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Flexible Two-Echelon Location Routing[ł]

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Abstract. This paper deals with an integrated routing problem in which a supplier delivers a commodity to its customers through a two-echelon supply network. Over a planning horizon, the commodity is first sent from a single depot to a set of Distribution Centers (DCs). Then, from the DCs, it is delivered to customers. Two sources of flexibility are analyzed: flexibility in network design and flexibility in due dates. The former is related to the possibility of renting any of the DCs in any period of the planning horizon, whereas the latter is related to the possibility of serving a customer between the period an order is set and a due date. The objective is to minimize the total cost consisting of the sum of the shipping cost from the depot to the DCs, the traveling cost from the DCs to the customers, the renting cost of DCs, and the penalty cost for unmet due dates. A mathematical programming formulation is presented, together with different classes of valid inequalities. Moreover, an exact method is proposed that is based on the interplay between two branch-and-bound algorithms. Computational results on randomly generated instances show the value of each of the two kinds of flexibility. Their combination leads to average savings of up to about 30%.

Keywords. Multi-depot vehicle routing, location routing, due dates, integrated logistics, flexibility.

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1. Introduction

The recent literature on Vehicle Routing Problems (VRPs) is evolving towards the study of ever more complex problems. This complexity stems from different sources, among which integration and flexibility are the most investigated ones.

Integrated VRPs include a broader set of decisions with respect to the pure routing ones. A classical example of integrated routing problems is the Inventory Routing Problem (IRP) where routing is integrated with inventory management (see Bertazzi and Speranza (2012, 2013); Coelho et al. (2013) for tutorials and surveys, and Archetti and Speranza (2016) for a study on the value of integration in IRPs). Other important examples are problems in which network design issues are integrated with routing decisions, such as in location routing problems (see Prodhon and Prins (2014) and Drexl and Schneider (2015) for recent surveys) and in two-echelon VRPs (see Cuda et al. (2015) for a recent survey and Guastaroba et al. (2016) for a more general survey on transportation problems with intermediate facilities). More classical integrated routing problems are the multi-depot vehicle routing problems (see Renaud et al. (1996); Cordeau et al. (1997); Lahyani et al. (2018)).

Flexibility is related to the possibility of relaxing some constraints in known problem settings. In VRPs defined over a planning horizon, the most studied kind of flexibility is on the time customers have to be served. In the pioneering paper by Francis et al. (2006) an extension of the Periodic VRP is studied where the service frequency becomes a decision of the model. It is shown that adding service choice can improve system efficiency and customer service. More recently, the savings that can be achieved by considering flexibility in the service time to customers have been analyzed in Archetti et al. (2015) and Darvish and Coelho (2018). Flexibility on the service time and quantity to be delivered to customers is studied in Archetti et al. (2017). Generally, in network design problems, decisions about nodes or arcs are assumed to be permanent, that is not time-dependent. The idea of renting warehouses for a very limited time period and in more flexible way, as opposed to fixed term contracts, is introduced and investigated in Darvish and Coelho (2018). In addition, recently, concepts such as on-demand warehousing have evolved from an idea to a working reality. By using cloud-based distribution platforms, several companies such as FLEXE, Stockspots, and Ware2Go offer on-demand and flexible warehousing services. These companies, in fact, act as a marketplace for unused warehouse spaces available in hundred of companies around the world (Cordón et al., 2015; Van der Heide et al., 2018).

In this paper a two-echelon distribution problem is studied which considers flexibility on the service time to customers and on the intermediate node locations. A supplier has to find a distribution plan to serve its customers through a two-echelon distribution network. A commodity is produced at a production plant, or stocked at the depot, and is distributed to a set of distribution centers (DCs). From the DCs the commodity is delivered to customers. A discrete planning horizon is considered and we refer to each period as a day. The supplier has the possibility to choose which DCs to use on a daily basis. In fact, the supplier is allowed to rent space in physical facilities managed by a third party. A single vehicle is available at each DC to perform the distribution to customers. Vehicles at DCs are homogeneous, i.e., they have the same capacity. Customers may place orders everyday. Each order has to be entirely fulfilled in a single delivery and is associated with a due date, which represents the latest delivery date. A penalty is defined for each order which is not satisfied within its due date. Products are shipped from the depot to the selected DCs with direct trips (visiting one DC only), and from DCs to customers via routes that possibly visit many customers. The supplier has to take four simultaneous decisions: which DCs to use in each day, when to satisfy the orders of customers, from which of the selected DCs to ship to the customers, and how to create vehicle routes from the selected DCs to the customers. The objective is to minimize the total cost consisting of the sum of the shipping cost from the depot to the DCs, the routing cost from the DCs to the customers, the renting cost of DCs, and the penalty cost for unmet due dates. We call this problem the Flexible Two-Echelon Location Routing Problem (FLRP-2E).

In this paper we aim at highlighting the advantages of two sources of flexibility:

- the possibility of selecting among the available DCs on a daily basis;
- the possibility of selecting the day when customer orders are satisfied, considering that either the due date is respected or a penalty is paid.

From the academic point of view, the FLRP-2E is an integrated problem that, as such, is related to several well-known problems, including location routing, inventory routing, and multi-depot VRPs. The FLRP-2E is an extension of the models presented in Archetti et al. (2015) and Darvish and Coelho (2018). Archetti et al. (2015) study the multi-period VRP with due dates. In this paper the problem is extended by adding intermediate facilities (DCs) where goods are stored and by considering the possibility of choosing among several DCs on a daily basis.

Darvish and Coelho (2018) study a multi-echelon lot sizing-distribution problem considering both delivery time windows and facility location decisions. A key difference between the FLRP-2E and that problem is the use of vehicle routes to manage the distribution to customers instead of direct shipments, which significantly enriches the problem setting investigated by Darvish and Coelho (2018). Moreover, classical location routing problems address a single echelon (Nagy and Salhi, 2007); extensions have been studied considering two echelons as well, e.g., the GRASP algorithm of Nguyen et al. (2012), the tabu search of Boccia et al. (2010), or the branch-and-cut and ALNS of Contardo et al. (2012).

The contributions of this paper are summarized as follows. (i) the FLRP-2E is introduced as a new integrated routing problem; (ii) a mathematical programming formulation is presented along with different classes of valid inequalities (iii) an exact method is proposed that is based on the interplay between two branch-and-bound algorithms that run in parallel; and finally, (iv) a large set of experiments is run on randomly generated instances to show the value of flexibility, in terms of due dates, of network design, and of their combination.

The results highlight the cost savings of both kinds of flexibility. In particular, it is shown that the combination of the two kinds of flexibility leads to a saving in total cost of up to about 30%. Computational and business insights based on this analysis are also provided.

The remainder of the paper is organized as follows. In Section 2 the problem is described and a mathematical formulation is presented together with different classes of valid inequalities. In Section 3 the exact method is proposed. The results of the computational experiments are presented in Section 4. Finally, some conclusions are drawn in Section 5.

2. Problem description and formulation

In this section the FLRP-2E is formally presented, a mathematical programming formulation is given and several classes of valid inequalities are proposed.

2.1. Problem description

In the FLRP-2E a supplier delivers a commodity to its customers through a two-echelon supply chain which consists of the supplier depot from which the commodity is shipped to a set of DCs. The supplier selects on a daily basis which DCs to use to distribute the commodity to its customers. The DCs are replenished by direct shipments from the depot. The commodity is then distributed from the DCs to the customers via routing. Without loss of generality, each day all DCs are available to be rented for a fee. Each customer may place an order in each day and a due date is associated with each of them. Each order must be satisfied within its due date, otherwise it is subject to a penalty to be paid per unit and per day of delay. For example, if an order is received on day 1 and a due date of 1 day is promised, the order could be delivered on day 1 or on day 2, but if it is delivered on day 3, then a penalty is due.

Let $\mathcal{T}$ indicate the discretized planning horizon, say in days. Let $\mathcal{C}$ represent the set of customers and $\mathcal{D}$ the set of DCs, each with a single vehicle available for the distribution. Customer $c \in \mathcal{C}$ orders a quantity d_c^t in day $t \in \mathcal{T}$. Any order can be fulfilled from any of the DCs within r days from the day the order is placed. Thus, an order placed at t has a due date $t + r$. Late orders are not lost but when fulfilled after the due date, a unitary penalty cost π is charged per day of delay. No order can be satisfied in advance, that is before it is placed. Let f_d be the daily fee for renting DC d, which covers the handling and inventory holding costs and the fixed cost of the vehicle that will be used for the distribution to the customers. If the same DC is rented for two or more consecutive days, it can hold inventory from one day to another up to a capacity C_d, $d \in \mathcal{D}$. When a DC is not rented in a given day, any remaining inventory is lost.

Each DC has a vehicle with capacity Q. The vehicle may visit several customers per day in a single trip, starting and ending at the same DC. An order must be satisfied with a single shipment. Different orders from the same customer in different days may be either bundled together or shipped separately from the same or different DCs, and/or in different days. Transportation costs are accounted for as follows. Each unit shipped from the depot to DC d costs s_d and is transported through vehicles of capacity W. Vehicle routes from DCs to customers incur a cost which is based on the distance traveled. A distance matrix c_{ij} is known, $i, j \in \mathcal{C} \cup \mathcal{D}$, where c_{ij} is the cost of routing from location i to location j. No transshipment between DCs is allowed, i.e., goods stored at a DC are distributed to customers only.

The objective of the FLRP-2E is to minimize the total cost of distribution, consisting of the sum of the shipping cost from the depot to the DCs, the traveling cost from the DCs to the customers, the renting cost of DCs, and the penalty cost for unmet due dates.

An example for the FLRP-2E is provided in Figure 1. The figure focuses on the second echelon of the distribution, i.e., from DCs to customers. There are 3 DCs denoted as α, β and γ, and

5 customers, denoted from A to E. The value of r is 2, that is each customer request can be satisfied within two days from the day in which it is placed. Customer A has a request of 5 units in day 1, 1 unit in day 2 and 5 units in day 3. Customer B requests 4 units in day 1, 1 unit in day 2 and 3 units in day 3. Customer C requests 3 units in day 1, 5 units in day 2 and 2 units in day 3. Customer D requests 4 units in day 1, 1 unit in day 2 and 4 units in day 3. Customer E requests 3 units in day 1 and 4 units in day 2. The capacity Q of the vehicles distributing goods from DCs is 15. The solution is such that nothing is delivered at day 1, so no DC is open. At day 2, DC γ is open and one route is performed delivering 6 units to A, 8 units to C and 1 unit to D. At day 3, DC γ is still open delivering 2 units to C, 8 to B and 5 to A. Moreover, DC β is open at day 3 with one route delivering 7 units to E and 8 units to D.

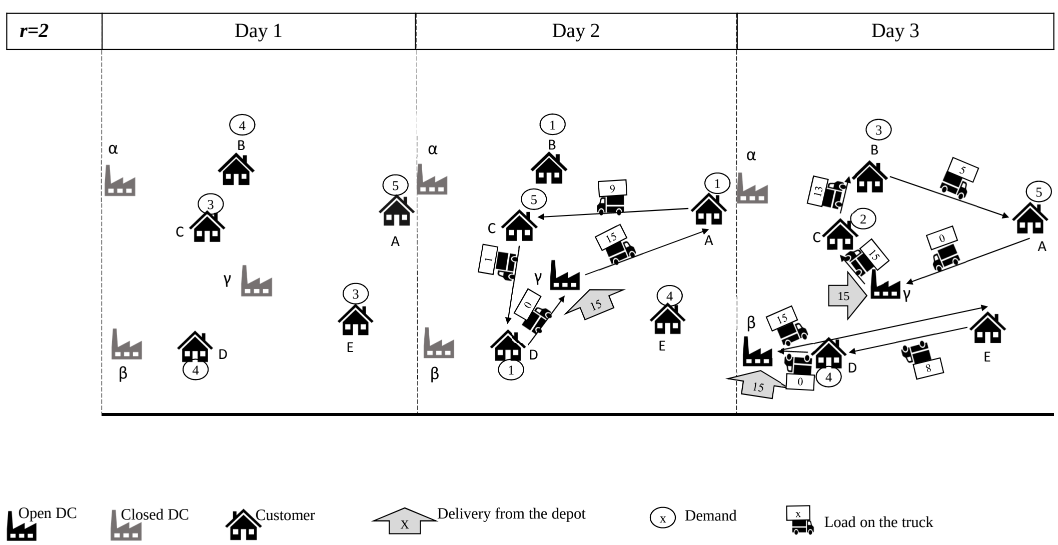

Figure 1: An example of the FLRP-2E.

2.2. Problem formulation

The proposed mathematical programming formulation for the FLRP-2E extends a commodity flow formulation initially proposed by Garvin et al. (1957) and extensively used in Salhi et al. (2014); Koç et al. (2016) and Lahyani et al. (2018).

The commodity flow formulation for the FLRP-2E makes use of the following variables:

- binary variables x_{ij}^{dt} indicate whether a vehicle from DC d traverses arc (i, j) in day t;
- binary variables y_i^{dt} take value 1 if and only if a vehicle from DC d visits node i in day t;
- continuous variables z_{ij}^{dt} represent the remaining load on the vehicle from depot d when traversing arc (i, j) in day t, i.e., after visiting node i and before visiting node j;
- continuous variables q_{id}^t indicate the quantity delivered to customer i from DC d in day t;
- continuous variables S_i^t represent the amount of goods backlogged for customer i in day t;
- binary variables w_d^t take value 1 if DC d is rented in day t;
- continuous variables I_d^t represent the amount of inventory in DC d in day t;
- continuous variables g_d^t represents the quantity shipped to DC d in day t;
- binary variables α_i^{tp} indicate whether the order of customer i in day t is satisfied in day p. These will be used to ensure that the delivery will not be split over several days.

The FLRP-2E is formulated as follows:

$$\text{minimize} \sum_{t\in\mathcal{T}}\sum_{d\in\mathcal{D}}\left(f_d w_d^t + s_d g_d^t + \sum_{i\in\mathcal{D}\cup\mathcal{C}}\sum_{j\in\mathcal{D}\cup\mathcal{C}} c_{ij}x_{ij}^{dt}\right) + \sum_{t=1}^{|\mathcal{T}|+1}\sum_{i\in\mathcal{C}}\pi S_i^t \tag{1}$$

subject to

$$\sum_{d\in\mathcal{D}} y_i^{dt} \leq 1 \quad i\in\mathcal{C}, t\in\mathcal{T} \tag{2}$$

$$y_i^{dt} \leq y_d^{dt} \quad i\in\mathcal{C}, d\in\mathcal{D}, t\in\mathcal{T} \tag{3}$$

$$\sum_{j\in\mathcal{D}\cup\mathcal{C}} x_{ij}^{dt} + \sum_{j\in\mathcal{D}\cup\mathcal{C}} x_{ji}^{dt} = 2y_i^{dt} \quad i\in\mathcal{D}\cup\mathcal{C}, i\neq j, d\in\mathcal{D}, t\in\mathcal{T} \tag{4}$$

$$\sum_{j\in\mathcal{D}\cup\mathcal{C}} x_{ij}^{dt} = \sum_{j\in\mathcal{D}\cup\mathcal{C}} x_{ji}^{dt} \quad i\in\mathcal{C}, i\neq j, d\in\mathcal{D}, t\in\mathcal{T} \tag{5}$$

$$y_d^{dt} \leq \sum_{i,j\in\mathcal{D}\cup\mathcal{C}, i\neq j} x_{ij}^{dt} \quad d\in\mathcal{D}, t\in\mathcal{T} \tag{6}$$

$$x_{ij}^{dt} = 0 \quad i\in\mathcal{C}, j\in\mathcal{D}, d\in\mathcal{D}, j\neq d, t\in\mathcal{T} \tag{7}$$

$$x_{ij}^{dt} = 0 \quad i \in \mathcal{D}, j \in \mathcal{C}, d \in \mathcal{D}, i \neq d, t \in \mathcal{T} \tag{8}$$

$$\sum_{i,j \in \mathcal{D} \cup \mathcal{C}} \left(z_{ji}^{dt} - z_{ij}^{dt} \right) = q_{id}^{t} \quad i \in \mathcal{C}, d \in \mathcal{D}, t \in \mathcal{T} \tag{9}$$

$$\sum_{i \in \mathcal{C}} z_{di}^{dt} = \sum_{i \in \mathcal{C}} q_{id}^{t} \quad d \in \mathcal{D}, t \in \mathcal{T} \tag{10}$$

$$z_{ij}^{dt} \leq Q x_{ij}^{dt} \quad i, j \in \mathcal{D} \cup \mathcal{C}, i \neq j, d \in \mathcal{D}, t \in \mathcal{T} \tag{11}$$

$$q_{id}^{t} \leq Q y_{id}^{t} \quad i \in \mathcal{C}, d \in \mathcal{D}, t \in \mathcal{T} \tag{12}$$

$$\sum_{d \in \mathcal{D}} \sum_{t' \leq t} q_{id}^{t'} \leq \sum_{t' \leq t} d_i^{t'} \quad i \in \mathcal{C}, t \in \mathcal{T} \tag{13}$$

$$S_i^{t+1} \geq \sum_{t' \leq t} d_i^{t'} - \sum_{d \in \mathcal{D}} \sum_{\substack{t' \leq t+r \\ t' \in \mathcal{T}}} q_{id}^{t'} \quad i \in \mathcal{C}, t \in \mathcal{T} \tag{14}$$

$$\sum_{i \in \mathcal{C}} S_i^1 = 0 \tag{15}$$

$$\sum_{p \geq t}^{|\mathcal{T}|+1} \alpha_i^{tp} = 1 \quad i \in \mathcal{C}, t \in \mathcal{T} \tag{16}$$

$$\sum_{d \in \mathcal{D}} q_{id}^{p} = \sum_{\substack{t \leq p \\ t \in \mathcal{T}}} \alpha_i^{tp} d_i^t \quad i \in \mathcal{C}, p \in \mathcal{T} \tag{17}$$

$$y_i^{dt} \leq w_d^t \quad i \in \mathcal{C} \cup \mathcal{D}, d \in \mathcal{D}, t \in \mathcal{T} \tag{18}$$

$$I_d^t \leq C_d w_d^t \quad d \in \mathcal{D}, t \in \mathcal{T} \tag{19}$$

$$I_d^t = I_d^{t-1} + g_d^t - \sum_{i \in \mathcal{C}} q_{id}^t \quad d \in \mathcal{D}, \quad t \in \mathcal{T} \setminus \{1\} \tag{20}$$

$$I_d^1 = g_d^1 - \sum_{i \in \mathcal{C}} q_{id}^1 \quad d \in \mathcal{D} \tag{21}$$

$$g_d^t \leq W w_d^t \quad d \in \mathcal{D}, t \in \mathcal{T} \tag{22}$$

$$x_{ij}^{dt} = 0 \quad i, j, d \in \mathcal{D}, t \in \mathcal{T} \tag{23}$$

$$y_i^{dt}, x_{ij}^{dt}, w_d^t \in \{0, 1\} \quad i, j \in \mathcal{D} \cup \mathcal{C}, d \in \mathcal{D}, t \in \mathcal{T} \tag{24}$$

$$\alpha_i^{tp} \in \{0, 1\} \quad i \in \mathcal{C}, t, p \in \mathcal{T} \tag{25}$$

$$I_d^t, z_{ij}^{dt}, g_d^t \in \mathbb{Z}^+ \quad i, j \in \mathcal{D} \cup \mathcal{C}, d \in \mathcal{D}, t \in \mathcal{T} \tag{26}$$

$$S_i^t, q_{id}^t \in \mathbb{Z}^+ \quad i \in \mathcal{C}, d \in \mathcal{D}, t \in \mathcal{T}. \tag{27}$$

The objective function (1) minimizes the total cost composed of the fixed renting cost of the DCs, shipping cost to the DCs, traveling cost to the customers, and the late delivery penalties.

Constraints (2) impose that a customer is visited at most once per day, and constraints (3) ensure that customers are visited only from the rented DCs. Constraints (4) and (5) are degree constraints. Constraints (6) link the visit to a depot with the arcs leaving from the depot. Constraints (7) and (8) forbid a vehicle to start a route from a DC and finish at another. Constraints (9) ensure the connectivity of a route, while constraints (10) guarantee that the quantity loaded on vehicles from all DCs is delivered to customers in the same day. Constraints (11) impose a bound on the z variables and ensure that vehicle capacities are respected. Constraints (12) link the delivery quantities with the DC used for delivery to that customer. Constraints (13) impose that no order can be satisfied in advance. Constraints (14) and (15) determine the amount of stockout. Constraints (16) and (17) ensure that the order of each customer is satisfied exactly once. Constraints (18) allow routes to start only from rented DCs, while constraints (19) impose capacity constraints on the selected DCs. Constraints (20) set the inventory level at each DC and constraints (21) are the special case of inventory balance equations for the first period. Constraints (22) guarantee that only rented DCs receive deliveries from the depot and the delivery respects the transportation capacity. Constraints (23) forbid vehicles to travel between DCs. Constraints (24)–(27) define the nature and bounds of the variables.

2.3. Valid inequalities

The proposed classes of valid inequalities are aimed at strengthening the formulation (1)–(27).

Valid inequalities (28) impose that the vehicles return empty to the DCs, breaking symmetries in the solutions that differ only in the quantity loaded:

$$z_{id}^{dt} = 0 \quad i \in \mathcal{C}, d \in \mathcal{D}, t \in \mathcal{T}. \tag{28}$$

Valid inequalities (29)–(30) forbid to transport a quantity of goods from one DC to another:

$$z_{ih}^{dt} = 0 \quad i \in \mathcal{C}, d, h \in \mathcal{D}, h \neq d, t \in \mathcal{T}, \tag{29}$$

$$z_{hi}^{dt} = 0 \quad i \in \mathcal{C}, d, h \in \mathcal{D}, h \neq d, t \in \mathcal{T}. \tag{30}$$

Valid inequalities (31)–(32) forbid links between a node and itself:

$$z_{ii}^{dt} = 0 \quad i \in \mathcal{D} \cup \mathcal{C}, d \in \mathcal{D}, t \in \mathcal{T}, \tag{31}$$

$$x_{ii}^{dt} = 0 \quad i \in \mathcal{C} \cup \mathcal{D}, d \in \mathcal{D}, t \in \mathcal{T}. \tag{32}$$

Valid inequalities (33) and (34) strengthen the link between routing and visiting variables:

$$x_{id}^{dt} \leq y_i^{dt} \quad i \in \mathcal{C}, d \in \mathcal{D}, t \in \mathcal{T}, \tag{33}$$

$$x_{di}^{dt} \leq y_i^{dt} \quad i \in \mathcal{C}, d \in \mathcal{D}, t \in \mathcal{T}. \tag{34}$$

Valid inequalities (35) exclude infeasible vehicle routes that visit customers assigned to two different DCs, and (36) are two-cycle elimination constraints:

$$x_{ij}^{dt} + y_i^{dt} + \sum_{\substack{h \neq d \\ h \in \mathcal{D}}} y_j^{ht} \leq 2 \quad i, j \in \mathcal{C}, i \neq j, d \in \mathcal{D}, t \in \mathcal{T}, \tag{35}$$

$$x_{ij}^{dt} + x_{ji}^{dt} \leq 1 \quad i, j \in \mathcal{C}, d \in \mathcal{D}, t \in \mathcal{T}. \tag{36}$$

Valid inequalities (37) state that total deliveries to all customers from all DCs up to day t' should not exceed the total capacities of all vehicles used during the t' days:

$$\sum_{i \in \mathcal{C}} \sum_{\substack{t' \leq t \\ t' \in \mathcal{T}}} \sum_{d \in \mathcal{D}} q_{id}^{t'} \leq \sum_{d \in \mathcal{D}} \sum_{\substack{t' \leq t \\ t' \in \mathcal{T}}} Q w_d^{t'} \quad t \in \mathcal{T}. \tag{37}$$

Valid inequalities (38) establish that the backlog has to be at least equal to the amount of exceeding demand with respect to the capacity of the vehicles used, while valid inequalities (39) state that the quantity delivered from a DC is bounded by the vehicle capacity multiplied by the number of days in which the DC is used:

$$\sum_{i \in \mathcal{C}} \sum_{t'=t}^{|\mathcal{T}|+1} S_i^{t'} \geq \sum_{t'=t}^{|\mathcal{T}|} \left(\sum_{i \in \mathcal{C}} d_i^{t'} - \sum_{d \in \mathcal{D}} Q w_d^{t'} \right), \quad t \in \mathcal{T}, \tag{38}$$

$$\sum_{i \in \mathcal{C}} \sum_{t'=t_1}^{t_2} q_{id}^{t'} \leq \sum_{t'=t_1}^{t_2} Q w_d^{t'}, \quad d \in \mathcal{D}, t_1, t_2 \in \mathcal{T}, t_1 \leq t_2. \tag{39}$$

Valid inequalities (40) impose that one route can start and end at a DC in each day, if the DC is rented:

$$2 y_d^{dt} \leq \sum_{\substack{j \in \mathcal{C} \\ j \neq d}} x_{dj}^{dt} + \sum_{\substack{j \in \mathcal{C} \\ j \neq d}} x_{jd}^{dt} \quad d \in \mathcal{D}, t \in \mathcal{T}. \tag{40}$$

Valid inequalities (41) and (42) link the quantities delivered to the customers with the visit:

$$y_i^{dt} \leq q_i^{dt} \quad i \in \mathcal{C}, d \in \mathcal{D}, t \in \mathcal{T}, \tag{41}$$

$$\frac{q_i^{dt}}{Q} \leq y_i^{dt} \quad i \in \mathcal{C}, d \in \mathcal{D}, t \in \mathcal{T}. \tag{42}$$

Valid inequalities (43) state that deliveries made from a DC in a day are less than the load of the vehicle from that DC to all customers:

$$\sum_{i \in \mathcal{C}} q_{id}^t \leq \sum_{i \in \mathcal{C}} z_{di}^{dt} \quad d \in \mathcal{D}, t \in \mathcal{T}. \tag{43}$$

Valid inequalities (44) are used to strengthen the connectivity of a route:

$$\sum_{j \in \mathcal{D} \cup \mathcal{C}} z_{ij}^{dt} \leq \sum_{j \in \mathcal{D} \cup \mathcal{C}} z_{ji}^{dt} - \sum_{d \in \mathcal{D}} q_{id}^t \quad i \in \mathcal{C}, d \in \mathcal{D}, t \in \mathcal{T}. \tag{44}$$

Finally, valid inequalities (45) strengthen the link between using an arc and the quantity delivered via that arc:

$$x_{ij}^{dt} \leq z_{ij}^{dt} \quad i, j, i \neq j \in \mathcal{C}, d \in \mathcal{D}, t \in \mathcal{T}. \tag{45}$$

3. Solution method

In this section an exact method is proposed which is based on the interplay between two branch-and-bound algorithms that run in parallel. We refer to it as Enhanced Parallel Exact Method (*EPEM*).

The main idea of the *EPEM* is that the formulation presented in Section 2.2, enhanced with the valid inequalities of Section 2.3, is solved by two different solution algorithms that run in parallel and exchange information. One of the two algorithms is a plain branch-and-bound, whereas the other is a Variable Mixed-integer programming Neighborhood Descent (VMND) algorithm. The VMND was proposed in Larrain et al. (2017) and then applied to other integrated routing problems (e.g., Larrain et al. (2017, 2018); Darvish et al. (2018); Deluster et al. (2018)). The algorithms exchange information every θ seconds. More precisely, if the plain branch-and-bound algorithm has found an improving solution in the last θ seconds, this solution is passed to the VMND. Note that the reverse flow of information is not performed, i.e., when the VMND finds an improving solution this is not passed to the plain branch-and-bound. This is due to the fact that some preliminary experiments showed better results in the version of the algorithm with a unilateral flow of information (from the plain branch-and-bound algorithm to the VMND).

The *EPEM* stops when either algorithm proves optimality or a time limit of μ is reached.

In what follows, we focus on the description of the VMND. We refer to the formulation strengthened with the valid inequalities, that is (1)-(45), as *strengthened formulation*.

3.1. The VMND algorithm

The VMND is a branch-and-bound algorithm embedding a local search heuristic to accelerate the search of high quality solutions. The main idea is as follows. The branch-and-bound algorithm is run on the strengthened formulation. Every time it finds a new incumbent solution, the local search heuristic is started. The heuristic is based on a guided fix-and-optimize procedure (see, for example, Neves-Moreira et al. (2018)) which takes as input the formulation and the incumbent solution, fixes the value of subsets of variables and optimizes over the remaining variables.

In the following we focus on the local search heuristic by describing the neighborhoods and the way in which they are explored.

Five neighborhoods are proposed. Each neighborhood is defined by the set of variables whose value is fixed to the one they take in the incumbent solution. The remaining variables are free. Once these sets are fixed, the corresponding solution is found by solving the strengthened formulation. Clearly, the higher is the number of the fixed variables, the easier is to solve the formulation. However, fixing many variables may prevent finding high-quality neighboring solutions. The neighborhoods we designed for the FLRP-2E are the following:

1. *DCs:* consider a DC $d \in \mathcal{D}$. Then, all routes starting from a DC different from d are fixed to the ones in the incumbent solution, i.e., the value of variables $x_{ij}^{d't}$ is fixed to the corresponding value in the incumbent solution for all $d' \in D, d' \neq d$, while the value of variables x_{ij}^{dt} is free. The neighborhood considers all DCs sequentially.

2. *Two days:* consider two days t and t', $t, t' \in \mathcal{T}, t \neq t'$. Then, all routes in days different from t, t' are fixed to the ones in the incumbent solution, i.e., the value of variables $x_{ij}^{dt''}$ is fixed as the corresponding value in the incumbent solution for all $t'' \in \mathcal{T}, t'' \neq t, t'$, while the value of variables x_{ij}^{dt} and $x_{ij}^{dt'}$ is free. The neighborhood considers all pairs of days.

3. *γ-closest neighbors:* consider a customer $i \in \mathcal{C}$ and its $\gamma - 1$ closest neighbors (distance-wise). Let Γ_i be the set composed by i and its $\gamma - 1$ closest neighbors. Then, the value of variables $x_{jj'}^{d't}$ is fixed to the corresponding value in the incumbent solution for all

$j, j' \in \mathcal{C} \backslash \Gamma_i$, while the value of variables $x_{jj'}^{dt}$ is free for $j \in \Gamma_i$ or $j' \in \Gamma_i$. The neighborhood considers all customers sequentially.

4. *Distance:* given a parameter τ, the value of variables x_{ij}^{dt} is fixed to the corresponding value in the incumbent solution for all $t \in \mathcal{T}, d \in D, i, j \in \mathcal{C}$ such that the distance between i and j is at most τ, while the value of the remaining x variables is free.

5. *One day:* similar to the two-days neighborhood with the difference that the routes of one day only at a time remain free. The neighborhood considers all days sequentially.

We noticed that the order in which we apply the local search operators has an effect on the quality of the solution obtained. After some preliminary experiments, the operators have been applied in the order of the above presentation.

Each neighborhood is applied for a maximum time of β seconds. However, as soon as a better solution is found, the exploration stops and, at the next iteration, the list of neighborhoods is explored from the beginning. The local search heuristic terminates once all neighborhoods have been explored without finding an improved solution.

4. Computational experiments

In this section we present the computational tests and results. The section is organized as follows. First, a description of the generated test instances is provided in Section 4.1 and the choice of the parameters of the solution method is described in Section 4.2. In Section 4.3 the performance of the *EPEM* is assessed by comparing it with a plain branch-and-bound algorithm. Then, the value of the two kinds of flexibility introduced in the FLRP-2E is assessed. In Section 4.4 the value of the network design flexibility and in Section 4.5 the value of the flexibility in the service times are analyzed, respectively. Finally, in Section 4.6 the effects of the combined kinds of flexibility are studied.

In the experiments we will refer to a *fixed network design* scenario, where an instance is solved imposing that the DCs selected in the first day remain unchanged throughout the planning horizon. Thus, in the fixed network design scenario we have $w_d^t = w_d^1$ for $t \in \mathcal{T}$ and $d \in \mathcal{D}$, i.e., if a DC d is rented or not rented in the first day, it remains rented or not rented for the entire planning horizon. In contrast with the fixed network design, we call *flexible network design* scenario the case where the FLRP-2E is solved. In order to better understand the difference between the fixed and the flexible network design scenario, Figure 2 provides a solution for the fixed network design scenario for the example provided in Figure 1. The figure reports, for the ease of comparison, the solutions for the two scenarios. In the fixed network scenario, DC γ is open at day 1 while α and β are not, and this decision is kept the same for days 2 and 3. The route performed at day 1 delivers 3 units to C, 4 to D, 3 to E and 5 to A. The route at day 2 delivers 5 units to C, 5 to B, 1 to A and 4 to E. The route performed at day 3 delivers 5 units to D, 2 to C, 3 to B and 5 to A.

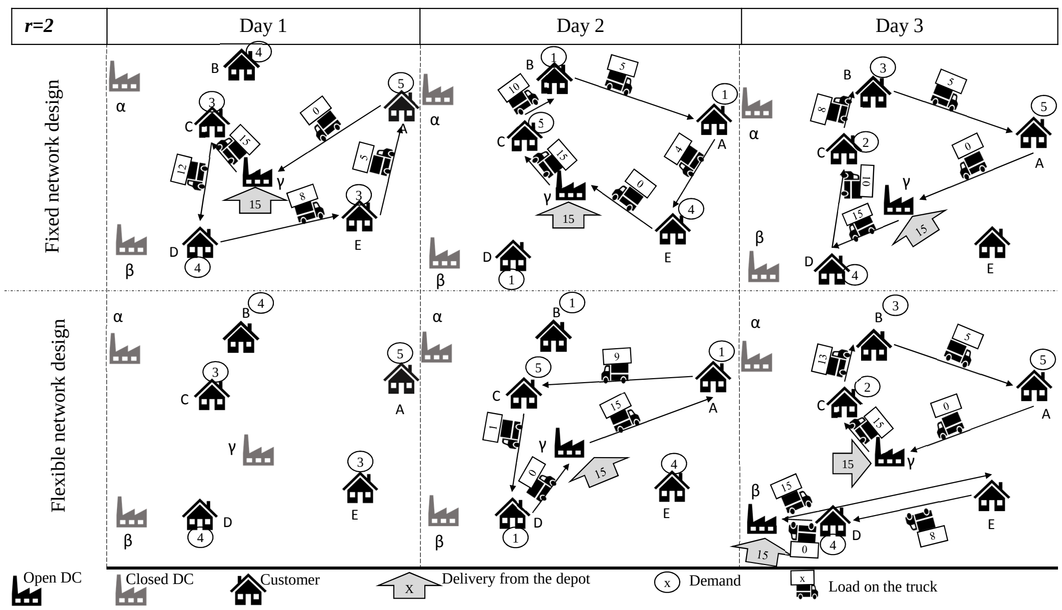

Figure 2: Fixed and flexible network design scenarios.

The strengthened formulation has been solved using CPLEX 12.8 and IBM Concert Technology in C++. No separation of constraints or valid inequalities is needed as they are all in polynomial number. The *EPEM* has been coded in C++ with CPLEX 12.8.

All experiments are conducted on an Intel Core i7 processor running at 3.4 GHz with 64 GB of RAM installed, with the Ubuntu Linux operating system. The maximum execution time is 10,800 seconds.

4.1. Instance generation

Instances for the FLRP-2E are randomly generated using the parameter values specified in Table 1, where the quantity D_{max} is equal to the total demand of the peak day ($\max_t \sum_{i\in\mathcal{C}} d_i^t$) whereas D_{min} is equal to the total demand of the day with the lowest demand ($\min_t \sum_{i\in\mathcal{C}} d_i^t$).

For each combination of number of customers and number of days (8 combinations), we first created instances with one DC only, then we added the second and third DCs in random locations. This process guarantees that the selection of multiple DCs is completely imputable to their convenience and not to different customer and DC locations. In addition, three different values for the capacity of the vehicle performing deliveries from DCs to customers are considered: tight, normal or loose. The capacity of the truck shipping the goods from the depot to the DCs (W) is set to a sufficiently large value so that flexibility in network design and/or service time is fully exploited. The penalty cost π is also set to a high value in order to force deliveries to take place within the due dates, if feasible. Finally, concerning due dates, we consider three cases: no flexibility ($r = 0$, i.e., the customer order has to be satisfied when the order is placed), next day delivery ($r = 1$) or delivery within two days ($r = 2$). For each combination of the above mentioned parameters five instances are generated by randomly choosing the values of C_i, d_i^t, f_i, s_i, X_i and Y_i, as specified in Table 1, for a total of 1,080 instances.

4.2. Parameters setting

The parameters needed to run the *EPEM* are calibrated through a set of preliminary experiments on a subset of instances. Table 2 provides an overview of these parameters and their values.

4.3. Performance of the Enhanced Parallel Exact Method

This section is devoted to assessing the performance of the *EPEM*. The solutions obtained with *EPEM* are compared with the ones from the plain branch-and-bound algorithm. The results are shown in Tables 3 and 4 for the fixed and flexible network design scenarios, respectively. In both tables, values are averaged over: number of customers, number of DCs, value of capacity and value of due dates r. Columns 2-4 refer to the plain branch-and-bound algorithm (*Plain B&B*) and report the average percentage optimality gap at the end of computation, the number of instances solved to optimality and the average computing time in seconds. The following columns refer to the *EPEM*. Columns 5-7 report the same information as columns 2-4. Column

Table 1: Input parameter values

Name	Parameter	Values
Days	$\|\mathcal{T}\|$	$\{3, 6\}$
DCs	D	$\{1, 2, 3\}$
Customers	C	$\{30, 40, 50, 60\}$
Vehicles	K	One per facility
Due dates	r	$\{0, 1, 2\}$
Demands	d_i^t	[0, 5]
DC rental fees	f_i	[100,150]
Shipping costs (plant-DC)	s_i	[1,5]
X coordinates	X_i	[0,100]
Y coordinates	Y_i	[0,100]
Shipping costs (DC-customers)	c_{ij}	$\left\lfloor \sqrt{(X_i - X_j)^2 + (Y_i - Y_j)^2 + 0.5} \right\rfloor$
Penalty cost	p	1,000
Inventory capacities	C_i	$[2, 3] \times D_{max}$
Full truckload capacity	W	D_{max}
Tight vehicle capacity	Q_T	D_{min}
Normal vehicle capacity	Q_N	$\left\lceil \frac{D_{min} + D_{max}}{2} \right\rceil$
Loose vehicle capacity	Q_L	D_{max}

Table 2: EPEM parameters

Name	Parameter	Value
Maximum time	μ	10,800 s
VMND Local search operator maximum time	β	200 s
Parallel exchange time	θ	1,200 s
The distance parameter	π	$\sum_{i,j \in \mathcal{D} \cup \mathcal{C}} (c_{ij} - \max_{c_{ij}})$
Number of closest neighbors	γ	$\frac{\text{Number of customers}}{5}$

8 provides the average percentage difference between the value of the solution (i.e., the value of the upper bound at the end of computation) provided by the *EPEM* and the one provided by the *Plain B&B* algorithm calculated as $(\frac{z_{EPEM}-z_P}{z_{EPEM}})$ where z_{EPEM} is the value of the solution provided by the *EPEM* and z_P is the value of the solution provided by the *Plain B&B* algorithm. The last two columns report the number of times $z_{EPEM} < z_P$ and $z_{EPEM} > z_P$, respectively.

Table 3: Comparison between *Plain B&B* and *EPEM* on fixed network design

	Plain B&B			*EPEM*					
	Av. % opt. gap	# opt.	Av. Time	Av. % opt. gap	# opt.	Av. Time	av. % UB diff.	# better	# worse
n									
30	1.31	155	5525.97	1.19	160	5434.80	-0.25	85	11
40	3.87	103	7252.61	2.52	115	6869.26	-1.72	131	15
50	13.81	90	7807.63	3.43	105	7448.53	-11.88	164	7
60	26.27	59	9040.89	5.16	75	8577.13	-24.70	201	3
# DCs									
1	0.60	206	5265.30	0.25	217	5044.16	-0.35	132	10
2	13.00	115	8097.21	3.29	139	7501.12	-10.89	213	8
3	20.28	86	8847.86	5.67	99	8691.88	-17.63	236	18
Capacity									
Tight	11.62	87	8749.75	2.46	102	8492.12	-10.03	233	17
Normal	10.47	135	7478.78	2.98	151	7090.69	-8.90	189	9
Loose	11.84	185	5999.44	3.78	202	5672.30	-9.98	159	10
r									
0	2.36	234	4745.88	1.16	254	4137.25	-1.94	99	6
1	17.31	91	8626.00	4.18	107	8436.24	-15.13	237	16
2	14.24	82	8837.08	3.88	94	8661.23	-11.83	245	14
All	11.31	407	7406.78	3.07	455	7082.43	-9.64	581	36

The results show that the *EPEM* clearly outperforms the *Plain B&B* algorithm. Indeed, on the fixed network design scenario (Table 3), the average optimality gap decreases from 11.31% to 3.07% and the number of instances solved to optimality increases from 407 to 455. The average computing time of the *EPEM* is lower. It deserves to be highlighted that the *EPEM* performs better than the *Plain B&B* algorithm especially on the large instances. In fact, focusing on the case with $n = 60$, the optimality gap decreases from 26.27% for the *Plain B&B* algorithm to 5.16% for the *EPEM* and the number of instances solved to optimality increases from 59 to 75. Very similar results are obtained for the case of flexible network design (Table 4). Thus, we can conclude that the *EPEM* is capable of providing high-quality solutions for small and (relatively) large size instances. Consequently, the following analysis is based on the results provided by the *EPEM*.

Table 4: Comparison between the *Plain B&B* and *EPEM* on flexible network design

	Plain B&B			*EPEM*					
	Av. % opt. gap	# opt.	Av. Time	Av. % opt. gap	# opt.	Av. Time	av. % UB diff.	# better	# worse
n									
30	1.87	156	5700.40	1.60	158	5524.91	-0.46	83	9
40	4.26	98	7381.87	2.55	106	7069.75	-2.11	142	6
50	12.55	89	7814.64	3.46	102	7371.16	-10.70	154	18
60	23.73	59	9149.36	3.85	85	8289.27	-22.23	187	9
# DCs									
1	0.52	210	5306.92	0.28	218	4946.33	-0.23	124	14
2	12.58	111	8261.87	3.08	133	7660.66	-10.44	208	14
3	18.71	81	8965.92	5.24	100	8584.33	-15.96	234	14
Capacity									
Tight	11.75	91	8650.38	3.02	101	8236.02	-9.84	234	18
Normal	10.51	145	7413.69	2.82	163	6996.97	-8.88	170	12
Loose	9.55	166	6470.64	2.76	187	5958.33	-7.92	162	12
r									
0	1.74	238	4724.90	0.99	259	4082.67	-1.20	84	8
1	16.46	87	8787.75	3.99	103	8464.59	-14.20	237	13
2	13.61	77	9022.05	3.62	89	8644.06	-11.22	245	21
All	10.60	402	7511.57	2.87	451	7063.77	-8.88	566	42

4.4. Flexibility in network design

Tables 5 and 6 present, for the fixed and flexible network design scenarios, respectively, the average costs and optimality gaps over the five instances with the capacity specified in the first column and the number of customers and days specified in the second and third columns. We set $r = 0$ in order to exclude any effect of due dates. The tables compare the results for different numbers of DCs. For each number of DCs the cost of the best solution found and the optimality gap are reported. In addition, for the case where the number of DCs is equal to 2 and 3, we report, in column 'guaranteed savings', the gap between the cost of the solution with the corresponding number of DCs and the lower bound of the solution with 1 DC. Note that, when the instance with 1 DC is solved to optimality, the guaranteed savings become the exact savings obtained. The guaranteed savings are calculated as $100 \times \dfrac{LB - Cost}{LB}$, where $Cost$ is the value reported in column 'Cost' and LB is the lower bound on the solution of the same instance with 1 DC.

Focusing on the fixed network scenario, Table 5 shows that the cost savings become more relevant when the vehicle capacity tightens. While the global average cost for all three vehicle capacity scenarios has a decreasing trend as the number of available DCs increases, the strongest influence of adding extra DCs is observed under the tight capacity scenario. Moreover, the savings are more substantial when moving from 1 to 2 DCs than when moving from 2 to 3 DCs. In fact, the average savings achieved with 2 DCs are 62% while in the case of 3 DCs is 63%. The optimality gap depends on the size of the instance and the capacity, but, in general, when more DCs are available, the problem becomes more difficult to solve. In general, optimality gaps are small with the only exception of instances with 60 customers, 6 days, normal capacity and 3 DCs. Similar considerations can be applied to the case of flexible network design (Table 6), where the average savings are 62% with 2 DCs and 64% with 3 DCs.

Table 7 compares the solutions obtained in the fixed network design scenario with the ones obtained in the flexible network design scenario for the case with 3 DCs. The guaranteed savings are calculated as in Table 5, with LB being equal to the lower bound of the solution obtained in the fixed network design scenario. As the table indicates, the flexible network design always yields lower costs. On average, the flexible network design reduces costs by 3%. The difference between the two scenarios is more significant with the normal vehicle capacity, for which we observe 6% savings in total cost.

4.5. Flexibility in due dates

In this section we assess the value of the flexibility in the due dates. The cost and difficulty in solving the problems are studied when no flexibility is allowed, i.e., $r = 0$, and with increasing flexibility, namely $r = 1$ and $r = 2$. We separate the analysis for fixed and flexible network designs in order to interpret the benefits coming from flexibility in due dates only. Results are presented in Table 8 for the fixed network design and in Table 9 for the flexible network scenario, considering 3 DCs in both cases. In both tables, the guaranteed savings are calculated by comparing the lower bound obtained with $r = 0$ with the solution value for the cases with $r = 1$ and $r = 2$.

In general, moving from $r = 0$ to $r = 1$ reduces the cost but makes the problem more difficult to solve, while the difficulty slightly decreases when moving from $r = 1$ to $r = 2$. For both network design scenarios, larger savings are achieved when changing from the case with no flexibility ($r = 0$) to next day delivery ($r = 1$), rather than from the next day delivery to a two-day delivery ($r = 2$). Comparing Tables 8 and 9, this difference is more significant in the flexible

Table 5: DC availability in the fixed network design with $r = 0$

Instances			# of $DC = 1$		# of $DC = 2$			# of $DC = 3$		
	Days	Customers	Cost	Gap (%)	Cost	Gap (%)	Guaranteed savings (%)	Cost	Gap (%)	Guaranteed savings (%)
Tight	3	30	60524.00	0.00	2488.00	0.00	95.37	2367.80	0.00	95.65
	3	40	59896.80	0.01	2798.20	0.00	93.49	2625.20	0.24	93.74
	3	50	40773.40	0.00	2894.80	0.07	91.94	2842.20	0.19	92.13
	3	60	101150.00	0.01	3306.20	0.30	90.90	3167.20	0.89	91.76
	6	30	303451.60	0.03	4966.60	0.50	98.31	4684.60	2.11	98.39
	6	40	297211.60	0.04	5822.40	0.89	97.76	5664.60	2.79	97.84
	6	50	296076.50	0.03	6406.50	0.70	97.64	6139.00	3.22	97.70
	6	60	210935.80	0.03	6363.20	9.37	95.94	6430.60	11.93	95.91
Average			168051.85	0.02	4328.79	1.50	95.11	4240.15	2.66	95.33
Normal	3	30	25350.00	0.00	2440.60	0.00	88.53	2331.80	0.00	89.12
	3	40	22539.20	0.00	2764.80	0.00	84.62	2592.20	0.00	85.26
	3	50	13205.20	0.00	2849.20	0.00	77.73	2816.80	0.00	78.01
	3	60	29909.80	0.01	3266.40	0.00	78.69	3130.00	0.40	80.34
	6	30	53531.80	0.00	4786.40	0.00	90.51	4519.60	0.46	90.98
	6	40	52986.00	0.04	5670.80	0.47	84.91	5482.60	0.57	85.57
	6	50	46591.20	0.01	6482.20	0.43	80.79	5755.80	2.77	83.05
	6	60	45283.80	0.02	6211.80	0.53	73.01	6296.00	30.63	72.66
Average			36174.63	0.01	4309.03	0.18	82.35	4115.60	4.35	83.12
Loose	3	30	2177.80	0.00	2023.60	0.00	6.73	1916.80	0.00	11.19
	3	40	2739.00	0.00	2303.60	0.00	15.06	2203.20	0.00	18.97
	3	50	2716.00	0.00	2396.80	0.00	9.49	2396.80	0.00	9.49
	3	60	3295.60	0.00	2830.00	0.00	12.62	2706.40	0.00	16.59
	6	30	4579.80	0.00	3913.60	0.00	13.86	3761.00	0.00	16.82
	6	40	4920.20	0.00	4820.40	0.00	1.79	4711.60	0.11	3.75
	6	50	5737.20	0.00	5563.60	0.00	2.54	4972.20	6.26	11.46
	6	60	5549.00	0.00	5374.40	0.00	2.84	5374.60	7.29	2.84
Average			3964.33	0.00	3653.25	0.00	8.12	3505.33	1.71	11.39
Global average			68567.90	0.01	4095.08	0.55	61.58	3953.69	2.91	63.01

Table 6: DC availability in the flexible network design with $r = 0$

Instances			# of $DC = 1$		# of $DC = 2$			# of $DC = 3$		
	Days	Customers	Cost	Gap (%)	Cost	Gap (%)	Guaranteed savings (%)	Cost	Gap (%)	Guaranteed savings (%)
Tight	3	30	60524.00	0.00	2361.20	0.00	95.61	2247.00	0.00	95.87
	3	40	59896.80	0.00	2660.80	0.00	93.82	2506.20	0.00	94.04
	3	50	40773.40	0.00	2760.20	0.00	92.33	2712.00	0.15	92.49
	3	60	101160.60	0.02	3169.60	0.27	91.26	3029.80	0.73	92.11
	6	30	303455.60	0.03	4841.00	0.42	98.35	4567.40	2.41	98.43
	6	40	297199.80	0.04	5699.20	0.97	97.81	5538.80	2.71	97.89
	6	50	270813.00	0.04	6479.00	0.80	97.29	5866.00	14.12	97.56
	6	60	211006.20	0.04	6245.80	2.91	96.02	6328.40	13.52	95.98
Average			168103.68	0.02	4277.10	0.67	95.31	4099.45	4.21	95.55
Normal	3	30	25350.00	0.00	2265.40	0.00	89.41	2161.60	0.00	89.98
	3	40	22539.20	0.00	2601.00	0.00	85.75	2444.80	0.00	86.33
	3	50	13205.20	0.00	2608.00	0.00	79.65	2582.60	0.00	79.87
	3	60	29912.60	0.01	3075.40	0.00	79.81	2944.00	0.91	81.45
	6	30	53531.80	0.00	4358.00	0.00	91.41	4147.40	0.75	91.75
	6	40	52989.60	0.05	5238.80	0.43	86.11	5094.80	0.90	86.67
	6	50	46597.40	0.03	5956.80	0.33	82.47	5306.20	16.97	84.48
	6	60	45293.00	0.04	5771.00	0.89	75.18	5775.40	10.21	75.18
Average			36177.35	0.02	3984.30	0.21	83.72	3807.10	3.72	84.46
Loose	3	30	2177.80	0.00	2023.60	0.00	6.73	1916.80	0.00	11.19
	3	40	2739.00	0.00	2303.60	0.00	15.06	2203.20	0.00	18.97
	3	50	2716.00	0.00	2396.80	0.00	9.49	2396.80	0.00	9.49
	3	60	3295.60	0.00	2830.00	0.00	12.62	2706.40	0.00	16.59
	6	30	4579.80	0.00	3913.60	0.00	13.86	3761.00	0.00	16.82
	6	40	4920.20	0.00	4820.40	0.18	1.79	4711.60	0.11	3.75
	6	50	5737.20	0.00	5563.60	0.00	2.54	4971.80	0.04	11.47
	6	60	5549.00	0.00	5374.40	0.02	2.84	5374.40	0.00	2.84
Average			3964.33	0.00	3653.25	0.02	8.12	3505.25	0.02	11.39
Global average			69415.12	0.01	3971.55	0.30	62.38	3803.93	2.65	63.80

Table 7: Fixed vs. flexible network design with $r = 0$

Instances			Fixed		Flexible		
	Days	Customers	Cost	Gap (%)	Cost	Gap (%)	Guaranteed savings (%)
Tight	3	30	2367.80	0.00	2247.00	0.00	5.11
	3	40	2625.20	0.24	2506.20	0.00	4.35
	3	50	2842.20	0.19	2712.00	0.15	4.41
	3	60	3167.20	0.89	3029.80	0.73	3.49
	6	30	4684.60	2.11	4567.40	2.41	0.89
	6	40	5664.60	2.79	5538.80	2.71	0.28
	6	50	6139.00	3.15	5866.00	14.12	3.16
	6	60	6430.60	11.93	6328.40	13.52	0.00
Average			4240.15	2.66	4099.45	4.21	2.71
Normal	3	30	2331.80	0.00	2161.60	0.00	7.34
	3	40	2592.20	0.00	2444.80	0.00	5.78
	3	50	2816.80	0.00	2582.60	0.00	8.36
	3	60	3130.00	0.40	2944.00	0.91	5.47
	6	30	4519.60	0.46	4147.40	0.75	7.87
	6	40	5482.60	0.57	5094.80	0.90	6.68
	6	50	5755.80	2.77	5306.20	16.97	5.14
	6	60	6296.00	30.63	5775.40	10.21	0.00
Average			4115.60	4.35	3807.10	3.72	5.83
Loose	3	30	1916.80	0.00	1916.80	0.00	0.00
	3	40	2203.20	0.00	2203.20	0.00	0.00
	3	50	2396.80	0.00	2396.80	0.00	0.00
	3	60	2706.40	0.00	2706.40	0.00	0.00
	6	30	3761.00	0.00	3761.00	0.00	0.00
	6	40	4711.60	0.11	4711.60	0.11	0.00
	6	50	4972.20	6.26	4971.80	0.04	0.00
	6	60	5374.60	7.29	5374.40	0.00	0.00
Average			3505.33	1.71	3505.25	0.02	0.00
Global average			3953.69	2.91	3803.93	2.65	2.85

network design. In other words, when the location of DCs is fixed, more flexibility does not have as significant cost saving effect as it has in a flexible network design scenario. Overall, while serving the orders the next day rather than on the same day reduces the cost by 13% in a fixed network design scenario, in a flexible network design the savings go up to 19%. Changing from next day delivery to delivery within two days leads to 5% additional savings in fixed networks and 9% in flexible ones.

Table 8: Value of flexibility in due dates for 3 DCs and fixed network design

Instances			$r = 0$		$r = 1$			$r = 2$		
	Days	Customers	Cost	Gap (%)	Cost	Gap (%)	Guaranteed savings over $r = 0$ (%)	Cost	Gap (%)	Guaranteed savings over $r = 0$ (%)
Tight	3	30	2367.80	0.00	2010.60	1.68	15.07	1898.60	2.56	19.80
	3	40	2625.20	0.24	2239.40	2.65	14.54	2059.60	2.84	21.42
	3	50	2842.20	0.19	2423.00	2.23	14.76	2218.60	2.27	22.04
	3	60	3167.20	0.89	2784.00	6.06	11.24	2571.60	6.08	17.96
	6	30	4684.60	2.11	3812.40	5.98	16.79	3551.40	5.13	22.48
	6	40	5664.60	2.79	4729.80	5.30	14.26	4441.40	7.35	19.39
	6	50	6139.00	3.15	4836.00	6.84	18.46	4505.20	7.21	23.92
	6	60	6430.60	11.93	5200.00	9.47	11.96	4824.00	8.36	17.37
Average			4240.15	2.66	3504.40	5.03	14.64	3258.80	5.23	20.55
Normal	3	30	2331.80	0.00	1971.00	0.95	15.45	1780.80	0.36	23.63
	3	40	2592.20	0.00	2222.80	2.96	14.31	1991.20	1.92	23.26
	3	50	2816.80	0.00	2406.00	4.08	14.76	2167.40	3.57	23.31
	3	60	3130.00	0.40	2708.60	8.86	13.16	2432.20	4.25	21.94
	6	30	4519.60	0.46	3590.80	4.09	20.14	3296.60	4.37	26.67
	6	40	5482.60	0.57	4538.80	10.14	16.86	4231.80	8.15	22.56
	6	50	5755.80	2.77	4685.40	9.28	16.25	4352.80	9.53	22.21
	6	60	6296.00	30.63	5284.60	15.74	1.64	4823.60	12.86	5.60
Average			4115.60	4.35	3426.00	7.01	14.07	3134.55	5.63	21.15
Loose	3	30	1916.80	0.00	1740.60	0.00	9.03	1693.40	0.25	11.47
	3	40	2203.20	0.00	2002.60	1.35	9.11	1937.20	4.20	11.97
	3	50	2396.80	0.00	2240.00	3.13	6.46	2133.40	5.74	11.25
	3	60	2706.40	0.00	2506.40	8.78	7.75	2406.00	8.24	11.11
	6	30	3761.00	0.00	3004.60	4.82	20.01	2854.20	8.17	24.16
	6	40	4711.60	0.11	4248.00	15.20	9.91	3956.80	14.12	15.72
	6	50	4972.20	6.26	4603.20	23.71	6.83	4206.60	18.78	13.18
	6	60	5374.60	7.29	5154.20	21.11	2.48	4733.60	18.68	8.80
Average			3505.33	1.71	3187.45	9.76	8.95	2990.15	9.77	13.46
Global average			3953.69	2.91	3372.62	7.27	12.55	3127.83	6.87	18.38

Table 9: Value of flexibility in due dates for 3 DCs and flexible network design

Instances			$r=0$		$r=1$			$r=2$		
	Days	Customers	Cost	Gap (%)	Cost	Gap (%)	Guaranteed savings over $r=0$ (%)	Cost	Gap (%)	Guaranteed savings over $r=0$ (%)
Loose	3	30	2247.00	0.00	1759.40	1.32	21.68	1573.80	2.24	29.95
	3	40	2506.20	0.00	2011.00	3.54	19.90	1795.80	3.67	28.40
	3	50	2712.00	0.15	2178.60	3.07	19.78	1971.40	3.87	27.47
	3	60	3029.80	0.73	2458.60	4.05	18.31	2230.40	4.22	25.80
	6	30	4567.40	2.41	3354.60	7.69	24.69	3043.80	6.02	31.76
	6	40	5538.80	2.71	4266.60	7.79	21.10	3918.60	7.87	27.60
	6	50	5866.00	14.12	4380.80	10.62	16.12	3959.60	9.29	22.82
	6	60	6328.40	13.52	4660.00	12.65	16.99	4277.80	11.73	22.34
Average			4099.45	4.21	3133.70	6.34	19.82	2846.40	6.11	27.02
Normal	3	30	2161.60	0.00	1680.60	0.84	22.24	1446.80	0.80	33.09
	3	40	2444.80	0.00	1970.40	2.94	19.52	1721.40	3.17	29.68
	3	50	2582.60	0.00	2100.80	3.50	19.03	1790.00	1.56	31.01
	3	60	2944.00	0.91	2438.40	4.31	16.42	2136.20	3.88	26.66
	6	30	4147.40	0.75	3052.20	8.11	25.74	2688.20	7.98	34.59
	6	40	5094.80	0.90	3965.20	9.13	21.55	3486.00	6.22	31.27
	6	50	5306.20	16.97	4164.20	9.91	12.92	3736.40	9.56	17.93
	6	60	5775.40	10.21	4613.00	13.94	13.95	4182.00	13.58	20.55
Average			3807.10	3.72	2998.10	6.58	18.92	2648.38	5.84	28.10
Tight	3	30	1916.80	0.00	1582.40	0.50	17.40	1342.60	0.32	29.88
	3	40	2203.20	0.00	1858.60	1.84	15.68	1594.40	2.49	27.64
	3	50	2396.80	0.00	2051.80	3.80	14.74	1735.20	1.37	27.97
	3	60	2706.40	0.00	2379.80	7.38	12.13	2005.60	2.99	25.98
	6	30	3761.00	0.00	2837.20	7.45	24.47	2438.80	6.15	35.11
	6	40	4711.60	0.11	3866.00	16.60	17.54	3427.00	9.12	27.34
	6	50	4971.80	0.04	4091.60	18.06	17.54	3622.60	11.93	27.05
	6	60	5374.40	0.00	4508.80	14.56	16.18	3965.00	10.14	26.36
Average			3505.25	0.02	2897.03	8.77	16.96	2516.40	5.56	28.42
Global average			3803.93	2.65	3009.61	7.23	18.57	2670.39	5.84	27.84

4.6. Fixed versus flexible network design with due dates

In this section we analyze the combined effect of both types of flexibility. Table 10 provides a general overview on the flexibility gained from the network design and the due dates in the case of 3 DCs. The solutions obtained from fixed and flexible network designs are compared by assuming different due dates. As shown in the table, the cost for both scenarios decreases as the due date increases. The cost of the flexible scenario is always lower than the one of the fixed scenario and the difference between the costs increases when the value of r increases. This is consistent with the results shown in the previous sections. The highest optimality gaps are related to the case of loose capacity and $r = 1$.

In order to have a better estimation of the savings achieved by combining the two kinds of flexibility, in Table 11 the most inflexible case, i.e., with $r = 0$ and a fixed network scenario, is compared with the most flexible one which has two-day delivery due date and a flexible network design, both with 3 DCs. Although the most flexible problem is harder to solve to optimality, average cost savings of 30% can be observed, with savings of up to 40%.

5. Conclusions

In this paper we introduced the Flexible Two-Echelon Location Routing Problem (FLRP-2E) and proposed a mathematical programming formulation along with different classes of valid inequalities. In addition, we presented an enhanced parallel exact algorithm capable of providing high-quality solutions for instances of small and medium size. Inspired by recent technological advances, the FLRP-2E extends several classes of routing problems and combines integration issues and flexibility issues coming from two sources: network design and delivery due dates. The results obtained from the experiments on randomly generated instances show the value of flexibility, both in terms of due date and network design.

This study opens different directions for future research. Being the FLRP-2E a highly complex problem, it would be worthwhile to design heuristic algorithms that can handle large size instances. Moreover, extensions of the problem to realistic settings would be interesting, such as for example the case where multiple vehicles are available for each DC.

Table 10: Cost of fixed and flexible designs for 3 DCs with different due dates

Instances			$r=0$				$r=1$				$r=2$			
			Fixed		Flexible		Fixed		Flexible		Fixed		Flexible	
	Days	Customers	Cost	Gap (%)	Cost	Gap (%)	Cost	Gap (%)	Cost	Gap (%)	Cost	Gap (%)	Cost	Gap (%)
Tight	3	30	2367.80	0.00	2247.00	0.00	2010.60	1.68	1759.40	1.32	1898.60	2.56	1573.80	2.24
	3	40	2625.20	0.24	2506.20	0.00	2239.40	2.65	2011.00	3.54	2059.60	2.84	1795.80	3.67
	3	50	2842.20	0.19	2712.00	0.15	2423.00	2.23	2178.60	3.07	2218.60	2.27	1971.40	3.87
	3	60	3167.20	0.89	3029.80	0.73	2784.00	6.06	2458.60	4.05	2571.60	6.08	2230.40	4.22
	6	30	4684.60	2.11	4567.40	2.41	3812.40	5.98	3354.60	7.69	3551.40	5.13	3043.80	6.02
	6	40	5664.60	2.79	5538.80	2.71	4729.80	5.30	4266.60	7.79	4441.40	7.35	3918.60	7.87
	6	50	6139.00	3.15	5866.00	14.12	4836.00	6.84	4380.80	10.62	4505.20	7.21	3959.60	9.29
	6	60	6430.60	11.93	6328.40	13.52	5200.00	9.47	4660.00	12.65	4824.00	8.36	4277.80	11.73
Average			4240.15	2.66	4099.45	4.21	3504.40	5.03	3133.70	6.34	3258.80	5.23	2846.40	6.11
Normal	3	30	2331.80	0.00	2161.60	0.00	1971.00	0.95	1680.60	0.84	1780.80	0.36	1446.80	0.80
	3	40	2592.20	0.00	2444.80	0.00	2222.80	2.96	1970.40	2.94	1991.20	1.92	1721.40	3.17
	3	50	2816.80	0.00	2582.60	0.00	2406.00	4.08	2100.80	3.50	2167.40	3.57	1790.00	1.56
	3	60	3130.00	0.40	2944.00	0.91	2708.60	8.86	2438.40	4.31	2432.20	4.25	2136.20	3.88
	6	30	4519.60	0.46	4147.40	0.75	3590.80	4.09	3052.20	8.11	3296.60	4.37	2688.20	7.98
	6	40	5482.60	0.57	5094.80	0.90	4538.80	10.14	3965.20	9.13	4231.80	8.15	3486.00	6.22
	6	50	5755.80	2.77	5306.20	16.97	4685.40	9.28	4164.20	9.91	4352.80	9.53	3736.40	9.56
	6	60	6296.00	30.63	5775.40	10.21	5284.60	15.74	4613.00	13.94	4823.60	12.86	4182.00	13.58
Average			4115.60	4.35	3807.10	3.72	3426.00	7.01	2998.10	6.58	3134.55	5.63	2648.38	5.84
Loose	3	30	1916.80	0.00	1916.80	0.00	1740.60	0.00	1582.40	0.50	1693.40	0.25	1342.60	0.32
	3	40	2203.20	0.00	2203.20	0.00	2002.60	1.35	1858.60	1.84	1937.20	4.20	1594.40	2.49
	3	50	2396.80	0.00	2396.80	0.00	2240.00	3.13	2051.80	3.80	2133.40	5.74	1735.20	1.37
	3	60	2706.40	0.00	2706.40	0.00	2506.40	8.78	2379.80	7.38	2406.00	8.24	2005.60	2.99
	6	30	3761.00	0.00	3761.00	0.00	3004.60	4.82	2837.20	7.45	2854.20	8.17	2438.80	6.15
	6	40	4711.60	0.11	4711.60	0.11	4248.00	15.20	3866.00	16.60	3956.80	14.12	3427.00	9.12
	6	50	4972.20	6.26	4971.80	0.04	4603.20	23.71	4091.60	18.06	4206.60	18.78	3622.60	11.93
	6	60	5374.60	7.29	5374.40	0.00	5154.20	21.11	4508.80	14.56	4733.60	18.68	3965.00	10.14
Average			3505.33	1.71	3505.25	0.02	3187.45	9.76	2897.03	8.77	2990.15	9.77	2516.40	5.56
Global average			3953.69	2.91	3803.93	2.65	3372.62	7.27	3009.61	7.23	3127.83	6.87	2670.39	5.84

Table 11: Comparison between the most inflexible and the most flexible scenarios

Instances			Most inflexible		Most flexible		
	Days	Customers	Cost	Gap (%)	Cost	Gap (%)	Guaranteed savings (%)
Tight	3	30	2367.80	0.00	1573.80	2.24	33.54
	3	40	2625.20	0.24	1795.80	3.67	31.51
	3	50	2842.20	0.19	1971.40	3.87	30.76
	3	60	3167.20	0.89	2230.40	4.22	28.90
	6	30	4684.60	2.11	3043.80	6.02	33.68
	6	40	5664.60	2.79	3918.60	7.87	29.15
	6	50	6139.00	3.15	3959.60	9.29	33.19
	6	60	6430.60	11.93	4277.80	11.73	24.22
Average			4240.15	2.66	2846.40	6.11	30.62
Normal	3	30	2331.80	0.00	1446.80	0.80	37.99
	3	40	2592.20	0.00	1721.40	3.17	33.76
	3	50	2816.80	0.00	1790.00	1.56	36.73
	3	60	3130.00	0.40	2136.20	3.88	31.32
	6	30	4519.60	0.46	2688.20	7.98	40.22
	6	40	5482.60	0.57	3486.00	6.22	36.44
	6	50	5755.80	2.77	3736.40	9.56	33.29
	6	60	6296.00	30.63	4182.00	13.58	11.23
Average			4115.60	4.35	2648.38	5.84	32.62
Tight	3	30	1916.80	0.00	1342.60	0.32	29.88
	3	40	2203.20	0.00	1594.40	2.49	27.64
	3	50	2396.80	0.00	1735.20	1.37	27.97
	3	60	2706.40	0.00	2005.60	2.99	25.98
	6	30	3761.00	0.00	2438.80	6.15	35.11
	6	40	4711.60	0.11	3427.00	9.12	27.34
	6	50	4972.20	6.26	3622.60	11.93	22.44
	6	60	5374.60	7.29	3965.00	10.14	20.63
Average			3505.33	1.71	2516.40	5.56	27.12
Global average			3953.69	2.91	2670.39	5.84	30.12

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