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A New Policy in Congestion Pricing : Why only Toll? Why not Subsidy?

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Abstract. Congestion pricing is seen as an effective policy to address network congestion. In such policies where money, people and authorities are involved, the success generally depends upon two factors: equity (being fair) and acceptability (to both people and authorities). The primary concern is the equity, for which “tradable credit scheme (TCS)” has been introduced and extensively studied in the literature. Nevertheless, due to the complexity of the trading schemes, the TCS has yet to find any foot in the real world. To this end, a novel idea of rewarding has substituted the trading component to be known as toll-and-subsidy scheme (TSS). The idea is to charge the drivers on some roads (toll) while rewarding them to use other alternative -and perhaps underutilized- roads (subsidy). The research of the TSS is in its infancy stage. The problem to be tackled in this study is as follows: given a set of roads constituting a cordon line around the CBD or a screen line, how much toll or subsidy should be assigned to each road? The problem is first transformed into a capacitated traffic assignment problem (CTAP). In order to solve the CTAP, we employ a new algorithm based on augmenting the travel time of roads links up to the level at which the traffic volumes do not exceed the target/capacity volumes. In order to demonstrate the practicality of the methodology, the method is coded in a leading commercial transportation planning software and is applied to a real dataset from the city of Winnipeg, Canada. We then discuss policy related applications of the toll and subsidy scheme.

Keywords. Equilibrium assignment, subsidies and tools, system optimal.

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1. INTRODUCTION

"Building more roads to prevent congestion is like a fat man loosening his belt to prevent obesity" (Lewis Mumford, 1955). The first question of interest is, apart from anecdotal evidence, how intense and crucial is the congestion problem? In one estimate, the cost of congestion in 2010 in the urban areas of the USA amounted to USD 101 billion (Grant-Muller and Xu, 2014; Hensher and Bliemer, 2014). A recent study shows that the avoidable cost of congestion for the Australian capital cities is estimated to be around AUD16.5 billion for the 2015 financial year, which is a significant increase from about AUD12.8 billion for 2010 (BITRE, 2015). It seems there is no ceiling cap for such costs. For instance, traffic congestion could cost the UK economy more than £300 billion over the next 16 years which represents a 63% increase in annual costs by 2030 (Alonze, 2015). The second question of interest is what is the solution to the congestion? By casting the transport in a supply-demand context, the congestion occurs when the demand outweighs the supply. Hence the solution to the congestion can be attained either by adding to the supply or by lowering the demand. In the same spirit, congestion pricing, originally proposed in the economic literature (Pigou, 1920), has emerged as a significant leverage to bring balance back to the supply-demand equation. Economists, who believe in the power of a free market, seek solutions in the cost and pricing mechanism when they see any supply-demand imbalance (Bagloee et al., 2014). Congestion pricing has been viewed as an effective tool to bring these two sides of equation to balance. In this article we propose an unorthodox scheme in which some roads are tolled and some other (under-utilized) roads are designated as subsidy roads. The aim is to encourage commuters to avoid highly congested roads and see alternative roads in the periphery area by means of credit incentive (subsidy).

The rest of the article is organized as follows.The relevant studies in the literature are reviewed in the next section. The concepts and underlining mathematical features of the methodology are elaborated in Section 3. Numerical results are presented in Section 4 followed by conclusion remarks in Section 5.

2. LITERATURE REVIEW

This section presents a comprehensive review through which we cover a variety subjects pertaining to the toll pricing, system optimal, credit based schemes etc.

2.1. Actual cost of congestion and system optimal

What we pay for transport is fuel cost, fare/tolls and value of travel time while we ignore other hidden costs imposed to the society such as environmental degradation –namely externality cost. Hence if we pay the actual cost of what we are consuming (mobility) the market will correct itself and no such imbalance would occur. In order to realize such a benefit the question now is what is the actual cost of using a road, so that if it is charged as a toll the congestion would disappear? The pioneers on this idea are (Beckmann, 1965; Vickrey, 1969; Walters, 1961), and the following summarizes their take on the matter.

Although, many traffic flow patterns could exist, it is widely believed that people seek to minimize their individual travel cost/time which results in a User Equilibrium (UE) traffic pattern. Unfortunately the UE is not the best pattern as far as total costs are concerned. The "best" traffic pattern is the one at which the costs of the entire transport system is minimized, referred to as the System Optimal (SO). The traffic congestion can be defined and formulated as the system cost. Therefore, forcing the traffic pattern from UE to SO - by the means of a pricing mechanism- can be regarded as a potential solution to the congestion. To do so, given a UE traffic pattern, it is mathematically proven (Sheffi, 1985), that if the marginal cost of using the roads is added to the travel time/cost, the UE flow pattern coincides with the SO. The marginal cost can represent some externality costs such as pollutions, inequity etc. (Bigazzi and Figliozzi, 2013). From mathematical point of view, it is easy to specify the marginal cost for each road (Sheffi, 1985).

2.2. First-best congestions pricing versus second-best and equity concerns

In reality, it is not possible to communicate the marginal costs of all roads of the network with the commuters (i.e. tolling all roads) and expect to see a full SO traffic flow (known as first-best congestion pricing (Yang and Huang, 2005)). Rather, a few roads, presumably the highly congested ones, are selected as tolled roads and fairly consistent toll rates are considered for them (known as second-best congestion pricing (Verhoef, 2002). A well-known example is the imposition of a toll on the links forming cordon lines that surround the central business districts (CBDs) or on the links forming some screen lines which bisect the network of the city. One of the main challenges in the pricing schemes is the equity (Zhu et al., 2014). Equity is concerned with the distribution of costs and benefits among members of society. A policy can be called equitable if it meets a normative standard of fairness (so equity means fairness). Following is one of the typical complaints: "I am living just one block after the cordon line in the CBD and my office is in the outer CBD. Beside the fact that you put my house inside the cordon line, I have no business in the CBD, so it is totally unfair to tax me". In the United States, many congestion pricing proposals have been rejected based on worries that they are inequitable (Ecola and Light, 2009).

The equity concerns raised when people feel what they are paying is more than what they are getting. The inequity makes the concept of congestion pricing a hard-sell to both public and traffic authorities which is also known as acceptability issue, (Daganzo and Lehe, 2015; Hensher and Bliemer, 2014; Liu and Huang, 2014; Wu et al., 2012)).

2.3. Credit-based scheme, an answer to equity concerns

One answer to the equity issue raised in the above typical complaint can be set out as follows: the authorities would issue a limited number of cordon-passing credits, at a reasonable price, to the people working or living in the CBD. Commuters who receive the credits can then sell them in a free market to other frequent commuters. Such a scheme is called "tradable credit scheme (TCS)". Although the TCS has been extensively studied in the past twenty years, there is no real implementation in the real world (Chu et al., 2014). The TCS is hampered by three crucial questions: who are eligible to receive the initial credit, what should be the initial price, what would be the trading mechanism.

Given the above-mentioned complexities involved with the TCS, recently, a novel idea of adding subsidy in the congestion pricing has been introduced. Again, another answer to the above typical complaint can be set out as follows: instead of traversing the main (tolled) roads, commuters can choose some other roads (un-tolled) slightly away from the main (tolled) roads. In addition, by traveling on these un-tolled roads commuters are given credits; so they are called subsidized roads[1].The concept is called “toll and subsidy scheme (TSS)”. In contrast with the TCS, there is no provision for trading the credit in the TSS, nor does it need to carefully pick up the eligible commuters which both are advantageous[2]. There are also a number of other advantages associated with such schemes: (1) the equity and acceptability features are highly upheld, (2) it would strongly rally the public behind the scheme, since it cannot be considered as a different form of taxation. (3) The scheme can be used to promote the public transport mode (4) districts in the vicinity of subsidized roads are expected to receive added-values which can be then exploited by zoning authorities in the land use planning initiatives (a practice called value-capture).

2.4. Toll and subsidy scheme (TSS) and capacitated traffic assignment

Although finding the locations of the tolled roads can also be cast in a mathematical programing framework, past experiences has shown that it is usually a decision to be made by the authorities considering many technical, social as well as political factors. One practical, intuitive and less computationally intensive intervention is via traffic control policies, which are discussed in the following exposition. In principle, congestion pricing aims to keep the traffic volumes under some certain levels. These levels could be simply set to prevent traffic jam on some specific part of the cities, such as down towns or the CBD. Traffic authorities largely have a fairly good understanding of acceptable level of traffic loads in major roads or total incoming traffic in the CBD. Or in a more insightful way, the acceptable level of the incoming traffic to the CBD can be found from fundamental diagram. Therefore, the location of the roads to be tolled and accepted level of traffic loads on them are assumed known and given in the TSS.

The only remaining issue is the pricing regime. The TSS’ pricing problem is articulated as follows: given a set of roads with capped traffic volume to be either tolled or subsidized, which one must be tolled which one must be subsidized and how much? The past studies have mathematically proven that this problem always has a solution (Chen and Yang, 2012). The main focus of this study is to present a methodology tailored to large sized networks to arrive at a pricing solution for the TSS. Since the congestion pricing is subject to a set of capped traffic volumes, we first transform the problem to a conventional “Capacitated Traffic Assignment Problem CTAP”. We then interpret the values of the Lagrange multipliers of the capacity constraints as the toll/subsidy values. To solve the resulting CTAP, we relax the capacity constraints by adding their Lagrangian multiplier values to the travel time of the respective road, vis inflated travel time (Bagloee and Sarvi, 2015a; Bagloee and Sarvi, 2015b). By doing so, the CTAP is transformed into a simple (un-capacitated) Traffic Assignment Problem (TAP) which is easier to solve. The solution algorithm is designed as an iterative process: the travel times are increased to the level at which the traffic volumes do not exceed the link capacities.

We present the toll-subsidy scheme in the context of congestion pricing and then the tradable schemes. The interested reader for a comprehensive review for the former subject

[1] In such schemes, these subsidized roads are largely underutilized, hence –by adding subsidy- they would take some traffic loads from already congested main (tolled) roads. Commuters can then spend the collected credit either to pass the main road, parking fee, public transport, metro, car’s annual registration fee, traffic fine etc.”

[2] Given the current available technology even in the developing countries, implementation of such schemes is not a big challenge

can consult with (de Palma and Lindsey, 2011; Tsekeris and Voβ, 2009; Yang and Huang, 2005) and for the latter with (Grant-Muller and Xu, 2014; Nie, 2012; Wang et al., 2012; Wu et al., 2012; Yang and Wang, 2011; Zhang et al., 2011; Zhu et al., 2014).

2.5. Literature on toll and subsidy scheme

This section is dedicated to a comprehensive review on the subject of toll-subsidy scheme which sometimes referred to as toll-cum-rebate or price-rewarding schemes as well.

Bernstein (1993) proposed a time-varying pricing scheme that includes a toll and a subsidy on the same routes. Commuters are charged if they arrive at the peak hour and are subsidized if they choose off peak-hour times. Over a simple network it was demonstrated that the scheme could reduce the equilibrium cost. Adler and Cetin (Adler and Cetin, 2001) discussed a redistribution scheme in which revenue collected from a desirable route is transferred to drivers on a less desirable route. Compared against the SO traffic pattern, they showed the redistribution scheme renders almost identical results. Guo and Yang (2010) investigated the existence of Pareto-improving[3] TSS in general networks with multiple users. In the presence of the restricting Pareto-improving condition, they show that, should there be any gap[4] to get the entire system better off (in terms of the costs); the toll-subsidy scheme problem always has a solution. Even in a more restrictive scheme where no subsidy exists, the Pareto-improving solution exists (Lawphongpanich and Yin, 2010; Wu et al., 2011). Furthermore, in a fashion reminiscent of the TSS, the revenue from tolling highway links is used to subsidize the fare adjustments on transit lines. In a controlled environment, Ben-Elia and Ettema (2011) investigated the impact of a reward scheme on the behaviour of commuters in the Netherlands. The results suggest that the reward scheme can be certainly effective in the short run, while it has yet to be investigated in the long run. It is worth noting that, recently, the idea of using rewards to change commuters' behaviour has been implemented in the Netherlands -though in limited scale- as part of a program called Spitsmijden (Ben-Elia and Ettema, 2011). Using a bi-level Stackelberg game approach, Maillé and Stier-Moses (2009) investigated a mechanism based on pure subsidy (without any toll) that aims at reducing congestion in urban networks. They concluded that the subsidies can significantly lower the social cost. Part of the budget for rebates may come from the savings that arise from the more efficient use of the system.

In conjunction with the toll-subsidy scheme, some scholars seek the revenue-neutral property, that is, the total toll-revenue collected is set equal to the total amount of subsidies paid (essentially, the average commuter pays nothing (Kockelman and Kalmanje, 2005)). As such, the authorities can attain an optimized system by merely redistributing the revenue without engaging in financial transfer to or from the commuters. Hence it cannot be referred to as a smart form of taxation. The concept is also called Robin Hood toll scheme (Hearn and Ramana, 1998), which can be characterized by the following simple equation: toll-subsidy=0. Compared to the tradable schemes (TCS) the Robin Hood method tries to obviate the "trade" part, this comes at a restrictive cost compared to what our method (TSS) offers. Moreover, in

[3] In a naïve congestion pricing, although the entire transport system are sought to be better off, some commuters might become worse off. To this end the Pareto-improving seek a pricing scheme based on which, no one becomes worse off.

[4] The gap refers to the difference in total travel time between the UE traffic pattern and that of the SO.

our approach we relax the equality sign in the equation, that is, the sum of the toll minus subsidy could be positive or negative (while in Robin Hood, this must equate to zero). This is aligned to what we are after (i.e. a certain roads associated with a certain target volumes to be achieved by too and subsidy), if we enforce Robin Hood method (adding additional constraints) the problem may become infeasible.

Regardless of the challenging task of the toll redistribution, merging the two ideas of revenue-neutral and Pareto-improving TSS, where everyone is a winner, is highly appealing. For a single origin–destination (O–D) pair, Eliasson (2001) showed that compared to the do-nothing scenario, in a revenue-neutral TSS, where the total travel time is reduced, no commuter is found to be worse off. Liu et al. (2009) and Nie and Liu (2010) examined the existence of Pareto-improving and revenue-neutral pricing scheme in a simple bi-modal network consisting of road and a parallel transit line. Results provided by Liu et al. (2009) suggests that the condition for every user being better off is weaker than the condition for total system travel time reduction. In other words, any revenue-neutral pricing scheme is Pareto-improving as long as it reduces the total system travel time. The results provided by Nie and Liu (2010) imply that the revenue-neutral property is a restricting condition which may compromise the existence of a solution. In other words, external subsidies may be required to make every commuter happy (Pareto-improving). Xiao and Zhang (2014) show that on a one-origin or one-destination network, a Pareto-improving, system-optimal and revenue-neutral TSS always exists. It turns out that such a scheme may not always exist for a multi-origin network, hence they sought the maximum possible revenue collected by the TSS problem subject to the Pareto-improving constrains. Given the results of the above studies, maintaining both the Pareto-improving and revenue-neutral properties is difficult; possibly the best bet is to drop the revenue-neutral in order to secure a solution. Chen and Yang (2012) proved that subject to nonnegative toll scheme and nonnegative cycle the TSS always has a solution. Considering that tolls and subsidies are represented by positive and negative values, then, a negative cycle is a distinct closed path along which the sum of the link travel costs is negative. If a negative cycle exists, commuters can make money merely by traversing along the negative cycle. This paper proves that the TSS -as we set out here- has always a solution, if, loosely speaking, the amount of toll is higher than that of subsidy.

3. THE MATHEMATICAL MODEL

As noted early, the problem to be undertaken is as follows: given a set of roads associated with capped volumes (let us call it capacity) identified to be tolled or subsidized, what is a pricing scheme. In other words, what is the price tag (toll or subsidy) of each road? In this section, we first articulate the mathematical expression of the problem and then propose the solution algorithm.

3.1. Formulation of the problem

The problem as set out above with the capped traffic volume essentially is a Capacitated Traffic Assignment Problem (CTAP). Consider $G(N,A)$ a traffic network as a graph which consists of N,A sets of nodes and links respectively on which $R \subset N \times N$ is set of origin-destination pairs. Set of roads nominated as toll/subsidy is denoted by $\overline{A} \subset A$. The CTAP can be formulated as a non-linear programing problem as follows (throughout the manuscript, all terms are non-negative unless otherwise stated):

[CTAP]:

$$\min \quad z(x) = \sum_{a\in A}\int_0^{x_a} t_a(x)\,dx \tag{1}$$

s.t.:

$$\sum_p f_{p,r} = q_r \qquad r \in R \;\rightarrow\; \text{Lagrange multiplier -- } w_r \tag{2}$$

$$f_{p,r} \ge 0 \qquad p \in P_r, r \in R \tag{3}$$

$$x_a = \sum_r \sum_p f_{p,r} . \delta^r_{a,p} ; \qquad a \in A, p \in P_r, \; r \in R \tag{4}$$

$$x_a \le C_a \qquad a \in \overline{A} \;\rightarrow\; \text{Lagrange multiplier -- } \beta_a \tag{5}$$

where z : is the Beckmann objective function to be minimized; x_a : is the traffic flow on link a ; q_r : is the travel demand pertaining to OD pair $r \in R$; $f_{p,r}$: is the traffic flow on path p connecting OD pair r ; P_r : is the set of all paths connecting OD pair r ; $\delta^r_{a,p}$: is the link-path incidence (1: if link a belongs to path p connecting OD pair r and 0 otherwise); C_a : is the capped traffic volume of link a .

It is important to note that other than the Beckmann function (equation (1)), there is no other objective function to be optimized, which makes the problem much easier. Otherwise, the conventional method is to formulate the problem as a non-convex, bi-level and NP-hard problem.

3.2. Karush-Kuhn-Tucker conditions and interpretation of Lagrangian multiplier values

It is proven that the CTAP subject to linear constraints is a strictly convex problem which renders a unique global optimal solution of link flows (Hearn, 1980; Inouye, 1987; Larsson and Patriksson, 1995; Marcotte and Patriksson, 2007; Nie et al., 2004; Patriksson, 1994). Let us consider w_r, β_a as Lagrangian multipliers for travel demand and capacity constraints respectively, hence the Karush-Kuhn-Tucker (KKT) conditions are established as:

$$f_{p,r}(u_{p,r} + \sum_{a\in A}\delta^r_{a,p}.\beta_a - w_r) = 0 \qquad p \in P_r, r \in R \tag{6}$$

$$\beta_a(C_a - x_a) = 0 \qquad a \in \overline{A} \tag{7}$$

$$u_{p,r} + \sum_{a\in A}\delta^r_{a,p}.\beta_a - w_r \ge 0 \qquad p \in P_r, r \in R \tag{8}$$

$$C_a - x_a \ge 0 \qquad a \in \overline{A} \tag{9}$$

$$f_{p,r} \ge 0 \qquad p \in P_r, r \in R \tag{10}$$

$$\beta_a \ge 0 \qquad a \in \overline{A} \tag{11}$$

$$\sum_p f_{p,r} = q_r \qquad r \in R \tag{12}$$

where $u_{p,r} = \sum_{a\in A}\delta^r_{a,p}.t_a$ is the total travel time perceived by commuters on path p connecting OD pair r . Let us define $\hat{u}_{p,r}$, an "inflated" travel time of the respective path as:

$$\hat{u}_{p,r} = \sum_{a\in A}\delta^r_{a,p}.(t_a + \beta_a) \qquad a \in \overline{A} \tag{13}$$

Introduction of (13) into (6) and (8) results in:

$$f_{p,r}(\hat{u}_{p,r} - w_r) = 0 \qquad p \in P_r, r \in R \tag{14}$$

$$\hat{u}_{p,r} - w_r \geq 0 \qquad p \in P_r, r \in R \tag{15}$$

With respect to equations (14) and (15), it can be proven that w_r is the travel time of the shortest path connecting OD pair r, hence, should any path take some traffic volume $(f_{p,r} > 0)$, it is certainly the shortest path. In other words the first principle of Wardrop holds and the global optimum solution of the CTAP is in fact user-equilibrium traffic flow. According to equations (7), (9) and (11), if the capacity constraint is binding, which means saturation $(x_a = C_a)$, then the corresponding Lagrangian multipliers are non-zero $(\beta_a > 0)$, otherwise it is zero. There are two terms contributing to the augmented travel time (Equation (13)): normal or cruise travel time (t_a) and beta (β_a). The beta can be interpreted as toll/subsidy value or alternatively the additional delay to prevent the respective road become over-saturated (i.e. traffic volume above the capacity). In the literature the beta is also interpreted as the waiting time caused by the queue built up in the oversaturated links (Larsson and Patriksson, 1995; Marcotte et al., 2004; Marcotte and Patriksson, 2007; Nie et al., 2004; Patriksson, 1994; Shahpar et al., 2008; Yang and Yagar, 1994, 1995*)*. As shown, the beta which represents the value of toll or subsidy is a positive number, while the subsidy is supposed to be a negative number. We will later handle this issue when an especial provision is devised to prevent any negative cycle.

3.3. Augmented travel time

Let us first assume that there exist only toll and no subsidy. Once we establish the formulation for the tolls we will then extend it to the subsidy as well. Also, consider the delay function as $t_a = t_a^0.(1 + f(x_a))$, where t_a^0 is the free-flow travel time perceived by commuters on link a and $f(x_a) \geq 0$ is a non-decreasing and convex function of x_a such that $f(x_a) = 0 \mid x_a = 0$. The aforementioned function can accommodate a variety of known delay functions including the widely used function proposed by the US Bureau of Public Roads (the BPR delay function) (BPR, 1964). As such, we can intuitively argue that, as long as the road is empty or uncongested, the travel time is the free flow travel time $(x_a \approx 0 \Rightarrow t_a \approx t_a^0)$; as the traffic builds up, t_a,the travel time increases - certainly higher than the free flow travel time. One can consider it as a factor of t_a^0 which is greater than 1 $(i.e.: (1 + f(x_a)) \geq 1)$. According to Equation (13), $\hat{t}_a$ the inflated travel time of "saturated" link a can be formulated as:

$$\hat{t}_a = (t_a^0 + b_a).(1 + f(x_a)) \tag{16}$$

$$\beta_a = \hat{t}_a - t_a = b_a.(1 + f(x_a)) \tag{17}$$

where b_a is an additional penalty in the free flow time in equation (16). In other words, b_a is the value of beta at $x_a = 0$; hence we referred to it as the "initial-beta".

Accordingly, the travel time in the CTAP is replaced with the augmented travel time $(t_a \leftarrow \hat{t}_a)$ and the capacity constraint is also dropped because the beta, the Lagrangian multiplier of the corresponding capacity constraint, now contributes to the travel time. Therefore, the CTAP is transformed to an un-capacitated TAP. If we had already known the global optimum value of the beta (in the CTAP) we would have just needed to solve the TAP using any known algorithm. Of course this is not the case; hence, the values of the (initial) betas are updated iteratively in the course of solving the TAP as explained in the following section. Moreover, the value of the beta is zero unless the corresponding road is oversaturated. In other words Equations (16) and (17) apply only to the oversaturated links to bring them down at their capacity levels (to become saturated). Hence, non-zero values are assigned to the initial-beta of the (over)saturated roads while they are set to be updated in the next iterations.

3.4. A heuristic method to update the initial-beta

A variety of algorithms have been proposed to solve the CTAP for which have recently presented a comprehensive review (Bagloee and Sarvi, 2015b). The inherent mathematical complexities associated with the capacity constraints have resulted in solution algorithms laden with a number of parameters to be calibrated, which is a prohibitive factor. In addition, arriving at an initial feasible solution on which to launch the algorithm is also a challenge. Alternatively, in this study the mathematical complexity of the CTAP is overcome by adopting an intuitive interpretation of capacity that is, the Lagrangian multipliers of the capacity constraints are interpreted as additional delay imposed on the oversaturated roads to make them saturated. Such an interpretation has also been employed in the congestion pricing, namely the "trial-and-error" methods (Meng et al., 2005; Yang and Huang, 2005; Yang et al., 2004; Yang et al., 2010; Zhou et al., 2015).

The concept embedded in Equations (16) and (17) is to lift the delay function of the oversaturated links until the traffic volume stabilizes at capacity level. The value of the initial-beta in the delay functions is updated iteratively, for which the main challenge is to progress towards convergence and user equilibrium. There is a strong correlation between the capacitated traffic assignment and the general theory of the congestion pricing (Yang and Huang, 2005) which is centred on charging commuters based on the marginal cost. Inspired by such an observation, the amount of additional delay to be added to obtain the inflated travel time is derived from the concept of marginal cost as follows (Beckmann et al., 1956; Patriksson, 1994; Sheffi, 1985*)* :

$$\tilde{t}_a(x_a) = t_a(x_a) + x_a.\,\partial t_a(x_a)/\partial x_a \tag{18}$$

where $\tilde{t}_a(x_a)$ is the marginal cost or travel time experienced by an additional commuter added to x_a who are already traversing link *a,* and $\partial t_a(x_a)/\partial x_a$ is the additional travel time experienced by each driver among x_a. The marginal cost enforces System Optimal flow - a better and uniformly - distributed traffic across the network - such that under-utilized roads will absorb the traffic discharged off the oversaturated links. Hence $x_a.\,\partial t_a(x_a)/\partial x_a$ -the additional delay imposed on the respective link - is considered as a template to update the initial-betas as follows:

$$\hat{t}_{a,C}^{(i)} = (t_a^0 + b_a^{(i)}).(1 + f(C_a)) \tag{19}$$

$$\nabla b_a^{(i)} = (x_a^{(i)} - C_a).\frac{\hat{t}_a^{(i)} - \hat{t}_{a,C}^{(i)}}{C_a} \tag{20}$$

$$b_a^{(i+1)} = b_a^{(i)} + \nabla b_a^{(i)} \tag{21}$$

where superscript i and a denote the current iteration and respective (over)saturated link; $\hat{t}_{a,C}^{(i)}$ represents travel time at capacity $(x_a = C_a)$ of the inflated delay function; $b_a^{(i)}$ is the initial-beta or an additional penalty to free-flow travel time $(b_a^{(i)})$; $\nabla b_a^{(i)}$ is the pace of the initial-beta computed at the current iteration, while $b_a^{(i+1)}$ is the updated initial-beta computed for the next iteration, initialized by zero $(b_a^{(1)} = 0)$.

The formulation provided in Equation (20) to update the pace values follows the spirit of marginal cost (i.e. $x_a.\partial t_a(x_a)/\partial x_a$). The "x" is replaced by the excess traffic $(x_a^{(i)} - C_a)$ and the slope of the delay function (dt/dx) is replaced by the slope of the inflated travel time minus the pace value normalized by the value of the capacity $(\hat{t}_a^{(i)} - \hat{t}_{a,C}^{(i)})/C_a$. It is an intuitive penalty added to over-saturated roads. The penalty is set proportional to the amount of the excessive volume while it is intensified by the slope.

In Figure 1, exhibition-1, the above formulations (Equations (19) to (21)) for three iterations on the (inflated) delay function is graphically shown. In the first iteration when there is no initial-beta $(b_a^{(1)} = 0)$, the volume stands at $x_a^{(1)} > C_a$, and $\nabla b_a^{(1)} > 0$ the pace is computed as shown graphically, which lifts the delay function for the next iteration $(b_a^{(2)} = 0 + \nabla b_a^{(1)})$. In the second iteration even with inflated travel time $(\nabla b_a^{(1)})$, the volume still stands above the capacity $(x_a^{(2)} > C_a)$. Hence the value of the pace $(\nabla b_a^{(2)} > 0)$ is adjusted again, ready for the third iteration: $(b_a^{(3)} = b_a^{(2)} + \nabla b_a^{(2)})$. The third iteration is executed and the volume stands at capacity $(x_a^{(3)} = C_a)$. The three key words or components of the proposed algorithm, beta (β_a), initial-beta (b_a) and the pace (∇b_a), are shown in the figure. During this progressive approach, if an (over)saturated road is found unsaturated -at an intermediate iteration- its corresponding penalty is nullified $(p_a^{(i+1)} = 0 \mid x_a^{(i)} < C_a)$ as shown in Figure 1, exhibit-2.

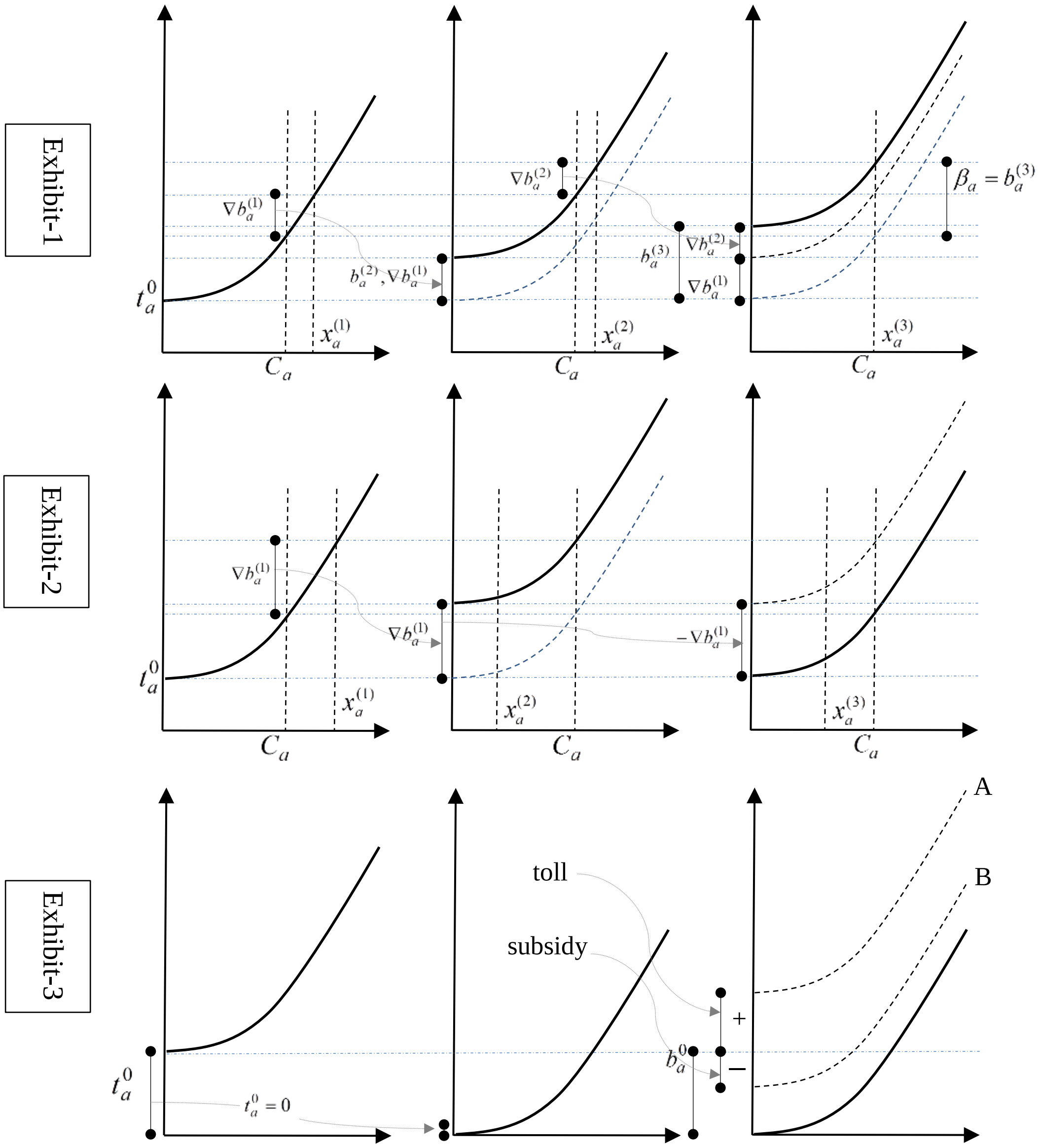

FIGURE 1 Conceptual representation of the proposed methodology for the road delay functions.

The above conventions for computing the initial-beta can be summarized by two rules:

$$b_a^{(i+1)} = \begin{cases} b_a^{(i)} + \nabla b_a^{(i)} \ \ from \ (20) & x_a^{(i)} \geq C_a \\ \\ 0 & x_a^{(i)} < C_a \end{cases} \tag{22}$$

The above setup complies with the KKT conditions at the stationary point of the capacitated assignment problem, in which the beta of the non-saturated roads must be set zero (Yang and Huang, 2005).

For ease of reference, the proposed methodology is referred to as the "Inflated Travel Time" (ITT) method, similar to the terminology used in the literature. In the context of the other methods, the numerical results showed promising convergence behaviour. Nevertheless the ITT is classified as a heuristic method since a formal mathematical proof for the convergence behaviour has yet to be provided.

3.5. Subsidy

We now can include the subsidy in the formulation. As noted early, a negative cycle may spoil the whole pricing process (the TSS). To avoid such a case, we ensure the roads' travel time to take always positive values. To this end, we first zero-out the free flow travel times of all the toll/subsidy roads. We then initialize the initial-beta to the respective free flow travel time. Effectively, nothing changes and delay functions remain intact[5]:

$$\begin{cases} b_a^{(0)} = t_a^0 \\ t_a^0 = 0 \end{cases} \qquad a \in \overline{A} \tag{23}$$

This process is graphically shown in Exhibit 3 of Figure 1. Let us have a close look at the Exhibition 3: in the end of the computations (the last iteration), due to the value of the beta (or the last initial-beta), the resulting delay function can be either A or B (above and below the free flow time axis). The beta minus the free flow time ($\beta_a - t_a^0$) can results in either a positive value (delay function A) or a negative value (delay function B) which represents toll or subsidy respectively. Moreover, the absolute value of the subsidy cannot exceed the free-flow-time. In other words, no road will take a negative travel time; hence no negative cycle will appear.

Therefore, given the outputs at the end of the last iteration (iteration n), the toll/subsidy value $(-t_a^0 \leq s_a \leq +\infty)$ can be calculated as follows:

$$s_a = \hat{t}_a^{(n)} - t_a^0.(1 + f(\overline{C}_a)) \qquad a \in \overline{A} \tag{24}$$

3.6. Termination Conditions

The proposed algorithm can be easily integrated into a conventional solution algorithm for the TAP such as FW. Hence, given fixed rates of the initial-betas, the solution algorithm itself needs to converge and meet its own termination criterion which is mainly driven by a relative gap. Boyce et al. (2004) and Dial (2006) recommend a relative gap of 0.0001 to ensure a perfect convergence to link flow stability. This criterion is adopted in our numerical evaluations. It is also expected that the initial betas show convergence behaviour over the successive iterations. Given the gradual built up of the initial-beta, a convergence criterion can be defined as pace values falling below a small enough value:

$$\max_a |\nabla b_a^{(i)} / \hat{t}_a| \leq \varepsilon \tag{25}$$

where ε is a small value called relative pace value. The numerical results showed that $\varepsilon = 1\%$ is sufficient to obtain reliable results. Consequently, the algorithm does not terminate unless both criteria: the relative gap and the relative pace values are met.

[5] In other words, though the toll values can theoretically be as high as infinity (i.e. $0 \leq toll \leq +\infty$), the absolute value of subsidy cannot be higher than the free flow travel time or $-t_a^0 \leq sunsidy \leq 0$

3.7. Capacity Feasibility

In case the travel demand is higher than the capacity of the network, the problem becomes infeasible. It is important for any algorithm to have some mechanism to detect and address the infeasibility cases. To this end, one can introduce a dummy node connected with all zones via un-capacitated links associated with high travel time. Therefore, the problem always remains feasible. As such, in the case of a residual traffic load on dummy links after a safely converged and terminated assignment, one can label it as a capacity-infeasible scenario.

The dummy links not only obviate any feasibility concern, they are also devised to replicate reality. Consider a single OD connected via a single road with capacity of 10 vehicles. Faced with 15 vehicles demand, the algorithm allows 10 vehicles take the road while 5 vehicles have no chance to do so; instead, they take on the dummy links as leftover flow. In reality, these excess 5 vehicles have no choice except changing their departure times which is also studied separately in the literature. In other words, the residual demand remains off the network until the next available traffic assignment interval, which is discussed in the literature on dynamic traffic assignment (Zhong et al., 2011).

3.8. A discussion on the convergence proof

Although the numerical results display the convergence behaviour of the proposed algorithm, we cast it as a heuristic method, since a formal convergence proof has yet to be developed.

The attempt to find the Lagrangian multiplier values of the capacity constraints iteratively is, in fact, equivalent to solving a partial dual transformation of the original CTAP (Meng et al., 2005). It can be shown that the gradient of the partial dual problem leads to a vector of excessive traffic volume for which the sub-gradient method is deemed to be a competent solution algorithm (Gustavsson et al., 2012; Larsson et al., 1996). The ITT algorithm is essentially a sub-gradient method, because the dual values are updated iteratively based on the excessive traffic volume.

A key component of the sub-gradient methods is the step-size which is uniformly applied to all roads and may vary across iterations. In order to ensure the convergence, the values of the step-size must comply with a set of rules (Anstreicher and Wolsey, 2009). Hence finding an appropriate scheme for the step-size is a challenge. Instead, we proposed an intuitive step-size mechanism (analogous to marginal cost) , exclusive to each road and iteration. We are yet to prove that the step-size devised for the ITT abides by the aforementioned rules. Nevertheless, due to the presence of both travel time and traffic volume in the proposed step-size, a variational inequality formulation may streamline the arguments for the proof.

4. NUMERICAL RESULTS

The methodology is coded in a “macro”, that is the programing language of EMME 3 (INRO, 2009) – a leading transportation planning software, in which the un-capacitated TAP is solved using the Frank-Wolfe (FW) algorithm. A desktop PC with a 3.60GHz CPU and 16 GB of RAM is employed. The ITT algorithm is applied to large-size network of city of Winnipeg, Canada which is widely used in the literature (Bar-Gera, 2015) (it is also provided by (INRO, 2009) as part of EMME 3). The case study consisted of 154 zones, 943 nodes and 3075 directional links with hourly travel demand of 56,219. The delay functions comply with the general format of BPR functions.

The city is bisected by two rivers which merge in the middle of the city. According to traffic surveys and field observations, the authorities have come up with a desirable traffic distribution over the west-north river crossings to avoid any major gridlock in the AM peak hour. Figure 2 depicts 11 bridges which are subject to the traffic control. In Table 1, characteristics of the crossings including the exiting traffic flow, t_a^0 free flow travel times, $\bar{C}_a$ physical capacities as well as C_a the capped traffic volume.

The algorithm runs for 237 iterations until the relative gap and the relative pace value falls below 0.0001and 0.01 respectively. The computation lasts only 15 seconds. The results including the traffic volumes and the inflated travel time as well as the toll/subsidy values are also shown in Table 1. The restrictions on the traffic volume are upheld, the slight violations –as shown under x_a/C_a column- is less than 1% that can be attributed to the fact that the ITT is a heuristic algorithm. Figure 3 demonstrates variations of traffic volume and the inflated travel times over the successive iterations pertaining to the roads 1, 2 and 3. The results indicate that the variation is volatile in early iterations but it then stabilizes as the results head towards the convergence. Similar erratic variations in solving the CTAP have also been reported in the literature.

Figure 4(a) illustrates traffic volume before and after implementation the toll-subsidy scheme. It is obvious that the traffic volumes are squeezed below the capped values. Variations of the pace values over successive iterations are depicted in Figure 4 (b). in this figure the maximum of the pace values (in absolute value) across the toll/subsidy roads are plotted. As can be seen, the pace values rapidly converge to zero. The overall convergence of the algorithm is depicted in Figure 4(b) in which the relative gaps (in percentage) as well as the Beckmann values are shown.

The last column in Table 1 presents the toll/subsidy value – in Canadian dollar unit- in which the value-of-time is assumed \$16.69 per hour (HDR, February 2015). The maximum toll and subsidy found to attain the target crossing volume are \$4.13 and \$0.26 (twenty six cents). Given the rapid expansion of communication technology, devises such as GPS are now indispensable gadget. Therefore communicating a set of diverse rates of toll/subsidy –as is the case in the output- should not be a problem[6].

[6] GPS devices nowadays can offer minimum route as well as toll-free routes. So it seems highly plausible to get the GPS devices programmed to advice on a variety of routes based on the users budget and toll prices

Figure 2. Winnipeg case study, locations of the river crossing nominated for the toll-subsidy scheme.

Table 1. Winnipeg case study, input and output values

Road ID	Road direction	road-direction ID	t_a^0: free flow travel time	$\bar{C}_a$: Physical Capacity	C_a: capped traffic volume	Existing traffic volume	x_a: traffic volume of the TSS toll-subsidy scheme	x_a/C_a (%)	$\hat{t}_a$: inflated travel time (min)	s_a: net toll/subsidy	monetary value of the net toll/subsidy ($)*
	A-B	1-1	0.77	2500	1800	828.93	1796.92	99.83	2.66	1.86	0.52
1	B-A	1-2	0.77	2500	1800	183.76	185	10.28	0.03	-0.74	-0.21
	A-B	2-1	0.84	750	750	722.32	727.05	96.94	0.03	-0.92	-0.26
2	B-A	2-2	0.84	750	750	762.35	750.24	100.03	7.08	6.11	1.7
	A-B	3-1	0.25	2500	1800	600.48	348.27	19.35	0.01	-0.24	-0.07
3	B-A	3-2	0.25	2500	1800	2759.71	1801.39	100.08	8.63	8.37	2.33
	A-B	4-1	0.35	2500	1800	1205.92	1800.78	100.04	7.2	6.84	1.9
4	B-A	4-2	0.35	2500	1800	84.03	108.25	6.01	0.01	-0.33	-0.09
	A-B	5-1	0.44	1750	1750	1641.31	1749.8	99.99	9.52	9.02	2.51
5	B-A	5-2	0.44	1750	1750	469.82	403.75	23.07	0.02	-0.42	-0.12
	A-B	6-1	0.23	2625	1800	2446.11	1802.87	100.16	12.18	11.95	3.32
6	B-A	6-2	0.23	1750	1750	785.81	592.48	33.86	0.01	-0.22	-0.06
	A-B	7-1	0.16	2500	1800	2313.34	1796.73	99.82	12.85	12.68	3.53
7	B-A	7-2	0.16	2500	1800	656.64	389.2	21.62	0.01	-0.15	-0.04
	A-B	8-1	0.23	1750	1750	2035.5	1751.26	100.07	11.73	11.47	3.19
8	B-A	8-2	0.23	2625	1800	783	752.7	41.82	0.01	-0.22	-0.06
	A-B	9-1	0.57	2625	1800	1683.56	1795.59	99.76	12.75	12.16	3.38
9	B-A	9-2	0.47	2625	1800	903.52	824.2	45.79	0.02	-0.45	-0.12
	A-B	10-1	0.30	2550	1800	1547.86	1092.88	60.72	0.01	-0.29	-0.08
10	B-A	10-2	0.27	3750	1800	3261.11	1802.74	100.15	15.13	14.86	4.13
	A-B	11-1	0.59	2500	1800	189.06	629.38	34.97	0.02	-0.57	-0.16
11	B-A	11-2	0.59	2500	1800	289.45	1505.82	83.66	0.02	-0.58	-0.16

* The value of time is assumed $16.69 per hour (Canadian dollar) : 0.40=1.45/60*16.69, the positive values are toll, the negative values are subsidy

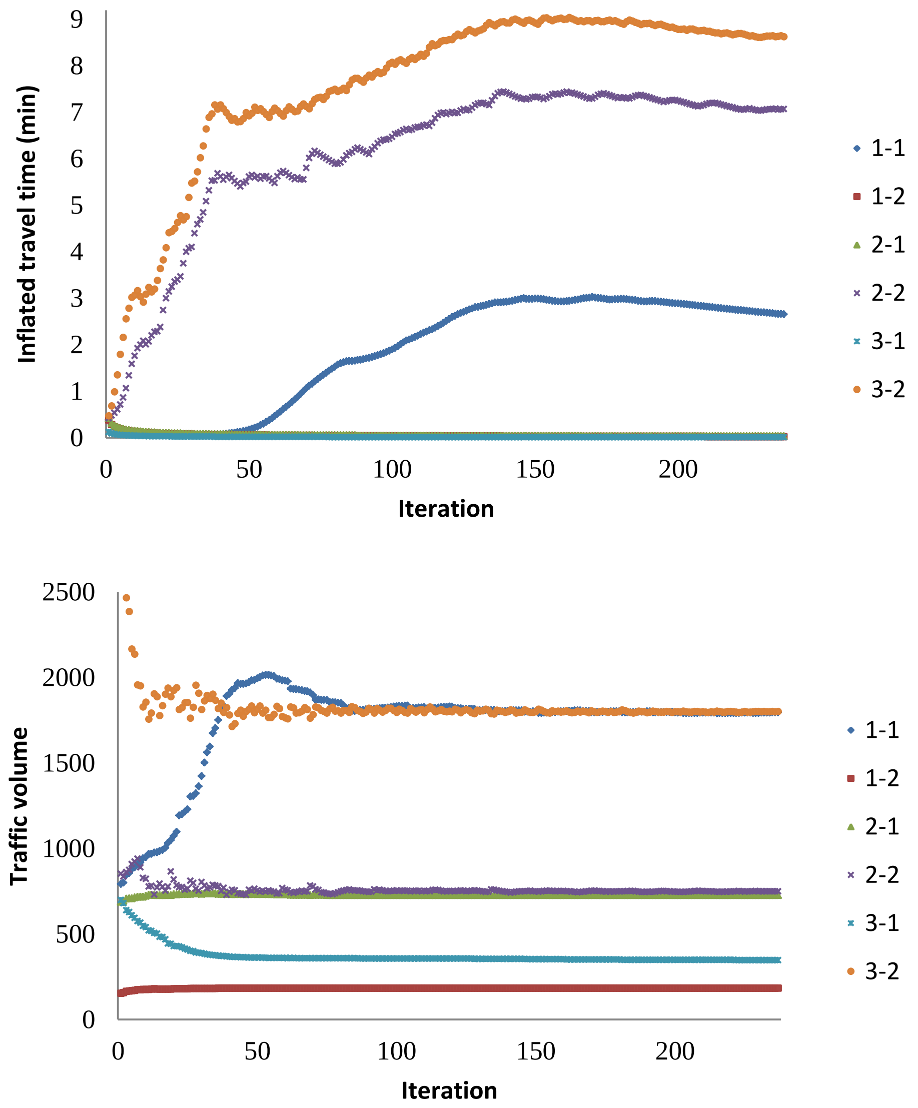

Figure 3. Winnipeg case-study, the first three bridges over successive iterations: (a) variation of the traffic volumes (b) variation of the inflated travel time

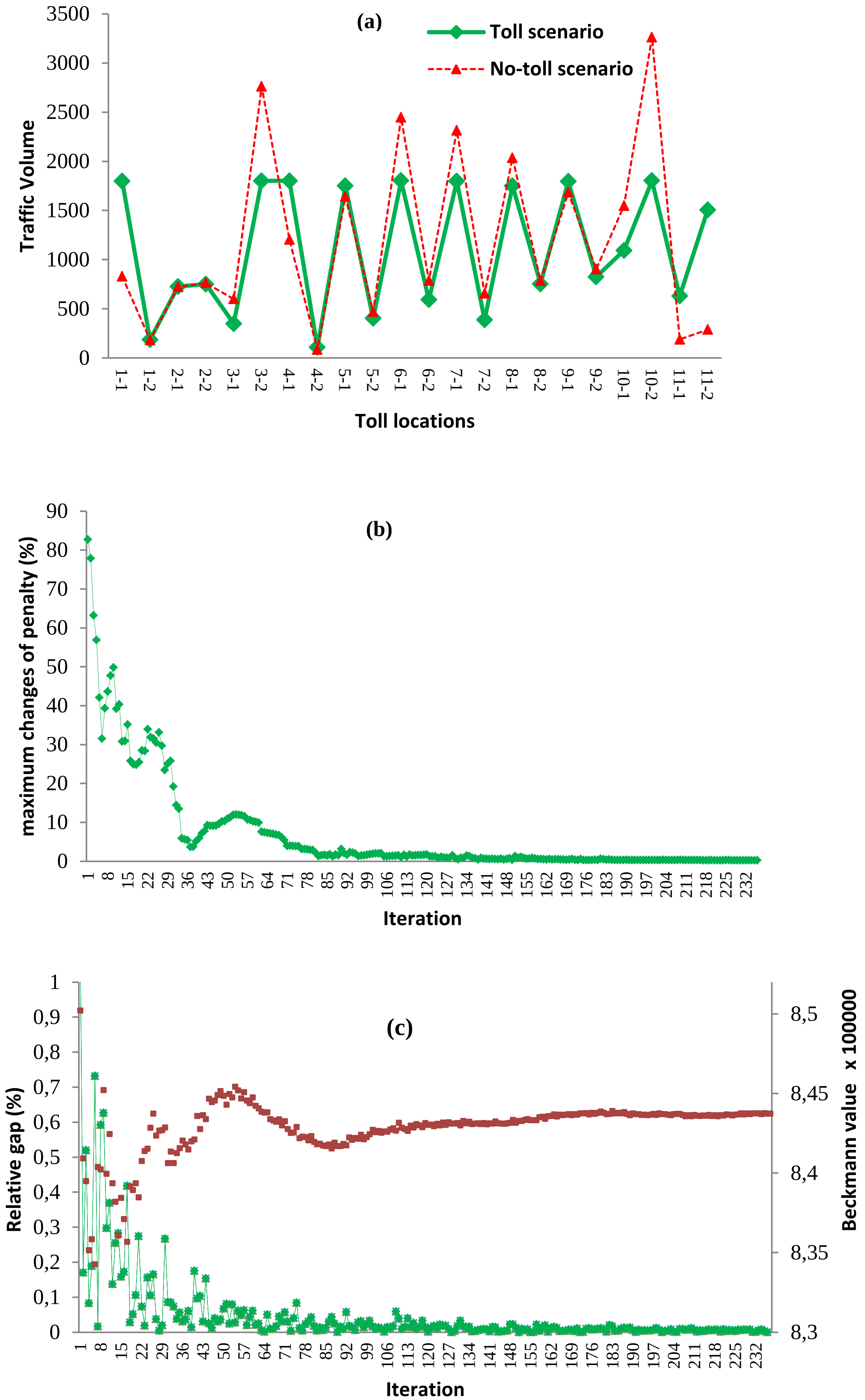

Figure 4. Winnipeg case-study, (a) traffic volume on the screen line before and after TSS (b) maximum of changes in penalty (c) the convergence behaviour

5. CONCLUSION

Congestion pricing is still a hotly debated subject, for which the new idea of toll-subsidy scheme (TSS) has recently received a surge of interest. The element of subsidy is an appealing factor that can greatly compensate some shortcomings related to equity and acceptability in the previous schemes. A fully-fledged TSS problem ought to be postulated as a bi-level programming problem which is highly intractable. Nonetheless, in practice, the problem is usually as simple as follows: the location of the tolled (or subsidized) roads are known, it could be a cordon line surrounding the CBD or screen lines bisecting the urban network. Even -based on the traffic survey or historical data- the target traffic volume on the tolled road could also be considered as a given input. Therefore, what the question of interest is what is the best pricing regime, or, which road should be tolled and which ones should be subsidized and how much? This is the problem that was studies in this research. Given the capped values of the target roads, we first transformed the problem to a conventional Capacitated Traffic Assignment Problem (CTAP). The Lagrangian values of the respective capacity constraints can be interpreted as the toll/subsidy. We then relaxed the capacity constraints by moving their Lagrangian values to the respective roads' delay function. In other words, the delay functions got an added term which needed to be iteratively updated in such a way the traffic volumes remain below the capped values. We referred to the process as "Inflated Travel Time (ITT)". The algorithm was then applied to the real dataset of the Winnipeg, Canada, in which 11 river crossings bridges were subject to the TSS. The methodology was coded in EMME 3 and was applied to the real network of the city of Winnipeg. The maximum values toll and subsidy were found to be 4 and 0.2 Canadian dollar.

The subsidy can also be well played as a leverage to address land use related problems. The area in the vicinity of subsidized road can be a magnet of business and job opportunity especially for blue-collar sectors. The immediate role of the subsidy can be attributed to the fact that it makes the pricing product more sellable to the public, since it brushes off the stain of being another form of taxation.

The concept of the subsidy does not need any payment mechanism. The collected subsidy can be later reimbursed in many different ways such as toll payment, parking fee, public transport ticket, traffic fine, annual car registration to name a few. It is also easy to protect the system from any mall practices, for instance total daily value of collected subsidies can be limited, and hence, no one can make money by simply driving in the city (a condition to ensure that the problem always has a solution (Chen and Yang, 2012)).

In the same spirit, (Morosan and Florian, 2015) have recently solved a distance based pricing with minimum and maximum toll rates based on which commuters are charged based on vehicle-kilometres-travelled (VKT). As such integration of the VKT based TSS can be a worthwhile research.

On other threads for future studies, the followings are worth mentioning. In this study we assumed that the travel demand is fixed and all commuters have identical perception about the toll/subsidy. In other words, it is a worthy attempt to extend the research to multiclass travel demand in which the value-of-time varies across different sector of the society. The elasticity of the demand to the changes in the pricing regime by itself deserves a further investigation. With the same spirit, consideration of the mutual impact of transit and private modes in the presence of congestion pricing needs to be further studied.

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