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A Dynamic Multi-Plant Lot-Sizing and Distribution Problem

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Abstract. In this paper we investigate a multi-plant, production planning and distribution problem for the simultaneous optimization of production, inventory control, demand allocation, and distribution decisions. The objective of this rich problem is to satisfy the dynamic demand of customers while minimizing the total cost of production, inventory and distribution. By solving the problem, we determine when the production needs to occur, how much has to be produced in each of the plants, how much has to be stored in each of the warehouses, and how much needs to be delivered to each customer in each period. On a large real dataset inspired by a case obtained from an industrial partner we show that the proposed integration is highly effective. Moreover, we study several trade-offs in a detailed sensitivity analysis. Our analyses indicate that the proposed scenarios give the company competitive advantage in terms of reduced total logistics cost, and also highlight more possibilities that become available taking advantage of an integrated approach toward logistics planning. These abundant opportunities are to be synergized and exploited in an interconnected open global logistics system.

Keywords. Physical internet, dynamic lot-sizing, integrated supply chain planning, production, inventory, distribution, optimization.

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1 Introduction

The world is changing rapidly and so do the supply chain management paradigms. Environmental concerns, economical crisis and rapid pace of technological changes have called for more efficient and effective supply chains. In response to the need for better supply chain management, the *Physical Internet* (PI) idea is introduced as an interconnected open global logistics system, which is increasingly gaining attention from research and practice. Inspired by the digital internet, researchers are developing the PI to move, store, realize, supply and use the goods similarly to how data is treated in the digital internet [31, 32]. Going global and exploiting a system as gigantic and holistic as the PI also requires a comprehensive view of different activities within a supply chain. In order to be able to exploit the opportunities the PI has to offer and to be an active member in this global open interconnected web, it is required that each company uses its own resources more efficiently. Short and medium-term decisions such as production planning and distribution might be the ones to benefit the most from or serve the PI better. By avoiding over or under production and improving transportation and distribution efficiencies, coordination and integration of different supply chain members are becoming the new source for competitive advantage. A real-world example of such effort in integrating supply chain decisions is the Kellogg Company which could save $35 to $40 million per year as a result of the joint optimization of production, inventory and distribution [14]. Another example comes from IBM that has developed an integrated plan for production and shipment of semiconductors and could gain 15% increase for on-time delivery, 2-4% increase in asset utilization, and 25-30% decrease in inventory [23]. Despite the fact that any cost minimization effort which is focused on only one area of supply chain and ignores other areas will often lead to increase in cost and not necessarily a reduction of global cost due to the lack of synergy [3], there is abundant literature available on any of the individual problems and all their well-known generalization. Integrated problems often present advantages from a practical point of view and are becoming the new standard [21, 22, 5].

Having seen the advantages gained by this holistic approach in supply chain management, researchers have shifted their interests towards integrating supply chain decisions that have typically been treated individually. For instance, the vehicle routing problem (VRP) and the lot-sizing problem (LSP) are two of the most well-known classical problems in supply chain management that have been studied separately for decades [3]. In the VRP several capacitated vehicles deliver products to a number of customers from a warehouse with the objective of minimizing the delivery cost, while the LSP is focused on the quantities and timing of production with the objective of production and inventory cost minimization [28]. Obviously, in practice, these two problems should not be treated separately, as production planning and distribution are interdependent and can benefit from a joint decision making.

1.1 Literature review

Since the seminal paper of Wagner and Whitin [38], several studies have investigated the dynamic LSP, some of which report close to optimal solutions for single plant, multi-product capacitated LSPs, but they have not been successful in solving large size instances [34]. In what follows we review some papers that have solved the LSP with the aim of finding a balance between the setup cost and the inventory holding cost [20].

Florian et al. [24] proved that the single-product capacitated problem is NP-hard, and Maes et al. [30] showed that finding a feasible production plan for a capacitated production system with no setup cost is an NP-complete problem. The LSP encompasses a wide variety of problems from continuous time scale, constant demand and infinite time horizon to discrete time scale, dynamic demand and finite time horizon lot-sizing models [26]. Several classifications have been presented in the literature for the LSP, for example, single or multi-level problems [10], with the most simple form of the dynamic LSP being the single-product uncapacitated problem [27]. LSPs are also categorized based on the number of products and number of plants considered, ranging from single to multi-product [10, 34, 27, 20, 1], and single or multi-plant [34], or based on how capacity constraints are treated [33, 28], i.e., uncapacitated [1] versus capacitated LSP [15].

Li et al. [29] identify two lines of research in LSP: in the first, the effort is to develop exact methods with mathematical programming, valid inequalities or tighter formulations; in the second, matheuristics are developed for large scale problems. Due to its complexity, various methods and techniques have been exploited to solve LSP; a Lagrangean-based heuristic for scheduling of lot sizes in a multi-plant, multi-product, multi-period capacitated environment with inter-plant transfers [34], matheuristic through a series of mixed-integer linear programs solved iteratively in a fix-and-optimize approach to solve a multi-level capacitated LSP with multi-period setup carry-over [33], and dynamic programming for the single-product LSP [19, 36]. A hybrid of an ant-based algorithm and mixed-integer linear programming method was used by Almeder [8] to solve a multi-level capacitated LSP; a tabu search and a variable neighborhood search heuristic was developed by Almada-Lobo and James [7] to solve the multi-product capacitated lot-sizing and scheduling problem with sequence-dependent setup times and costs; finally, a hybrid of genetic algorithms and a fix-and-optimize heuristic was used to solve the capacitated LSP with setup carry-over [25].

The problem studied in this paper is on one hand closely related to the production-routing problem (PRP), which integrates lot-sizing, inventory and distribution decisions simultaneously and on the other hand, it deals with the LSP with time windows. Both delivery and production time windows are considered in the literature [1, 6], and the PRP has been thoroughly studied recently [2, 5]. The PRP is solved mostly by heuristics: greedy randomized adaptive search procedure [13], memetic algorithm [12], reactive tabu search [11] are to name a few. Archetti et al. [9] propose a hybrid algorithm to solve a production and distribution of a single-product single-plant system to minimize the total cost of production, inventory replenishment of a set of retailers and transportation costs. A common recent approach in dealing with integrated planning problems is to decompose the problems into subproblems and to exploit heuristics to solve them. For instance, Adulyasak et al. [4] use an adaptive large neighborhood search combined with a network flow algorithm. Chen [18] proposes a heuristic approach to solve dynamic multi-level capacitated lot-sizing problems with and without setup carryovers. Steinrücke [37] uses relax-and-fix decomposition methods to solve a multi-stage production, shipping and distribution scheduling problem in the aluminum industry. Camacho-Vallejo et al. [16] consider a two-level problem for planning the production and distribution in a supply chain in which the upper level minimizes the transportation cost while the lower level deals with the operation costs occurring at plants for which a scatter search-based heuristic is proposed. The reduce and optimize approach is used by Cárdenas-Barrón et al. [17] to solve the multi-product multi-period inventory lot-sizing with supplier selection problem. Akbalik and Penz [6] propose a mixed integer linear program and a pseudo-polynomial dynamic program to solve time window LSP and ultimately to compare time window with just-in-time policy. Finally, by means of dynamic programming, Absi et al. [1] solve single item uncapacitated LSP with production time windows.

1.2 Contributions and organization of the paper

In this paper we solve a problem inspired by a real case with an integrated cost saving approach, in which we optimize a multi-plant multi-period problem to determine production scheduling and lot sizes, inventory quantities as well as aggregated plans for demand delivery within a promised time window. This problem arises in a company that produces and sells furniture, who has shared data and insights with us. Inventory and production capacities are limited and once a changeover takes place at a production site, a fixed setup cost is incurred. Thus, we consider a rich generalization of the dynamic LSP. This paper contributes to the literature by considering a multi-plant capacitated LSP in which one has to fulfill the demand of customers within a promised time window, determining production dates and quantities, inventory levels, demand satisfaction and distribution. This problem generalizes many variants of the transportation problem and network flow problems.

Making the right lot-sizing decisions becomes even more significant in interconnected logistic networks. By pooling capacities and enabling horizontal collaboration, interconnected logistics networks seek higher performance levels [35]. By optimizing lot-sizing decisions, we show the benefits of an integrated approach to supply chain decision making. Besides, we demonstrate the resource sharing potentials a case company could offer to the interconnected logistic networks. Finally we also highlight the importance of the cost of transportation which justifies the need for utilizing the synergy in an open interconnected network, such as that advocated by the PI initiative.

The remainder of this paper is organized as follows. In Section 2 we describe the problem and introduce the real case study. Section 3 presents the mathematical formulation of the problem. We present the results of extensive computational experiments and sensitivity analysis in Section 4, followed by our conclusions in Section 5.

2 Problem description

The motivation for this problem stems from a company's needs that produces and sells furniture. The plants are located in Canada and the US and its customers are spread all over North America. The demand is categorized in two groups: from large retail stores where the products are sold directly to customers, and online customers who visit the company's website and place orders over the internet. In this paper we focus only on the second category of orders, in which all demand is satisfied from one of the warehouses associated with each of the plants.

Whenever a plant starts production, a fixed setup cost is incurred. Therefore, the company must determine how to plan its daily production and inventory levels across different plants. By considering a dependable demand forecast, and production and warehouse capacities, the company develops a production plan to fulfill the demand. Since this demand forecast is highly precise, we consider the demand to be deterministic by relying on these accurate forecasts. Moreover, given the very long planning horizon of our problem, we can assume that the time it takes to produce an item as well as the lead-time to be negligible. A solution to this problem identifies the periods in which production takes place as well as the production quantities, the amount that has to be stored and which customers to serve in each of the periods from each of the warehouses.

We define a plant set $\mathcal{N}_p$ and a customer set $\mathcal{N}_c$, such that $\mathcal{N} = \mathcal{N}_p \cup \mathcal{N}_c$. The plant set $\mathcal{N}_p$ contains p plants, and a fixed setup cost f_i ($i \in \mathcal{N}_p$) is incurred whenever a production batch starts, and a variable cost u_i is paid for each unit produced. A warehouse located close to each plant stores all the products. A holding cost h_i is incurred per unit held in warehouse $i \in \mathcal{N}_p$ per period. Production at the plants are limited to b_i units per period and inventory at the warehouses are limited to s_i units. The production is sent to customers $i \in \mathcal{N}_c$ from the warehouses. Customers represent the final demand, meaning that they do not hold any inventory and there is no transshipment between them. The planning horizon is e periods long, typically days over one year, and at the beginning of each time period $t \in \mathcal{T} = \{1, \ldots, e\}$, the demand of customer i is d_i^t, which needs to be fulfilled within r periods, i.e., r is the time window or the maximum lateness allowed. Inventories are not allowed to be negative and all demand must be satisfied, i.e., backlogging is not permitted. We also assume, without loss of generality, that the initial inventory is zero for all warehouses. Once the delivery dates for each customer is determined, the products are posted to the customer using a third party logistics provider. As a consequence no consolidation is allowed and the company pays c_{ij} as the postal fee from warehouse i to customer j for each unit of product. Products can be sent directly to the final customers from any of the warehouses, and transfers between warehouses are also allowed, costing c_{ij}, with i and $j \in \mathcal{N}_p$.

A schematic view of the model for an instance with two plants and three customers is depicted in Figure 1. This figure shows how inventories evolve in each plant, and how dispatches are allowed between plants or from plants to customers.

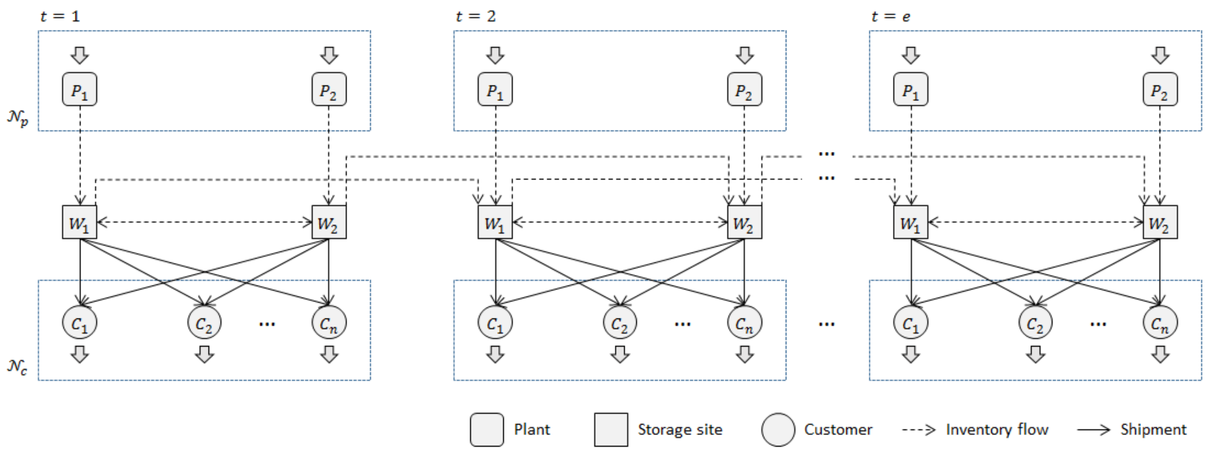

Figure 1: An instance of the dynamic multi-plant lot-sizing and distribution problem with two plants and three customers

The objective of this problem is to minimize the total of production, inventory holding and delivery costs. All demand must be satisfied by the end of the planning horizon, all capacities and the maximum lateness must be respected.

3 Mathematical formulation

In order to model the problem as an integer linear program, let y_i^t be a binary variable equal to one if and only if plant $i \in \mathcal{N}_p$ is active in period t. Let p_i^t be the quantity produced at plant i in period t, q_{ij}^t be the quantity delivered from warehouse $i \in \mathcal{N}_p$ to customer or warehouse $j \in \mathcal{N}$ in period t, and I_i^t be the inventory level of warehouse $i \in \mathcal{N}_p$ measured at the end of period t.

Table 1 summarizes the notation used in our model:

Table 1: Notations of the model

Parameters	
b_i	Production capacity of plant i (unit of products)
c_{ij}	Shipping cost from node i to node j ($/unit)
d_i^t	Demand of customer i in period t (unit of products)
f_i	Fixed setup cost in plant i ($)
h_i	Unit holding cost in warehouse i ($/period)
r	Time window or the maximum lateness allowed (periods)
s_i	Inventory capacity of warehouse i (unit of products)
u_i	Unit variable cost of production in plant i ($)
Sets	
$\mathcal{N}_c$	Set of customers
$\mathcal{N}_p$	Set of plants
$\mathcal{N} = \mathcal{N}_p \cup \mathcal{N}_c$	Set of all nodes
T	Set of periods
Variables	
p_i^t	Quantity produced at plant i in period t
q_{ij}^t	Quantity delivered from warehouse $i \in \mathcal{N}_p$ to customer or warehouse $j \in \mathcal{N}$ in period t
I_i^t	Inventory level at warehouse $i \in \mathcal{N}_p$ in period t
y_i^t	Binary variable equal to one if and only if plant $i \in \mathcal{N}_p$ is active in period t
Indices	
k, t	Period index
i, j	Node index

The problem can be formulated as follows:

$$\text{minimize} \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{N}_p} \left(u_i p_i^t + f_i y_i^t + h_i I_i^t + \sum_{j \in \mathcal{N}, i \neq j} c_{ij} q_{ij}^t \right) \tag{1}$$

subject to

$$p_i^t \leq b_i y_i^t \quad i \in \mathcal{N}_p \quad t \in \mathcal{T} \tag{2}$$

$$I_i^t \leq s_i \quad i \in \mathcal{N}_p \quad t \in \mathcal{T} \tag{3}$$

$$I_i^t = I_i^{t-1} + p_i^t - \sum_{j \in \mathcal{N}, i \neq j} q_{ij}^t + \sum_{j \in \mathcal{N}_p, i \neq j} q_{ji}^t \quad i \in \mathcal{N}_p \quad t \in \mathcal{T} \tag{4}$$

$$I_i^0 = 0 \quad i \in \mathcal{N}_p \tag{5}$$

$$\sum_{k=1}^{t} \sum_{j \in \mathcal{N}_p} q_{ji}^k \geq \sum_{k=1}^{t-r} d_i^k \quad i \in \mathcal{N}_c \quad t \in \mathcal{T} \tag{6}$$

$$\sum_{k=1}^{t} \sum_{j \in \mathcal{N}_p} q_{ji}^k \leq \sum_{k=1}^{t} d_i^k \quad i \in \mathcal{N}_c \quad t \in \mathcal{T} \tag{7}$$

$$\sum_{k \in \mathcal{T}} \sum_{j \in \mathcal{N}_p} q_{ji}^k = \sum_{k \in \mathcal{T}} d_i^k \quad i \in \mathcal{N}_c \tag{8}$$

$$y_i^t \in \{0, 1\} \tag{9}$$

$$p_i^t, I_i^t, q_{ij}^t \in \mathbb{Z}^*. \tag{10}$$

The objective function (1) minimizes the total cost of production setup and variable costs, inventory holding cost and transportation cost. Constraints (2) ensure that the production capacity is respected, while constraints (3) guarantee that the inventory level does not exceed the capacity of the warehouse. Constraints (4) and (5) are respectively inventory conservation and initial conditions, and constraints (6) and (7) enforce that the customer demand is satisfied within the r periods time window. Specifically, constraints (6) impose that the total demand up to period $t - r$ must be delivered by time t, thus allowing the demand satisfaction to a maximum lateness of r periods, and constraints (7) guarantee that no demand is satisfied in advance. Finally, constraints (8) ensure that all the demand is satisfied by the end of the planning horizon. Constraints (9) and (10) define the domain and nature of the variables. Figure 2 shows how a feasible delivery plan looks like, satisfying constraints (6)–(8). Graphically, the accumulated curve of a feasible delivery plan for a given customer has to move between the accumulated demand and the accumulated expiring demand curves; the latter corresponds to the accumulated demand, displaced by r time units. The accumulated delivery curve has to match the total demand by the end of the horizon.

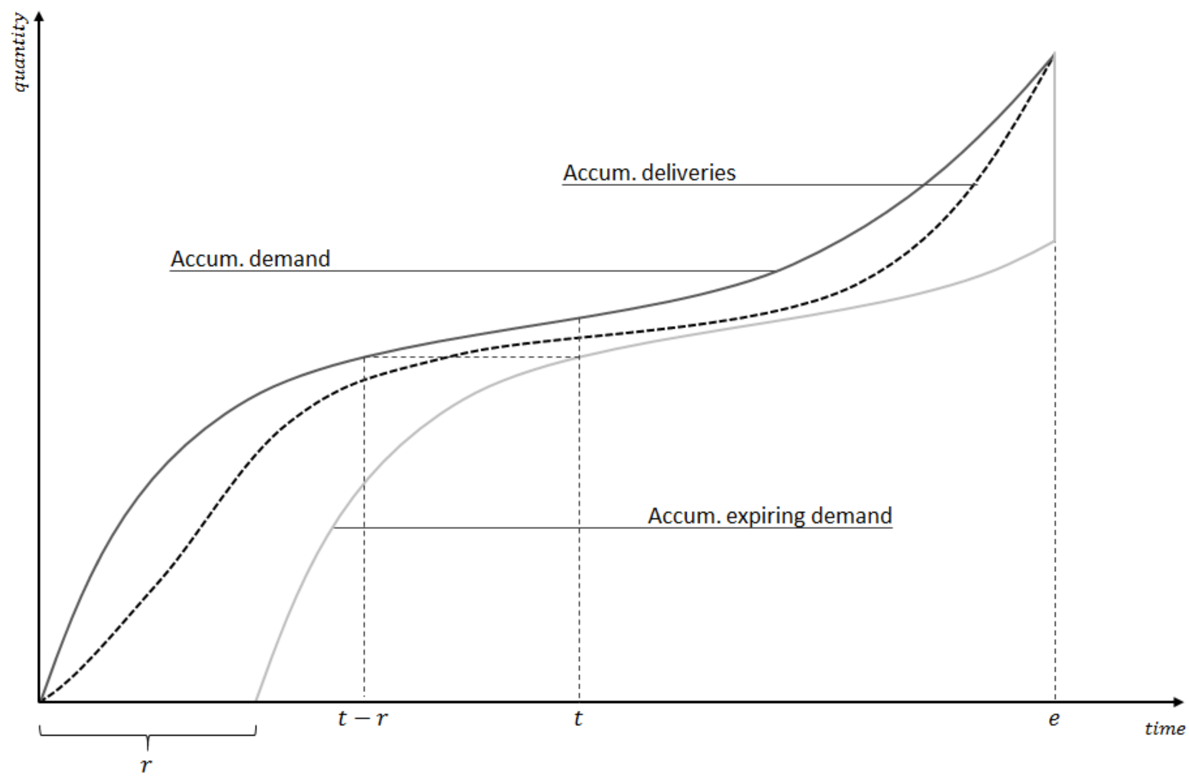

Figure 2: Feasibility of a delivery plan with respect to the maximum lateness allowed

4 Computational experiments

The mathematical model was implemented into a branch-and-bound algorithm coded in C++, using IBM CPLEX 12.6 as the MIP solver. All computations were executed on a grid of Intel Xeon™ processors running at 2.66 GHz with up to 48 GB of RAM installed per node, with the Scientific Linux 6.1 operating system. A time limit of one hour was imposed on the execution of each run.

An industrial partner has provided us with a large amount of data regarding production costs, production and inventory capacities, shipping costs, and daily demand for a year-long planning horizon. We have used this information to evaluate our method under different production scenarios. In our dataset, periods represent days and we have a year long planning horizon of available data. Customers are spread all over North America;

there are in total 66 customers aggregated over the US states and Canadian provinces. The company operates two plants, significantly far away from each other. Company practices dictate that the time window r varies between three and four days, depending on the area. We test different values of r for the whole customer set. In our testbed, there exists 364 daily periods. All data has been obfuscated for confidentiality reasons, and our resulting instances can be shared upon request.

We start our computational analysis by comparing some feasible production plans inspired by those a manager could employ to control the production. These do not represent current company practices, but are inspired from them. We report in Table 2 different scenarios, that range from producing every period, every other period, every three periods, and the global optimized plan. Other plans, such as every four periods, have been tested as suggested by our industrial partner, however, no feasible solution could be obtained for any other time window scenarios, which could be due to the fact that a bigger time window for production makes the problem significantly bigger and consequently the problem becomes very difficult to be solved to optimality. Table 2 reveals that the obtained optimal scenario gives the company competitive advantage in terms of serving customers faster since for any promised time windows r the optimized scenario yields lower total cost compared to the other three possible scenarios. The optimized scenario provides solutions which are on average 17%, 4% and 4% less costly in comparison to the other three production scenarios. We observe that when producing every day, the optimal solution is to immediately ship the products to customers so as not to incur any inventory costs, thus the solution cost does not change with respect to r. When the production is intermittent, the best cases are observed for slightly bigger time windows. We observe that due to production and inventory capacities, it is impossible to produce every three days and guarantee deliveries within a tight time window. Finally, as expected, the optimized scenario yields the best production plan such as to minimize total costs. Moreover, we observe from Table 2 that the optimized plan can offer next-day delivery guarantee with a slight increase in the total cost with respect to the other time windows, only 5% more costly than the minimum cost obtained with $r = 10$. This constitutes the first important insight obtained from our research.

Table 2: Total costs for different production scenarios

Production scenario	Time window r					
	1	2	3	5	7	10
Every day	**6,366,591**	**6,366,591**	**6,366,591**	**6,366,591**	**6,366,591**	**6,366,591**
Every other day	5,713,911	5,703,504	5,696,257	5,688,965	**5,687,841**	**5,687,841**
Every three days	—	—	—	5,662,420	5,571,270	**5,540,990**
Optimized	5,625,782	5,527,674	5,453,789	5,380,042	5,349,342	**5,331,363**
Gap (%)	2.95	2.52	1.74	0.81	0.50	0.16

The results reported in Table 2 were obtained after solving the model described in Section 3 using CPLEX for one hour. Although, given the size of the problem, CPLEX was not always able to yield an optimal solution, the optimality gap is very low, on average only 1.45%, with a maximum of 2.95%.

In order to better understand the impact of each part of the objective function to the costs obtained in Table 2, we provide a decomposition of each individual cost as a percentage of the total cost as presented in Table 3.

In all scenarios the transportation cost is the main contributor to total cost, given high quantities and distances. Although the same number of units needs to be produced and transported, neither transportation nor variable production costs are constant due to the choice of plant, their different parameters, and distances to customers. As expected, when plants produce every day the setup costs are significant, but no inventory is kept at the warehouses. Once plants operate only every other day, the setup costs decrease significantly, giving more relative weight to the transportation cost. We observe that when production is set to happen at every three periods, then production capacities come into play. Thus, these cases with $r = 1$, 2 and 3 yields infeasible solutions. Finally, the

Table 3: Percentage of contribution to the total cost per component

Production scenario	Cost component	Time window r					
		1	2	3	5	7	10
Every day	Inventory holding	0.00	0.00	0.00	0.00	0.00	0.00
	Setup	21.44	21.44	21.44	21.44	21.44	21.44
	Variable production	5.36	5.36	5.36	5.36	5.36	5.36
	Transportation	73.20	73.20	73.20	73.20	73.20	73.20
Every other day	Inventory holding	0.10	0.02	0.02	0.01	0.00	0.00
	Setup	12.01	12.03	12.05	12.06	12.07	12.07
	Variable production	6.07	6.06	6.04	6.01	6.00	6.00
	Transportation	81.82	81.89	81.89	81.92	81.93	81.93
Every three days	Inventory holding	—	—	—	1.57	0.23	0.11
	Setup	—	—	—	8.08	8.21	8.26
	Variable production	—	—	—	6.39	6.47	6.46
	Transportation	—	—	—	83.96	85.09	85.18
Optimized	Inventory holding	1.74	1.15	0.50	0.16	0.04	0.01
	Setup	9.23	8.28	7.74	6.87	6.45	6.12
	Variable production	6.02	6.12	6.24	6.32	6.38	6.42
	Transportation	83.00	84.45	85.52	86.66	87.13	87.44

gap between the variable cost of production and the setup cost is the minimum in the optimized solution while the inventory holding cost has the biggest percentage of the total cost in the optimized scenario. It is in this scenario that the transportation cost plays a bigger role, representing more than 85% of the total cost of the system.

Further analysis of the optimized solution for the time window currently used by the company, i.e., $r = 3$, reveals interesting and useful information justifying the potential gains the company could obtain offering its idle capacities on an open logistics network. We have selected a plant randomly and have identified two distinct periods of high versus low usage. This has allowed us to measure how much of the production and inventory capacities were used by the company and how much could be offered to an open logistics networks as that proposed by the PI. As depicted in Figure 3 during the high demand period (a), the production is up to the capacity while the inventory capacity is available; on the other hand, in low production period (b), the idle inventory and production capacities are ready to be offered to other companies on the interconnected logistics network.

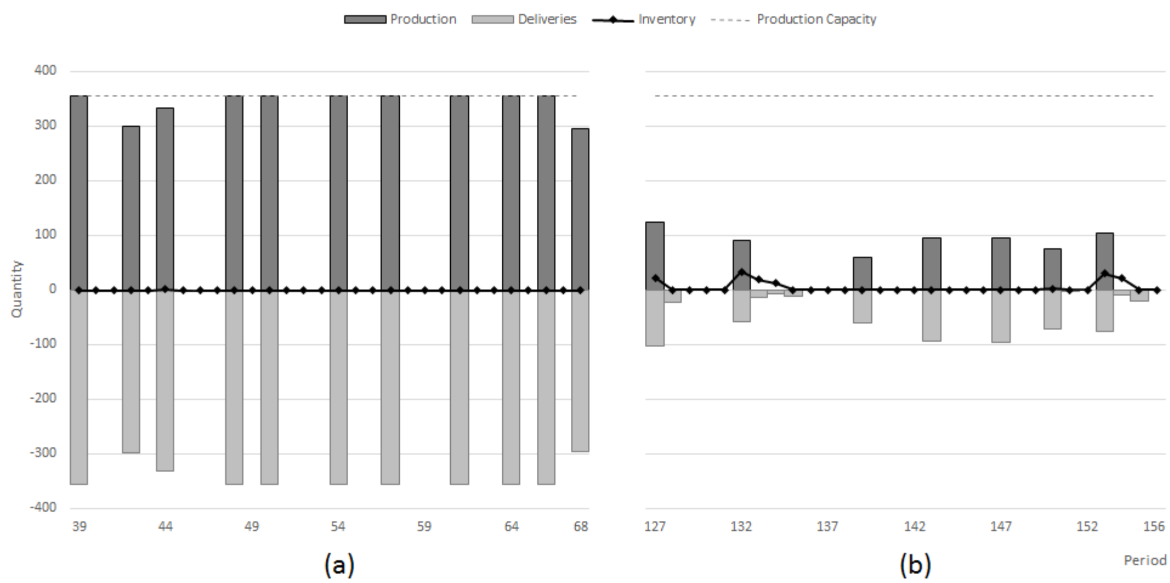

Figure 3: An example of production and inventory capacities in high (a) versus low (b) demand periods

In order to analyze how changes in cost or capacity parameters of the model affect the total cost, a number of alternative parameter scenarios are now assessed. Table 4 summarizes the percentage change in the objective function when either decreasing the costs and production capacity by half, or increasing them by a factor of two. We observe that decreasing costs by half will have a beneficial impact on the total cost, but on different degrees. Since the transportation cost is the main contributor to the total cost, any reduction in transportation costs will have the most noticeable impact on the bottom line. We note that the slight increase in cost when the inventory holding cost is halved in the case of a time window of $r = 10$ periods is due to the noise in the computations because not all optimal solutions could be obtained, as explained earlier. By decreasing the production capacity by half, the plants need to be active more often in order to satisfy the demand and this incurs an increase in the total cost of about 5%. Likewise, when costs are doubled, the total costs always increase, but in different degrees. Finally, if the capacities are doubled, the opposite effect is observed: due to relatively low inventory costs, plants can operate less often, reducing the total cost.

Table 4: Percentage of cost changes with respect to the optimized solutions

Parameters	Cost component	Time window r					
		1	2	3	5	7	10
0.5	Inventory holding	−1.31	−1.08	−0.61	−0.15	−0.12	0.03
	Setup	−5.17	−4.59	−4.10	−3.40	−3.20	−3.09
	Variable production	−3.16	−3.13	−3.07	−3.13	−3.06	−3.17
	Transportation	−41.32	−42.80	−42.87	−43.27	−43.46	−43.74
	Production capacity	3.08	3.51	4.42	5.12	5.57	5.88
2.0	Inventory holding	0.97	0.28	0.26	0.21	0.12	0.00
	Setup	8.69	7.67	7.16	6.58	6.18	6.07
	Variable production	6.34	5.73	5.98	6.37	6.46	6.38
	Transportation	82.85	83.96	85.43	86.54	87.11	87.44
	Production capacity	−0.84	−1.09	−1.13	−1.64	−2.09	−2.41

The sensitivity analysis shown in Table 4 only considers the individual effects of the cost components. In order to detect possible synergies arising when these factors are combined, we have performed experiments on all possible pairwise combinations of cost components. In Table 5 we show that of the transportation and setup costs, as these are the two most significant cost components identified in Table 4. We have solved the instances where transportation and setup costs are altered simultaneously. In Table 5 we compare the combined effects with the sum of individual effects. A quick look at this table shows that the combined effects for smaller time windows are almost linear, i.e., the combined effects are very close to the sum of the individual effects. However, when the time windows get larger, the cost increase by combined effects can be even 10% smaller than the sum of the individual costs increase in our instances. Hence, once more we witness the positive effect of integrated decision making approach.

Table 5: Combined effects for the transportation and setup costs

Time window r	Trasnportation cost	Setup cost	Additive effect (%)	Combined effect (%)	Difference (%)
1	0.5	0.5	−46.49	−46.35	−0.14
		2	−32.63	−32.45	−0.18
	2	0.5	79.37	77.71	1.66
		2	91.53	91.31	0.22
2	0.5	0.5	−47.39	−47.81	0.41
		2	−35.13	−35.96	0.83
	2	0.5	79.37	76.53	2.84
		2	91.64	88.76	2.87
3	0.5	0.5	−46.97	−48.40	1.43
		2	−35.72	−37.42	1.70
	2	0.5	81.33	75.94	5.39
		2	92.59	86.98	5.60
5	0.5	0.5	−46.67	−49.07	2.40
		2	−36.69	−39.54	2.85
	2	0.5	83.14	75.15	8.00
		2	93.12	84.86	8.26
7	0.5	0.5	−46.66	−49.41	2.75
		2	−37.66	−40.41	2.75
	2	0.5	84.35	74.87	9.48
		2	93.51	83.95	9.57
10	0.5	0.5	−46.83	−49.57	2.75
		2	−37.66	−40.92	3.26
	2	0.5	84.35	74.73	9.63
		2	93.51	83.50	10.01

5 Conclusions

We have solved a real multi-plant, production planning and distribution problem in which production, inventory, demand and distribution decisions are optimized simultaneously. A mathematical formulation for a rich dynamic LSP problem with delivery time windows is proposed. The problem is defined using a large real dataset inspired by a case obtained from an industrial partner. We have showed that our method is highly effective; elaborate and extensive sensitivity analysis are presented to compare various production scenarios as well as changes in the parameters of the model.

The contributions of this work to PI literature are twofold. First, we demonstrate the potentials of resource sharing in a real case company. Total of idle capacities from different companies gathered together in an interconnected logistics network provides enormous opportunities for businesses to benefit economically, environmentally and socially. Second, the results indicate that the most important cost component in this problem is the transportation cost. Exploiting PI-enabled open distribution centers to store products closer to the final customers and PI-enabled mobility web is expected to dramatically reduce the storage and transportation costs.

Further extensions of our work could include multi-products cases and transition costs, which would appear as an additional setup cost when switching products, as well of integrating different lead times per product and per facility. Another interesting extension is related to the consolidation of the transportation costs when delivering the goods to account for economies of scale.

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