Aaker and Jacobson, 1987). The focal firm's profit is monotonically decreasing with the hazard rate for a high impact crisis. This is an expected result because if the impact is high the advertising will be low, post-recall goodwill erosion will be high and consequently the profit is low. The non focal firm's profits are complementary to that of the focal firm. This follows from our model's mathematical structure. The managerial implications of the above observations are significant. When two similar firms compete, a low impact recall can be beneficial for the firm at fault.




ɵ1(0) =.1; ɵ2(0) =.1; ρ(0)=.5;

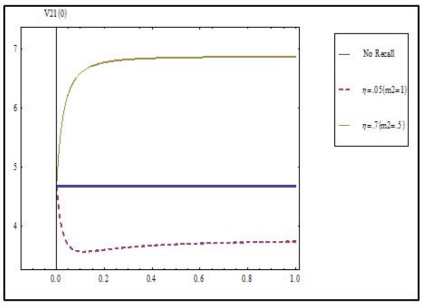



ɵ1(0) =.13; ɵ2(0) =.1; ρ(0)=.65;

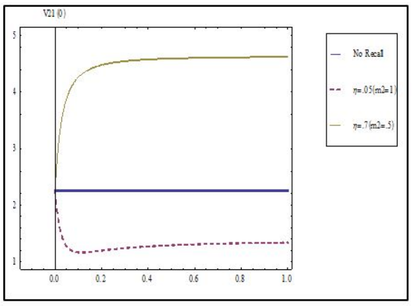



ɵ1 (0) =.1; ɵ2 (0) =.13; ρ (0) =.35

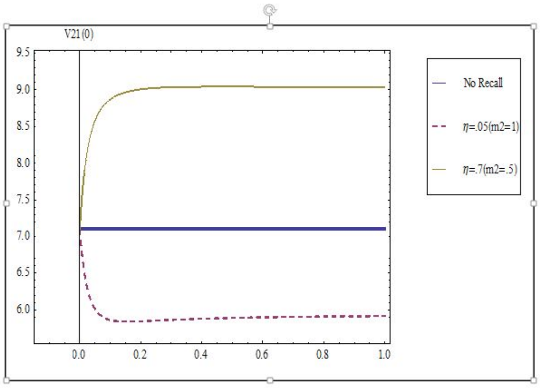
Figure 8: V11(0) -long-term profit for the focal firm




$\theta 1$ (0) =.1; $\theta 2$ (0) =.1; $\rho$ (0) =.5




$\theta 1$ (0) =.13; $\theta 2$ (0) =.1; $\rho$ (0) =.65




$\theta 1$ (0) =.1; $\theta 2$ (0) =.13; $\rho$ (0) =.35

Figure 9: V21(0) -long-term profit for the non focal firm

## 4.2. Can Hazard Myopia be Profitable?

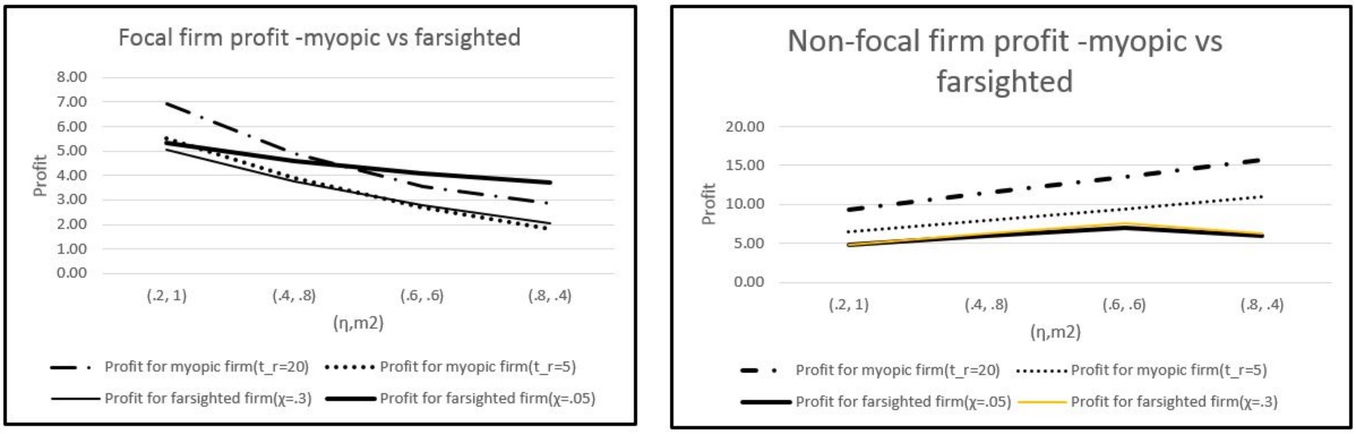
We ask whether a myopic focal firm or non-focal firm can be better off in terms of profit under any circumstances during a recall. To compute the profit of the myopic firms we need to assume some recall time $t_r$ as the profit depends on the time of recall. An early recall, $t_r = 5$ or average normal time recall $t_r = 20$ are considered. The farsighted focal firms' profit will also depend on $\chi$ which the myopic firm never foresees. The profits we consider are given in tables 7 and 8.

| $\eta, m_{12}$ | Profit for myopic firm($t_r$ = 20) | Profit for myopic firm($t_r$ = 5) | Profit for farsighted firm($\chi$ = .05) | Profit for farsighted firm($\chi$ = .3) |
|---|---|---|---|---|
| (.2, 1) | 6.92258 | 5.52507 | 5.3349 | 5.07491 |
| (.4, .8) | 4.90894 | 3.90697 | 4.59584 | 3.78435 |
| (.6, .6) | 3.54813 | 2.678 | 4.06619 | 2.79197 |
| (.8, .4) | 2.84013 | 1.83816 | 3.72173 | 2.06443 |

Table 7: Focal firm profits- Myopic and farsighted

| $\eta, m2$ | Profit for myopic firm($t_r$ = 20) | Profit for myopic firm($t_r$ = 5) | Profit for farsighted firm($\chi$ = .05) | Profit for farsighted firm($\chi$ = .3) |
|---|---|---|---|---|
| (.2, 1) | 9.34 | 6.54 | 4.85 | 4.81 |
| (.4, .8) | 11.47 | 8.01 | 6.02 | 6.26 |
| (.6, .6) | 13.59 | 9.47 | 7.06 | 7.59 |
| (.8, .4) | 15.72 | 10.94 | 6.02 | 6.26 |

Table 8: Non-focal firm profits- Myopic and farsighted




Focal firm profit-myopic vs farsighted

Non-focal firm profit-myopic vs farsighted

Figure 10: Expected profits -hazard myopic vs farsighted firms

**Observation 3:**

(a) *A farsighted focal firm with high hazard rate (.3 in this case) is less profitable than a myopic firm when a recall occurs later($t_r$ = 20) in the planning horizon. For an early high impact recall, the far-sighted firm is better off than the myopic firm.*

(b) *The myopic non-focal firm is always better off than the farsighted non-focal firm. The myopic non-focal firm's profit decreases with an early recall.*

The result that a focal firm may be profitable by ignoring a potential risk, is surprising. However, this is true only if the hazard rate of the firm is low or the recall occurs at a later stage in the planning horizon. For a high impact and high hazard rate, a focal firm is more profitable if advertising decisions are taken while accounting for the potential risk.

Theoretically, this is consistent with our model because we have shown in section 4.2 that for a high hazard rate, the post-crisis advertising is always higher. Since a myopic advertising policy always has the same structure as the post-crisis advertising, myopic advertising is always higher as compared to far-sighted advertising. This results in higher goodwill and hence higher goodwill dependent demand and profit. Profit margin has a vital influence on the policy and hence we have considered margin erosion for high impact recalls while evaluating the value function of the myopic ( or farsighted) firms. However, margin erosion under consideration is indicative. If it so happens that the $\eta$ = .8 impact recall decreases the margin to .2 (for example due to high recall costs) instead of .4, the profit decreases further.

From a managerial perspective, the results show that it is important for a firm to estimate the likelihood of a crisis and also potential damage. A focal firm may ignore low impact crisis while taking advertising decisions(as they will make more profit while doing so). However, if a high impact or highly likely recall is anticipated, it is much better to incorporate the hazard rate while making the equilibrium advertising policy. The non-focal firm's management, on the other hand, would be better off ignoring the rival's hazard rate in their decision. Having said this, it has been seen that non-focal firms, especially belonging to the same country of origin of the focal firm or selling similar products are often negatively affected by a spillover effect of the recall. Consumers get sceptical about the non-focal firm's products, consequently decreasing its profit. In such cases, it may be advantageous to consider the crisis likelihood. Furthermore, if the focal firm's advertising drops due to high margin loss, it may decrease advertising and the non-focal firm's advertising efforts will be way above those of the focal firm. Consumers may also perceive the higher advertising by the non-focal firm as opportunistic behaviour and have formed a negative mindset and might not buy the competitor's products (Gao et al., 2015). The examination of the effects of consumer's negative perception or spillover is beyond the scope of this study.

### 4.3. Equilibrium advertising and firm performance for two focal firms

The experiments conducted for two focal firms are categorized by two situations - $i^{th}$ *firm recalls first* or $i^{th}$ *firm recalls second*. The most important parameters that we are varying for these experiments are :

$$\chi_i, \chi_j, \eta_i, \eta_j, m_{i1}, m_{i2}, m_{i3}$$

Advertising for all three regimes are not affected by all the parameters that we are varying for our experiments. To easily visualize the effect of the above mentioned paramters on the the equilibrium advertising, we present the following table 8, where "Yes" corresponding to a parameter means that the corresponding advertising is a function of the parameter. For example, $A_{i1}$ is a function of $\chi_i, \chi_j, \eta_i, m_{i1}, m_{i2}, \hat{m}_{i2}, m_{i3}$.

| Description | Equilibrium Advertising | Parameters | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $\chi_i$ | $\chi_j$ | $\eta_i$ | $\eta_j$ | $m_{i1}$ | $m_{i2}$ | $\hat{m}_{i2}$ | $m_{i3}$ |
| 1st Period Ad | $A_{i1}$ | Yes | Yes | Yes | | Yes | Yes | Yes | Yes |
| 2nd Period Ad - $i$ recalls first | $A_{i2}$ | | Yes | | | | Yes | | Yes |
| 2ndd Period Ad - $i$ recalls second | $\hat{A}_{i2}$ | Yes | | Yes | | | | Yes | Yes |
| 3rd Period Ad | $A_{i3}$ | | | | | | | | Yes |

Table 9: Parameters Affecting Equilibrium Advertising

We follow some assumptions about the margins in the different periods.

*Assumptions about margins*

1. If the $i^{th}$ firm recalls first, we consider $m_{i1} > m_{i2} = m_{i3}$
2. If the $i^{th}$ firm recalls second then $m_{i1} = m_{i2} > m_{i3}$

An important point to note is that if the there is no change in margin in the second and third regimes, for the $i^{th}$ firm, then $A_{i2} = A_{i3}$. This follows from the fact,

$$A_{i2} - A_{i3} = \frac{k(m_{i2} - m_{i3})(\alpha\gamma + \beta_i(r + \chi_j))}{\mu_i(r + \chi_j)(r + \delta_i + \chi_j)}. \tag{27}$$

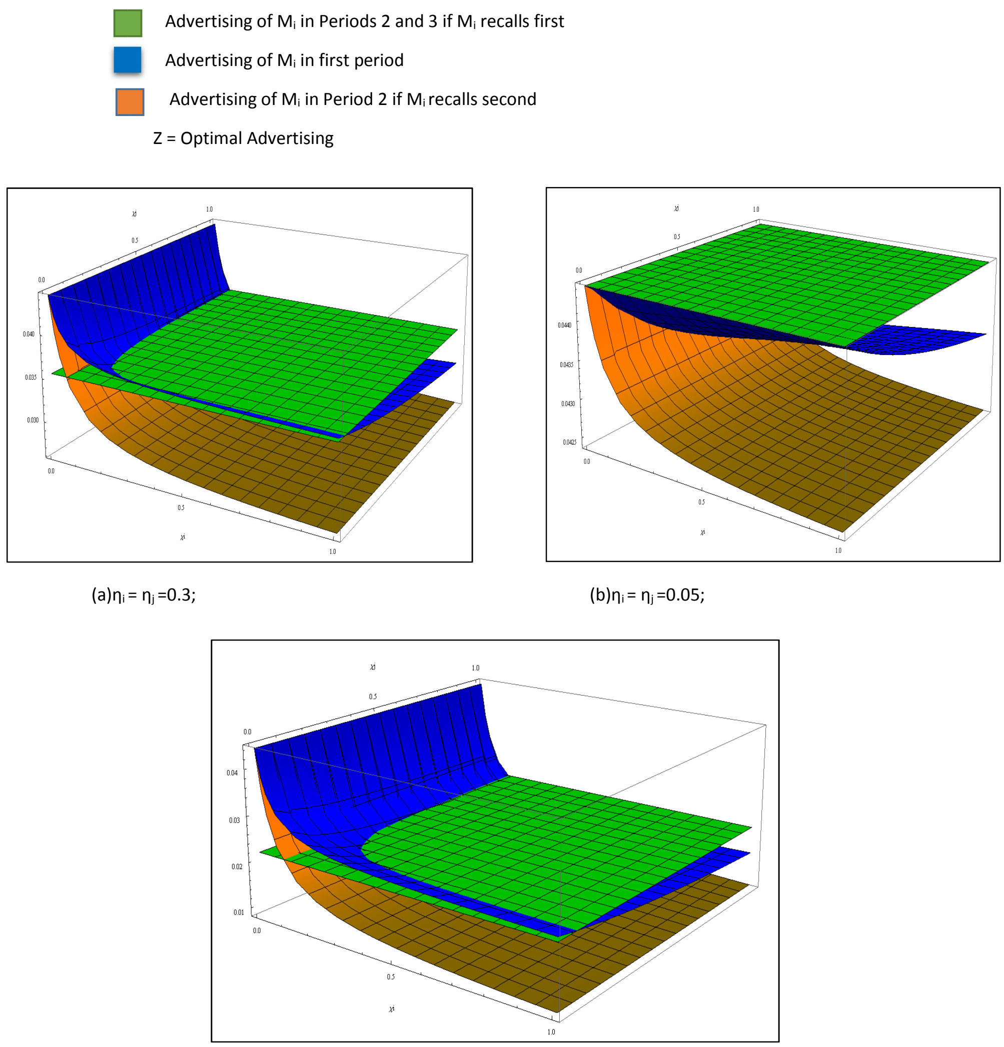
The complexity of the analysis of the equilibrium advertising decisions and the performance of the two focal firms case is multiplied due to the dimension of the parametric space. We now have the extra parameters, for hazard rate, damage effect and margins. Our goal is to reduce the analytical complexity without reducing the dimension of the problem. While many possible cases are imperative, we have focused on the cases which we believe to be most feasible. The assumptions about the margins is an example of such feasibility. Similar to the one focal firm case, we consider the combinations when damage and crisis are high or low and the benchmark case when damage or crisis equals 0.3. Thus there are four parameters $\chi_i, \chi_j, \eta_i, \eta_j$ and these take values high(0.7) or low (.05). We have thus $2^4+1$(benchmark case) $=$ 17 cases to consider. However, from table 9, we notice that the goodwill damage of the $j^{th}$ firm, $\eta_j$, does not affect the advertising decision of the $i^{th}$ firm. Hence, the number of cases reduce to $2^3+1=9$. While Table 10 gives the values of the parameters and the corresponding equilibrium advertising, we present the most important cases diagrammatically -

*Benchmark Case* ($\eta_i = \eta_j = .3$);
*High Impact Recall* ($\eta_i = \eta_j = .7$);
*Low Impact Recall* ($\eta_i = \eta_j = .05$).

We perform 3D plot in mathematica and vary the equilibrium advertising over all values of $\chi_i$ and $\chi_j$. Therefore, these figures not only cover all the cases in the 8 scenarios, but also spans over all likelihood scenarios of both the rivals. This makes our findings more robust.

| Scenarios | | $\chi_i$ | $\eta_i$ | $\chi_j$ | $m_{i1}$ | $m_{i2}$ | $m_{i3}$ | $A_{i1}$ | $A_{i2}$ | $A_{i3}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | $m_{i2} = m_{i3}$ | 0.05 | 0.05 | 0.05 | 1 | 1 | 1 | 0.041999 | 0.044444 | 0.044444 |
| 2 | | 0.05 | 0.05 | 0.8 | 1 | 1 | 1 | 0.044218 | 0.044444 | 0.044444 |
| 3 | | 0.05 | 0.8 | 0.05 | 1 | 0.5 | 0.5 | 0.023304 | 0.022222 | 0.0222222 |
| 4 | | 0.05 | 0.8 | 0.8 | 1 | 0.5 | 0.5 | 0.022307 | 0.022222 | 0.0222222 |
| 5 | | 0.8 | 0.05 | 0.05 | 1 | 1 | 1 | 0.04087 | 0.044444 | 0.044444 |
| 6 | | 0.8 | 0.05 | 0.8 | 1 | 1 | 1 | 0.042473 | 0.044444 | 0.044444 |
| 7 | | 0.8 | 0.8 | 0.05 | 1 | 0.5 | 0.5 | 0.00547 | 0.022222 | 0.0222222 |
| 8 | | 0.8 | 0.8 | 0.8 | 1 | 0.5 | 0.5 | 0.012896 | 0.022222 | 0.0222222 |

Table 10: Equilibrium advertising in different cases




(a) $\eta_i = \eta_j = 0.3$; (b) $\eta_i = \eta_j = 0.05$; (c) $\eta_i = \eta_j = 0.7$;

Figure 11: Equilibrium Advertising in Three Regimes - Two Focal Firms

Noting that the rival's damage, $\eta_j$, does not affect the advertising policies of firm $M_i$, the following cases are equivalent with respect to advertising:
i) $\eta_i = .05; \eta_j = .7 \quad \eta_i = .05; \eta_j = .05$
ii) $\eta_i = .7; \eta_j = .7 \quad \eta_i = .7; \eta_j = .05$

**Observation 4**:

(a) *$M_i$ recalls first*:

- *During a low impact recall, irrespective of the likelihoods of recall of each firm, $M_i$'s equilibrium advertising is always higher in the second and third regimes than in the first regime.*
- *If recall impact is the benchmark or high, the first period advertising of $M_i$ is higher than the second or third period advertising for all values of $\chi_i$ if the rival's crisis likelihood $\chi_j$ is low (Figure 11 (a) & (c)).*

(b) *$M_i$ recalls second*:

- *The first regime advertising of $M_i$ is always higher than the second regime advertising irrespective of recall impact or likelihood of each firm.*
- *For moderate and high likelihood of a crisis, the post-crisis (third regime) advertising of $M_i$ is higher than it's pre-crisis advertising.*

When $M_i$ recalls first, in the second regime $M_i$ already knows the impact of the recall and that only his rival has a chance of recall in future. Also, a low impact recall means no or very little margin erosion. Therefore, the firm $M_i$ uses higher advertising as a double-edged sword to overcome any negative effect of its own recall and also to react to the possible higher advertising by the rival. On the other hand, a moderate or high crisis impact makes $M_i$ more cautious and his first-period advertising is higher if the rival's crisis likelihood is low. This is consistent with Proposition 7. $M_i$ tries to make the most profit out of the market in the first regime when there is a high impact high likelihood recall possibility. However, if the rival also has a high possibility of high impact recall, $M_i$'s advertising level is low in the first period as it can potentially benefit from the competitor's misfortune.

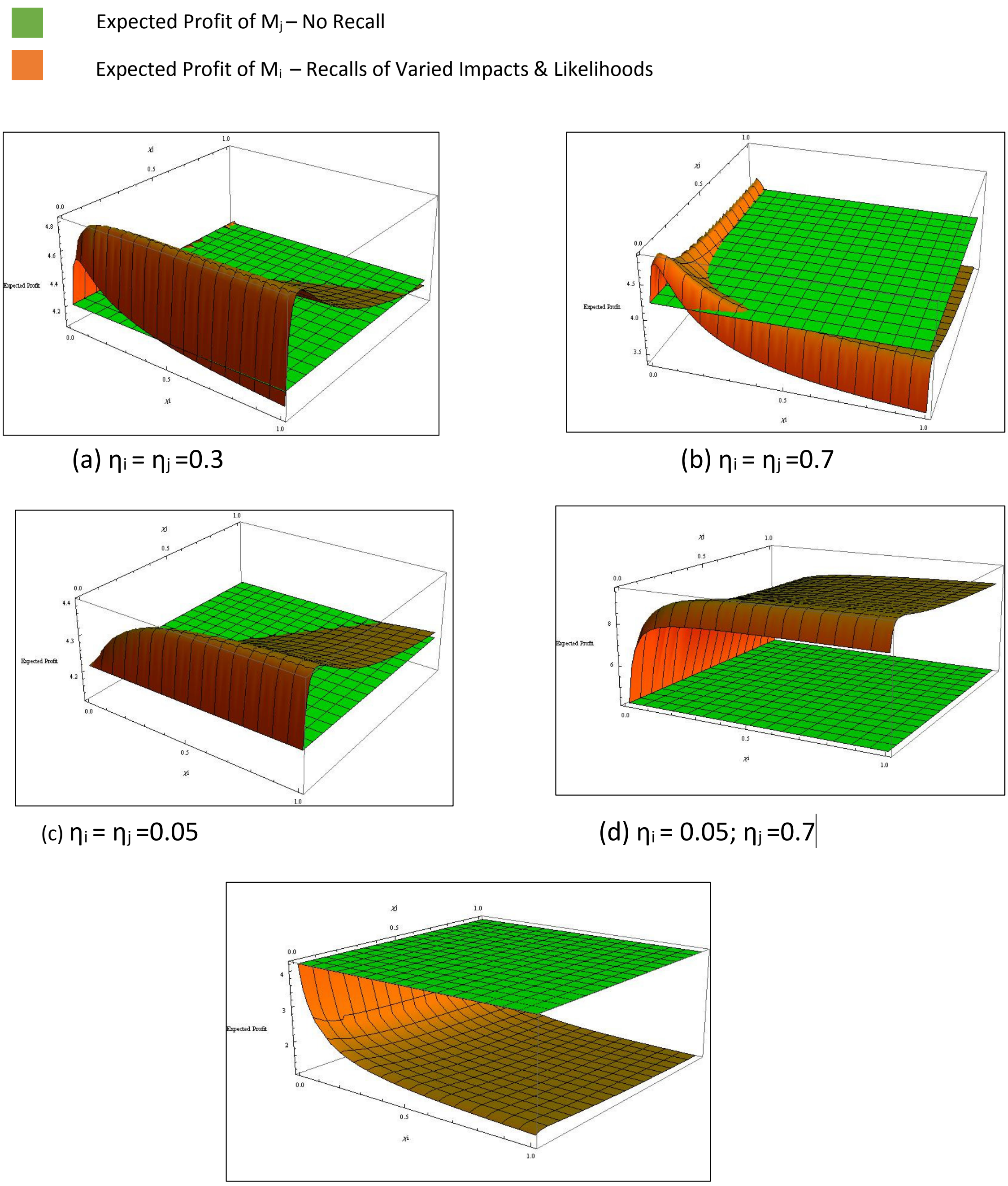
If $M_i$ recalls second, it's second regime advertising level drops. In the second period, $M_i$'s rival has already issued a recall and is presumably coping with the negative effects. Moreover, $M_i$ knows that there is a possibility of a recall that it itself might issue in the future. Therefore, $M_i$'s advertising drops in the second period as low advertising can well give $M_i$ the optimal profit while its rival suffers from a recall and at the same time $M_i$ higher post-crisis advertising (third regime) helps it to overcome the effects of its own recall.

**Observation 5:**

(a) *Irrespective of all crises likelihoods, the no recall scenario is strictly more profitable for $M_i$ only when $M_i$ has a very high impact recall and $M_j$ has a low impact recall. On the other hand, irrespective of all crises likelihoods, the no recall scenario is strictly less profitable for $M_i$ when $M_i$'s recall impact is low and $M_j$'s recall impact is high (Figures 12 (d) & (e).*

(b) *When both firms have the same impact of recall*:

- *In a high impact recall scenario, $M_i$'s profit is higher than the no recall scenario when $M_i$'s crisis likelihood is low.*
- *Surprisingly, when there is a low impact recall, the firm profit is increasing with its own hazard rate.*




(a) $\eta_i = \eta_j = 0.3$ (b) $\eta_i = \eta_j = 0.7$

(c) $\eta_i = \eta_j = 0.05$ (d) $\eta_i = 0.05$; $\eta_j = 0.7$

(e) $\eta_i = 0.7$; $\eta_j = 0.05$

Figure 12: Firm profits in Presence of Two Firms

The results show that during a competition between two focal firms, the no recall scenario may not always be the maximum profitable. If the expected damage caused by own recall is small, a firm will have no incentive to avoid a recall as long as the competitor is more risky than the firm itself. When a firm is at a risk of causing a high damage, it is best to make efforts to avoid the recall because a no recall scenario is more profitable for it. Consequently, the firm may seek to invest in quality or strategy in order to avoid a recall. When the expected recall damage is equal for both the firms, a firm should aim to keep the recall probability low when the damage is high. On the other hand if expected damage is low for both the firms, a firm has no incentive to reduce the crisis likelihood as a high likelihood of crisis increases it's profit. The main takeaway from our findings is that in the presence of a risky competitor a firm should try to avoid a recall if the expected damage for itself or both the firms is high and consequently invest in other areas of quality or possibly supplier management. If the firm's own risk is low, the firm can invest more in advertising without making any effort to avoid the recall.

## 5. Manegerial Implications and Conclusion

The marketing literature has investigated whether advertising should increase after a product recall. The findings have revealed conflicting results. While some studies emphasize the need for increased pre-crisis advertising, others found support for the increased post-crisis advertising. One common conclusion in the product recall literature highlights the firms loss of goodwill as a result of the recall (Craig and Thomas, 1996; Cleeren et al., 2013). Hence, in this study, we built a theoretical model with goodwill based demand function and analyzed advertising decision models under the lens of differential game theory.

In this study, we make some significant contributions to the existing literature related to product recall. First, we re-investigated not only the advertising decisions of firms in a competitive market but also the effect of a product recall on firm profit. We found that whether a firms' advertising level will increase after the product recall, depends on the the severity and likelihood of the recall. This claim is consistent with the empirical findings of Gao et. al. (Gao et al., 2015). We also found that when similar firms compete or the focal firm is bigger, a small crisis likelihood increases the focal firm's profit. This is consistent with another study which presents price competition in presence of product recall (Rubel, 2018). Second, we have introduced the concept of "hazard myopia", a firm's inability to foresee recall occurrence. This concept differs from the conventional myopia (ignoring state evolution) in the context of differential games. We showed how to solve a hazard myopic firm's decision problem and found an interesting case when hazard myopia can be more profitable than envisioning a recall. Our study enhances the literature by studying the equilibrium advertising policies and profits of two farsighted focal firms.

Although price is not a decision variable in our model, the profit margin has a very important role to play when it comes to advertising decision making. Our analytical and numerical findings give a number of insights which are important from a managerial point of view. We summarize our key findings. The investigation results answer our research questions and provide new managerial insights in addition to verifying previous research results.

**Managerial Implications**: In a duopoly market with a product recall risk, it is important for the management of both firms to understand if the rival is risky or not because a firm's advertising strategy depends on the rival's product recall likelihood. By risk we mean the combination of recall likelihood and intensity of the damage. Thus, a firm should estimate its own and the rival's recall likelihood and possible damage caused by any expected recall. The damage should not only be measured in terms of the goodwill-loss, but also in monetary value. Since we showed that advertising levels depend on profit margins, the monetary value measure of a recall damage will enable the firm to evaluate the profit margin and advertise accordingly. It is also important to know how much a damage can potentially erode a profit margin. Advertising decisions are seldom the only decisions made. Product pricing is another important decision. The prevalence of profit margins of the different regimes in the expressions of our equilibrium advertising decisions shows that price might also be used as a potent tool for mitigating the effects of a product recall. When a recall is expected to cause low damage, managers may well ignore the chances of recall and advertise myopically. Managers know that a low likelihood of recall can increase profit. Therefore, at the beginning of the planning horizon, if a focal firm's management knows that the firm's crisis likelihood is high, they can put efforts like quality maintenance or inspection to minimize the chances of a recall and signal these efforts to the market. Consequently, its rival also estimates a low crisis likelihood for the focal firm. This may soften competition and increase profit. Hazard myopia might look attractive as it generates high long-term profit, but, it is important for the firm's management to estimate the hazard rate rather than ignoring it because a high hazard rate might be disastrous. When both firms have different likelihoods of product recall, it is important for the managements of the firms to estimate the likelihoods and the expected damage severity. The effectiveness of the resulting advertising policies in the three regimes depend on this knowledge likelihood and expected damage.

Our work can be extended in a number of directions. First, we have not considered the recall costs as a separate cost in our problem. Instead, we assumed that the margin will drop in the post-crisis period due to possible recall costs. Modelling recall costs as a function of the recall damage and the recall time can result in valuable insights. Second, the damage caused by the recall is assumed to be a constant fraction by which the brand image is eroded. This ensures the mathematical tractability of our model. However, this damage might not be a constant factor and can be a decreasing function of time. This is because as the product overcomes the pernicious consequences of the recall, the damaging effect might fade after some time i.e., there can be recovery. Empirical research shows that as products move further into the life cycle, the chances of recall decreases (Hora et al., 2011). Other distributions with a decreasing hazard rate can be considered. Third, we have not considered channel leadership in our model. In fact, due to the structure of our game, the Stackelberg game and the Nash game coincide. The firms' advertising strategies and performance can be analyzed for a model where there is a market leader. This will help in finding if market leadership gives any advantage during a recall. Last, it is challenging but possible to investigate multiple decision like price and advertising or quality in the same recall scenario.

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## References

Aaker, D. A., Jacobson, R., 1987. The role of risk in explaining differences in profitability. Academy of Management Journal 30 (2), 277–296.

Achenbaum, A., 1989. How to breathe new life into brands. Advertising Age 60 (18), 24–70.

Ataman, B., Ülengin, B., 2003. A note on the effect of brand image on sales. Journal of Product & Brand Management 12 (4), 237–250.

Balasubramanian, S., Bhardwaj, P., 2004. When not all conflict is bad: Manufacturing-marketing conflict and strategic incentive design. Management Science 50 (4), 489–502.

Benchekroun, H., Martín-Herrán, G., Taboubi, S., 2009. Could myopic pricing be a strategic choice in marketing channels? a game theoretic analysis. Journal of Economic Dynamics and Control 33 (9), 1699–1718.

Boukas, E.-K., Haurie, A., Michel, P., 1990. An optimal control problem with a random stopping time. Journal of optimization theory and applications 64 (3), 471–480.

Chao, G. H., Iravani, S. M., Savaskan, R. C., 2009. Quality improvement incentives and product recall cost sharing contracts. Management science 55 (7), 1122–1138.

Chen, Y., Ganesan, S., Liu, Y., 2009. Does a firm's product-recall strategy affect its financial value? an examination of strategic alternatives during product-harm crises. Journal of Marketing 73 (6), 214–226.

Chintagunta, P. K., Vilcassim, N. J., 1992. An empirical investigation of advertising strategies in a dynamic duopoly. Management science 38 (9), 1230–1244.

Cleeren, K., 2015. Using advertising and price to mitigate losses in a product-harm crisis. Business Horizons 58 (2), 157–162.

Cleeren, K., Dekimpe, M. G., Helsen, K., 2008. Weathering product-harm crises. Journal of the Academy of Marketing Science 36 (2), 262–270.

Cleeren, K., Dekimpe, M. G., van Heerde, H. J., 2017. Marketing research on product-harm crises: a review, managerial implications, and an agenda for future research. Journal of the Academy of Marketing Science 45 (5), 593–615.

Cleeren, K., Van Heerde, H. J., Dekimpe, M. G., 2013. Rising from the ashes: How brands and categories can overcome product-harm crises. Journal of Marketing 77 (2), 58–77.

Craig, S. N., Thomas, R. J., 1996. A strategic approach to managing product recalls. Harvard Business Review 74 (5), 102–112.

Dawar, N., Pillutla, M. M., 2000. Impact of product-harm crises on brand equity: The moderating role of consumer expectations. Journal of marketing research 37 (2), 215–226.

Dockner, E. J., Jorgensen, S., Van Long, N., Sorger, G., 2000. Differential games in economics and management science. Cambridge University Press.

Eilert, A. M., 2013. Brand reputation and product recall(doctoral dissertation) retrieved from http://scholarcommons.sc.edu/etd/ 453.

Eilert, M., Jayachandran, S., Kalaignanam, K., Swartz, T. A., 2017. Does it pay to recall your product early? an empirical investigation in the automobile industry. Journal of Marketing 81 (3), 111–129.

Fenech, S., 2017. Samsung kicks off ad campaign to regain customer trust after Note7 recall. http://www.techguide.com.au/news/samsung-kicks-off-ad-campaign-regain-customer-trust-note7-recall/, [Online; accessed 11-February-2019].

Friedman, H. H., Friedman, L., 1976. Advertising myopia. Journal of Advertising 5 (2), 29–31.

Gao, H., Xie, J., Wang, Q., Wilbur, K. C., 2015. Should ad spending increase or decrease before a recall announcement? the marketing–finance interface in product-harm crisis management. Journal of Marketing 79 (5), 80–99.

Govindaraj, S., Jaggi, B., Lin, B., 2004. Market overreaction to product recall revisited—the case of firestone tires and the ford explorer. Review of Quantitative Finance and Accounting 23 (1), 31–54.

Haurie, A., Moresino, F., 2006. A stochastic control model of economic growth with environmental disaster prevention. Automatica 42 (8), 1417–1428.

Hora, M., Bapuji, H., Roth, A. V., 2011. Safety hazard and time to recall: The role of recall strategy, product defect type, and supply chain player in the us toy industry. Journal of Operations Management 29 (7-8), 766–777.

Kalaignanam, K., Kushwaha, T., Eilert, M., 2013. The impact of product recalls on future product reliability and future accidents: Evidence from the automobile industry. Journal of Marketing 77 (2), 41–57.

Kamien, M. I., Schwartz, N. L., 2012. Dynamic optimization: the calculus of variations and optimal control in economics and management. Courier Corporation.

Karray, S., Zaccour, G., 2005. A differential game of advertising for national and store brands. In: Dynamic games: Theory and applications. Springer, pp. 213–229.

Ketchen Jr, D. J., Wowak, K. D., Craighead, C. W., 2014. Resource gaps and resource orchestration shortfalls in supply chain management: The case of product recalls. Journal of Supply Chain Management 50 (3), 6–15.

Kim, C. K., Chung, J. Y., 1997. Brand popularity, country image and market share: an empirical study. Journal of International Business Studies 28 (2), 361–386.

Levitt, T., 2008. Marketing myopia. Harvard Business Press.

Lindsay, M., 1990. Establish brand equity through advertising. Marketing News 24 (2), 16.

Liu, Y., Shankar, V., 2015. The dynamic impact of product-harm crises on brand preference and advertising effectiveness: An empirical analysis of the automobile industry. Management Science 61 (10), 2514–2535.

Meenaghan, T., 1995. The role of advertising in brand image development. Journal of product & brand management 4 (4), 23–34.

Nair, A., Narasimhan, R., 2006. Dynamics of competing with quality-and advertising-based goodwill. European Journal of Operational Research 175 (1), 462–474.

Nerlove, M., Arrow, K. J., 1962. Optimal advertising policy under dynamic conditions. Economica, 129–142.

Prasad, A., Sethi, S. P., 2004. Competitive advertising under uncertainty: A stochastic differential game approach. Journal of Optimization Theory and Applications 123 (1), 163–185.

Reuters, 2016. How Samsung Has Tried to Minimize Damage After the Galaxy Note 7 Recall. http://fortune.com/2016/10/06/samsung-smartphone-recall-damage/, [Online; accessed 11-February-2019].

Rubel, O., 2018. Profiting from product-harm crises in competitive markets. European Journal of Operational Research 265 (1), 219–227.

Rubel, O., Naik, P. A., Srinivasan, S., 2011. Optimal advertising when envisioning a product-harm crisis. Marketing Science 30 (6), 1048–1065.

Sethi, S. P., 1983. Deterministic and stochastic optimization of a dynamic advertising model. Optimal Control Applications and Methods 4 (2), 179–184.

Sharma, M., 2015. Marketing myopia-a mousetrap that marketer should avoid. International Research journal of Management Sociology Humanities 6 (4), 488–491.

Taboubi, S., Zaccour, G., 2002. Impact of retailer's myopia on channel's strategies. In: Optimal Control and Differential Games. Springer, pp. 179–192.

Tajitsu, N., 2017. How Samsung Has Tried to Minimize Damage After the Galaxy Note 7 Recall. https://www.reuters.com/article/us-takata-bankruptcy-japan/japanese-airbag-maker-takata-files-for-bankruptcy-gets-chinese-backing-idUSKBN19G0Z [Online; accessed 11-February-2019].

Van Heerde, H., Helsen, K., Dekimpe, M. G., 2007. The impact of a product-harm crisis on marketing effectiveness. Marketing Science 26 (2), 230–245.

Walsh, F., 2006. Salmonella outbreak costs Cadbury £20m. https://www.theguardian.com/business/2006/aug/03/food.foodanddrink, [Online; accessed 11-March-2019].

Wowak, K. D., Boone, C. A., 2015. So many recalls, so little research: a review of the literature and road map for future research. Journal of Supply Chain Management 51 (4), 54–72.

Zu, Y., Chen, L., 2017. Myopic versus far-sighted behaviors in dynamic supply chain coordination through advertising with reference price effect. Discrete Dynamics in Nature and Society 2017.