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**Rahma Lahyani**
**Leandro C. Coelho**
**Mahdi Khemakhem**
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**Frédéric Semet**

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**Rahma Lahyani[1,2], Leandro C. Coelho[3,4,*], Mahdi Khemakhem[2], Gilbert Laporte[3,5], Frédéric Semet[1,3]**

[1] Laboratoire d'Automatique, Génie Informatique et Signal (LAGIS), École Centrale de Lille, Cité Scientifique, BP 48, 59651 Villeneuve d'Ascq, France

[2] LOgistique, Gestion Industrielle et de la Qualité (LOGIQ), Institut Supérieur de Gestion Industrielle de Sfax, University de Sfax, Route de l'Aéroport Km 0,5, B.P. 1069, 3029 Sfax, Tunisie

[3] Interuniversity Research Centre on Enterprise Networks, Logistics and Transportation (CIRRELT)

[4] Department of Operations and Decision Systems, Université Laval, 2325 de la Terrasse, Québec, Canada G1V 0A6

[5] Department of Management Sciences, HEC Montréal, 3000 chemin de la Côte-Sainte-Catherine, Montréal, Canada H3T 2A7

**Abstract.** We introduce, model and solve to optimality a rich multi-product, multi-period and multi-compartment vehicle routing problem with a required compartment cleaning activity. This real-life application arises in the olive oil collection process in Tunisia, where regional collection offices dispose of a fleet of vehicles to collect one or several grades of olive oil from a set of producers. For each grade, the quantity offered by a producer changes dynamically over the planning horizon. We first provide a mathematical formulation for the problem, along with a set of known and new valid inequalities. We then propose an exact branch-and-cut algorithm in order to solve the problem. We evaluate the performance of the algorithm on real data sets under different transportation scenarios to demonstrate to our industrial partner the advantages of using multi-compartment vehicles.

**Keywords**. Vehicle routing with compartments; incompatible products; valid inequalities; branch-and-cut.

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* Corresponding author: Leandro.Coelho@cirrelt.ca

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# 1 Introduction

We introduce, model and solve a real-world application of a multi-product, multi-period and multi-compartment vehicle routing problem (MPPC-VRP) arising in the collection of olive oil in Tunisia. In 2012, that country was the fourth largest exporter of olive oil worldwide, with an export production of 163,000 tons. This amount was expected to increase in 2013 according to the General Directorate for Research at the Ministry of Agriculture. For climatic and geographical reasons, olive groves are rather widespread in the central part of the country, as shown in Figure 1. Collecting olive oil is particularly important during the four-month production season. It mobilizes considerable human and material resources, and timeliness is crucial in this operation. Producers work non-stop 24 hours a day in order not to damage the harvest. On any given day, olive oil collection is carried out over six periods lasting almost 24 hours in total. This activity is performed by a fleet of capacitated heterogeneous vehicles, often with compartments of equal or different sizes, all equipped with a debit meter, enabling the decision maker to have full knowledge of the load contained in each compartment at all times. The oil must be collected before the producer runs out of storage space. A good forecast is available for the production rate of each product by each producer.

Olive oil comes in three different grades known as *extra*, *virgin*, and *lampante*. The top two grades with superior tastes are extra and virgin, which are suitable for consumption, whereas lampante oil is mostly destined for industrial uses. The transportation is regulated by law in order to protect the natural flavors of the oils. In particular, the different grades must be kept separate during transportation, hence the need to have multi-compartment vehicles. It is forbidden to load superior grades immediately after lampante oil in the same compartment, unless it has been cleaned before the changeover. The cleaning activity generates a cost and takes time.

Routing problems with a cleaning activity have not been widely studied from a scientific perspective, but similar constraints appear in other contexts. Thus, Oppen et al. [26]

**Figure 1:** Map of Tunisia pinpointing producers and regional offices locations (Source: Google Maps, accessed March 2014)


⌂: Regional offices, (1-9, A-G): Producers

consider the problem of transporting different types of live animals from farms to slaughterhouses by means of multi-compartment vehicles. They add time between consecutive tours to allow for unloading and disinfection of the vehicles. Hvattum et al. [19] deal with a tank allocation problem arising in the shipping of bulk oil and chemical products by tanker ships. They consider that a cleaning activity is required if two incompatible products are assigned to the same compartment within less than three trips.

The use of fleet with several compartments is common in fuel and oil distribution [2, 7, 9, 10, 13, 31, 33] and in some maritime applications [4, 17, 19, 32]. Transporting oil and fuel with multi-compartment vehicles is more challenging and interesting from a scientific point of view than transporting food, where dry, refrigerated and frozen commodities can be pre-assigned to suitable compartments. In this case, the loading problem reduces to a simple capacity checking procedure [11, 12, 24, 25]. In contrast, in fuel transportation, a routing problem and a compartment assignment problem must be solved jointly. For more details on the loading aspect arising in vehicle routing problems with compartments, see Lahyani et al. [23] and Pirkwieser et al. [29]. Multi-compartment problems have also been investigated in the context of inventory-routing [6, 26, 28, 34]. Other multi-period problems dealing with the joint optimization of transportation and inventory include [1, 3, 18].

Multi-compartment vehicles are often not equipped with debit meters, which implies that a compartment must satisfy at most one request. This scenario has been extensively studied [7, 8, 9, 10, 30, 34]. Coelho and Laporte [5] introduce and define the generalized case where compartments may be equipped with debit meters, and the load of a compartment may be split between different customers. They also distinguish between the cases where customers may or may not receive the visit of more than one vehicle per period. The authors assess the difficulty of the problem with split compartments and multiple visits per period. Derigs et al. [11] provide a literature review on the VRP with compartments as it arises in food and petrol delivery.

In this paper, we introduce other complicating constraints reflecting the complexities of the olive oil industry. The mathematical model developed for this application considers two vehicle types which differ in the number and sizes of the compartments. In the olive oil industry, a producer typically offers several products in different quantities in each period. The grades and quantities offered depend on the producer. All offers must be picked up in each period. These assumptions, together with the presence of the cleaning activity, increase the difficulty of the problem and make it very difficult to solve by exact algorithms for most instances of realistic sizes.

From a scientific perspective, we introduce, model, and solve exactly a difficult and rich variation of the well-known vehicle routing problem [22]. Part of the complexity of the problem comes from the requirement to clean the compartments, depending on the assignment and scheduling decisions. From a practical standpoint, we provide a tool that can assist managerial decision making at the tactical and operational levels. In particular, we are able to compare several transportation alternatives for the service provider and to evaluate the potential routing savings yielded by replacing single compartment vehicles with multi-compartment ones.

The remainder of this paper is organized as follows. In Section 2 we provide a formal description of the problem and we introduce a mathematical model complemented with known and new valid inequalities. The branch-and-cut algorithm is described in Section 3. We present computational results on real and artificially generated instances in Section 4, followed by conclusions in Section 5.

# 2 Mathematical description of the problem

We first introduce some notation, followed by a mathematical model and valid inequalities.

## 2.1 Notation

The MPPC-VRP is defined on an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ is the vertex set and $\mathcal{E}$ is the edge set. The set $\mathcal{V} = \{0, \ldots, n\}$ contains the locations of the depot 0 and of the producers $\mathcal{V}' = \{1, \ldots, n\}$. The set $\mathcal{E} = \{(i, j) : i \in \mathcal{V}, j \in \mathcal{V}', i < j\}$ defines the edges of the problem. A routing cost $\alpha_{ij}$ is known for each edge $(i, j) \in \mathcal{E}$. The problem is defined over a finite planning horizon $\mathcal{T} = \{1, \ldots, p\}$, and a set $\mathcal{K} = \{1, \ldots, K\}$ of heterogeneous vehicles is available at the depot. Each vehicle $k$ contains a set $\mathcal{L}^k$ of compartments equipped with a debit meter, and each compartment $l \in \mathcal{L}^k$ has a capacity $Q^{lk}$. The use of vehicle $k$ incurs a fixed cost $\beta^k$. Identical products collected from different producers can be loaded into the same compartment provided there is sufficient capacity. Also, each producer may be visited by more than one vehicle in any given period.

A set $\mathcal{M} = \{1, \dots, M\}$ of products are offered. For ease of notation, we assume that products are ordered according to their grade, and if the product of the lowest grade, i.e., $M$, is loaded into a compartment of a vehicle, it contaminates the compartment and a cleaning procedure is required to load any product $m < M$ ($m \in \mathcal{M}$) in subsequent periods. If a cleaning procedure is undertaken, it generates a cost $\gamma$. Each producer $i \in \mathcal{V}'$ offers a quantity $d_i^{mt}$ of product of type $m$ in period $t$ with which is associated a transportation request. Not all producers offer all types of products, and the available quantities change from one period to the next.

## 2.2 Mathematical model

We now provide an integer linear programming formulation for the MPPC-VRP, followed by known and new valid inequalities used to strengthen the mathematical model. The decision variables are defined as follows. The routing variables $x_{ij}^{kt}$ are equal to the number of times edge $(i, j)$ is used by vehicle $k$ in period $t$; the visiting variables $y_i^{kt}$ are binary and equal to one if and only if vertex $i$ is visited by vehicle $k$ in period $t$; the assignment variables $z_{im}^{lkt}$ are equal to one if and only if product $m$ of producer $i$ is loaded into compartment $l$ of vehicle $k$ in period $t$; likewise, the variables $w_m^{lkt}$ are equal to one only if product $m$ is loaded in compartment $l$ of vehicle $k$ in period $t$. Finally, the variables $u^{lkt}$ are binary and indicate whether or not compartment $l$ of vehicle $k$ is cleaned at the beginning of period $t$.

Recall that not all producers offer all products at each period. Nevertheless, we define variables $z_{im}^{lkt}$ for all combinations of producers and products, because this simplifies the formulation. If a producer $i$ does not offer product $m$ in period $t$, we then set $z_{im}^{lkt} = 0$ for all compartments $l$ and vehicles $k$. Finally, we need to define the set $\mathcal{T}_s^t$ containing the periods elapsed between $s$ and $t$, i.e., $\mathcal{T}_s^t = \{s + 1, \dots, t - 1\}$.

The problem can then be modeled as follows:

$$\text{minimize} \sum_{t\in\mathcal{T}}\sum_{k\in\mathcal{K}}\sum_{(i,j)\in\mathcal{E}} \alpha_{ij}x_{ij}^{kt} + \sum_{t\in\mathcal{T}}\sum_{k\in\mathcal{K}}\sum_{j\in\mathcal{V}'} \beta^k y_0^{kt} + \sum_{t\in\mathcal{T}}\sum_{k\in\mathcal{K}}\sum_{l\in\mathcal{L}^k} \gamma u^{lkt} \tag{1}$$

subject to

$$\sum_{i\in\mathcal{V}'} d_i^{mt} z_{im}^{lkt} \le Q^{lk} \quad k\in\mathcal{K}, l\in\mathcal{L}^k, m\in\mathcal{M}, t\in\mathcal{T} \tag{2}$$

$$z_{im}^{lkt} \le y_i^{kt} \quad i\in\mathcal{V}', k\in\mathcal{K}, l\in\mathcal{L}^k, m\in\mathcal{M}, t\in\mathcal{T} \tag{3}$$

$$z_{im}^{lkt} \le w_m^{lkt} \quad i\in\mathcal{V}', k\in\mathcal{K}, l\in\mathcal{L}^k, m\in\mathcal{M}, t\in\mathcal{T} \tag{4}$$

$$w_m^{lkt} \le \sum_{i\in\mathcal{V}'} z_{im}^{lkt} \quad k\in\mathcal{K}, l\in\mathcal{L}^k, m\in\mathcal{M}, t\in\mathcal{T} \tag{5}$$

$$\sum_{m\in\mathcal{M}} w_m^{lkt} \le 1 \quad k\in\mathcal{K}, l\in\mathcal{L}^k, t\in\mathcal{T} \tag{6}$$

$$\sum_{j\in\mathcal{V}, i<j} x_{ij}^{kt} + \sum_{j\in\mathcal{V}, j<i} x_{ji}^{kt} = 2y_i^{kt} \quad i\in\mathcal{V}, k\in\mathcal{K}, t\in\mathcal{T} \tag{7}$$

$$\sum_{i\in\mathcal{Z}}\sum_{j\in\mathcal{Z}, i<j} x_{ij}^{kt} \le \sum_{i\in\mathcal{Z}} y_i^{kt} - y_z^{kt} \quad \mathcal{Z}\subseteq\mathcal{V}', z\in\mathcal{Z}, k\in\mathcal{K}, t\in\mathcal{T} \tag{8}$$

$$\sum_{k\in\mathcal{K}}\sum_{l\in\mathcal{L}^k} z_{im}^{lkt} = 1 \quad i\in\mathcal{V}', m\in\mathcal{M}, t\in\mathcal{T} \tag{9}$$

$$w_M^{lks} + w_m^{lkt} \le 1 + u^{lkt} + \sum_{t'\in\mathcal{T}_s^t} u^{lkt'} \quad k\in\mathcal{K}, l\in\mathcal{L}^k, m\in\mathcal{M}\backslash\{M\}, s,t\in\mathcal{T} \tag{10}$$

$$x_{0j}^{kt} \in \{0,1,2\} \quad j\in\mathcal{V}', k\in\mathcal{K}, t\in\mathcal{T} \tag{11}$$

$$x_{ij}^{kt} \in \{0,1\} \quad i,j\in\mathcal{V}', k\in\mathcal{K}, t\in\mathcal{T} \tag{12}$$

$$y_i^{kt}, z_{jm}^{lkt}, w_m^{lkt}, u^{lkt} \in \{0,1\} \quad i\in\mathcal{V}, j\in\mathcal{V}', k\in\mathcal{K}, l\in\mathcal{L}^k, m\in\mathcal{M}, t\in\mathcal{T}. \tag{13}$$

The objective function (1) minimizes the sum of the routing cost, the vehicle fixed cost and the compartment cleaning cost. Constraints (2) ensure that the capacity of each compartment is not violated. Constraints (3)−(5) link the variables $y_i^{kt}$, $w_m^{lkt}$, and $z_{jm}^{lkt}$. Specifically, constraints (3) ensure that a product from a producer is loaded into a given compartment of a vehicle at a given period only if the producer is served by the vehicle. Constraints (4) and (5) guarantee that a compartment is allowed to carry a product in a given period only if the vehicle visits a producer offering that product in the same period. Constraints (6) ensure that each compartment carries at most one type of product at a time. Constraints (7) and (8) are degree and subtour elimination constraints, respectively. Constraints (9) ensure that all the quantities being offered are collected. Constraints (10) mean that a cleaning operation is performed if necessary. The term $\sum_{t' \in \mathcal{T}_s^t} u^{lkt'}$ keeps track of each compartment cleaning operations for the interval between periods $s$ and $t$. Constraints (11)$-$(13) define the integer and binary nature of the variables.

### 2.3 Valid inequalities

The formulation defined by (1)$-$(13) is sufficient to model the MPPC-VRP. We can, however, strengthen it through the inclusion of valid inequalities in the form of symmetry breaking constraints and additional cuts imposing upper bounds on the integer variables. The first ones are related to the period in which a cleaning operation takes place. For example, suppose a contaminating product is transported by a given compartment in period $t = 1$, and the next use of this compartment is to carry a higher grade product in period $t = 5$. Then, at least four optimal solutions exist, by cleaning the compartment in period 2, 3, 4, or 5. In order to avoid such symmetries, we impose constraints (14) which postpone the cleaning operation as much as possible:

$$u^{lkt} \leq \sum_{m \in \mathcal{M}} w_m^{lkt} \quad k \in \mathcal{K}, l \in \mathcal{L}^k, t \in \mathcal{T}. \tag{14}$$

We also integrate the vehicle and the compartment symmetry breaking constraints for the first period, when no cleaning operation is necessary. These constraints are inspired from those proposed in [5]. Constraints (15) and (16) are valid if the considered vehicles and compartments are identical. We define the set $\mathcal{K}' \subset \mathcal{K}$ containing only the homogeneous vehicles of $\mathcal{K}$. Note that in the olive oil industry as in petrol distribution, vehicles with different compartments capacities are seldom used in order to reduce the imbalance of loaded vehicles on the road [11]. These symmetry breaking constraints are

$$\sum_{i\in\mathcal{V}} y_i^{k1} \leq \sum_{i\in\mathcal{V}} y_i^{k-1,1} \quad k \in \mathcal{K}'\backslash\{1\} \tag{15}$$

and

$$w_h^{lk1} \leq \sum_{m\in\mathcal{M}} w_m^{l-1,k,1} \quad k \in \mathcal{K}, l \in \mathcal{L}^k\backslash\{1\}, h \in \mathcal{M}. \tag{16}$$

Constraints (15) rank identical vehicles according to the index of the producers served. In particular, they ensure that among vehicles of the same type, vehicle $k$ cannot serve any customer if vehicle $k-1$ has not already been used in the first period. Constraints (16) state that if deliveries are performed using compartment $l$, then compartment $l-1$ has already been used. These rules cannot be generalized to the remaining periods because they may impact the cleaning operation and ultimately affect the solution cost.

We also make use of a known set of useful cuts to enforce logical relationships between routing, visiting and assignments variables. For more details on logical inequalities for routing problems, see Coelho and Laporte [5] and Gendreau et al. [15]. These cuts are as follows:

$$x_{0i}^{kt} \leq 2y_i^{kt} \quad i \in \mathcal{V}', k \in \mathcal{K}, t \in \mathcal{T} \tag{17}$$

$$x_{ij}^{kt} \leq y_i^{kt} \quad i, j \in \mathcal{V}', k \in \mathcal{K}, t \in \mathcal{T} \tag{18}$$

$$y_i^{kt} \leq y_0^{kt} \quad i \in \mathcal{V}', k \in \mathcal{K}, t \in \mathcal{T} \tag{19}$$

$$y_i^{kt} \leq \sum_{m\in\mathcal{M}} \sum_{l\in\mathcal{L}^k} z_{im}^{lkt} \quad i \in \mathcal{V}', k \in \mathcal{K}, t \in \mathcal{T} \tag{20}$$

$$y_i^{kt} \leq \sum_{m\in\mathcal{M}} \sum_{l\in\mathcal{L}^k} w_m^{lkt} \quad i \in \mathcal{V}', k \in \mathcal{K}, t \in \mathcal{T} \tag{21}$$

$$\sum_{i\in\mathcal{V}'} y_i^{kt} \leq \sum_{i\in\mathcal{V}'} \sum_{m\in\mathcal{M}} \sum_{l\in\mathcal{L}^k} z_{im}^{lkt} \quad k \in \mathcal{K}, t \in \mathcal{T}. \tag{22}$$

Constraints (17), (18) and (19) are referred to as routing cuts, as a way to ensure that if a producer $i$ is visited, i.e., (17) holds, or a producer $j$ is the successor of a producer $i$, i.e., (18) holds on the route of vehicle $k$ in period $t$, then the producer $i$ to be visited, i.e.,

$y_i^{kt} = 1$. Constraints (18) can also be considered as subtour elimination constraints (8) when $|\mathcal{Z}| = 2$. Inequalities (19) guarantee that if vehicle $k$ visits producer $i$ in period $t$, then the depot must be included in the route of vehicle $k$ in period $t$. Through constraints (20) and (21), we ensure that if a producer $i$ is visited in period $t$ by vehicle $k$, then at least one product $m$ should be loaded in some compartment of vehicle $k$ in that period. Constraints (22) strengthen the relationships between the collection routes, products and compartments. Specifically, a collection route using vehicle $k$ in period $t$ exists to ensure the pickup of some products from a producer $i$ and load them in some compartment of that vehicle. We also note that constraints (22) are the sum over the locations of (20). Even if these constraints are redundant in this context, they are known to help the mathematical programming solvers to derive new cuts and improve the overall algorithmic performance [6, 16, 20, 21].

# 3 Branch-and-cut algorithm

We have implemented a branch-and-cut algorithm capable of solving the formulation just presented. All variables of the formulation are explicitly handled by the algorithm. Since the number of constraints (17)−(22) is polynomial, they are added a priori to the model. In the sequel, we will show how each subset of inequalities impacts its solution. In contrast, we cannot generate all subtour elimination constraints (8) a priori since their number is exponential. These are dynamically generated as cuts as they are found to be violated. The formulation is then solved by branch-and-cut as follows. At a generic node of the search tree, a linear program with relaxed integrality constraints is solved, a search for violated constraints is performed, appropriate valid inequalities are added to eliminate subtours, and the current subproblem is then reoptimized. This process is reiterated until a feasible or dominated solution has been reached, or until no more cuts can be added. At this point, branching on a fractional variable occurs. We provide in Algorithm 1 a sketch of the branch-and-cut scheme.

**Algorithm 1** Branch-and-cut algorithm

At the root node of the search tree, generate and insert all valid inequalities into the program.
$z^* \leftarrow \infty$.
Termination check:
**if** there are no more nodes to evaluate **then**
    Stop with the incumbent and optimal solution of cost $z^*$.
**else**
    Select one node from the branch-and-bound tree.
**end if**
Subproblem solution: solve the LP relaxation of the node and let $z$ be its cost.
**if** the current solution is feasible **then**
    **if** $z \geq z^*$ **then**
        Go to the termination check.
    **else**
        $z^* \leftarrow z$.
        Update the incumbent solution.
        Prune the nodes with a lower bound larger than or equal to $z^*$.
        Go to the termination check.
    **end if**
**end if**
Cut generation:
**if** the solution of the current LP relaxation violates any cuts **then**
    Identify connected components as in Padberg and Rinaldi [27].
    Determine whether the component containing the producer is weakly connected as in Gendreau et al. [14].
    Add violated subtour elimination constraints (8).
    Go to the subproblem solution.
**end if**
Branching: branch on one of the fractional variables.
Go to the termination check.

# 4 Computational experiments

In this section we describe the computational experiments carried out in order to assess the performance of our model and algorithm. We provide in Section 4.1 details of the real instances we have obtained from our Tunisian partner, and the instances generated with a different fleet composition. In Section 4.2 we describe the results of computational experiments performed to evaluate the effectiveness of the cuts and valid inequalities, and we compare our solutions with those corresponding to the current situation.

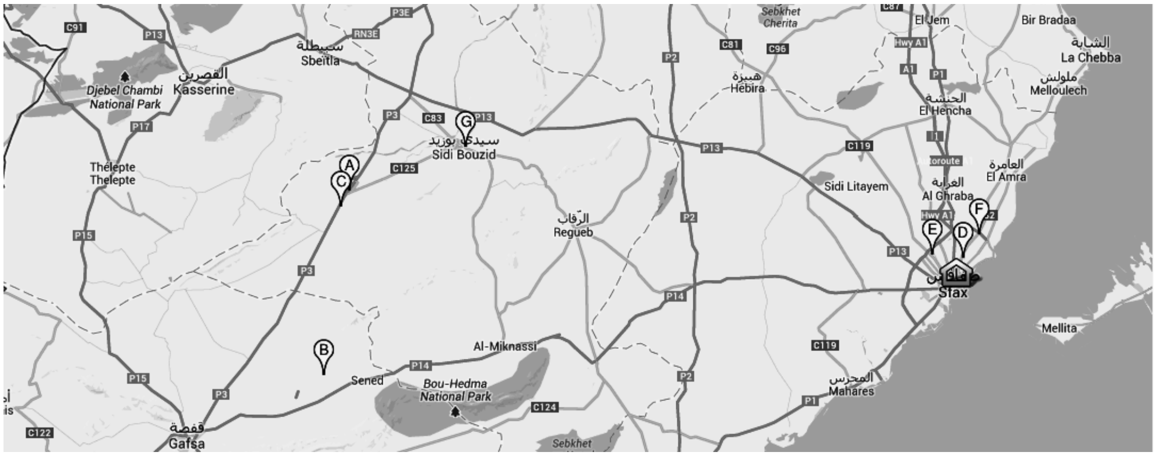
We have coded the algorithm in C++ using IBM CPLEX Studio 12.5.1 as the MIP solver. All computations were executed on a grid of Intel Xeon™ processors running at 2.66 GHz with up to 96 GB of RAM installed per node, with the Scientific Linux 6.1 operating system. All instances and detailed results are available from `http://www.leandro-coelho.com`.

## 4.1 Instance generation

We have created a set of five instances based on real data gathered from industrial partners in the regions of Sfax and Kairouan in Tunisia. In terms of size, we handle instances with one depot and up to 45 transportation requests loaded in three or four vehicles. The product quantities being offered are either obtained from our partner, or estimated when these could not be made publicly available for confidentiality reasons. There is no restriction on the number of producers that can be visited on a route. The quantities of available extra, virgin and lampante oils represent 56%, 30% and 14% of the total available production, respectively. The routable network dataset is constructed using real travel distances. Only driving distances provided by the regional office of Sfax take traffic congestion and the road state into account. The geographical locations of the producers of this case study are listed in Table 1 and depicted in Figures 2 and 3.

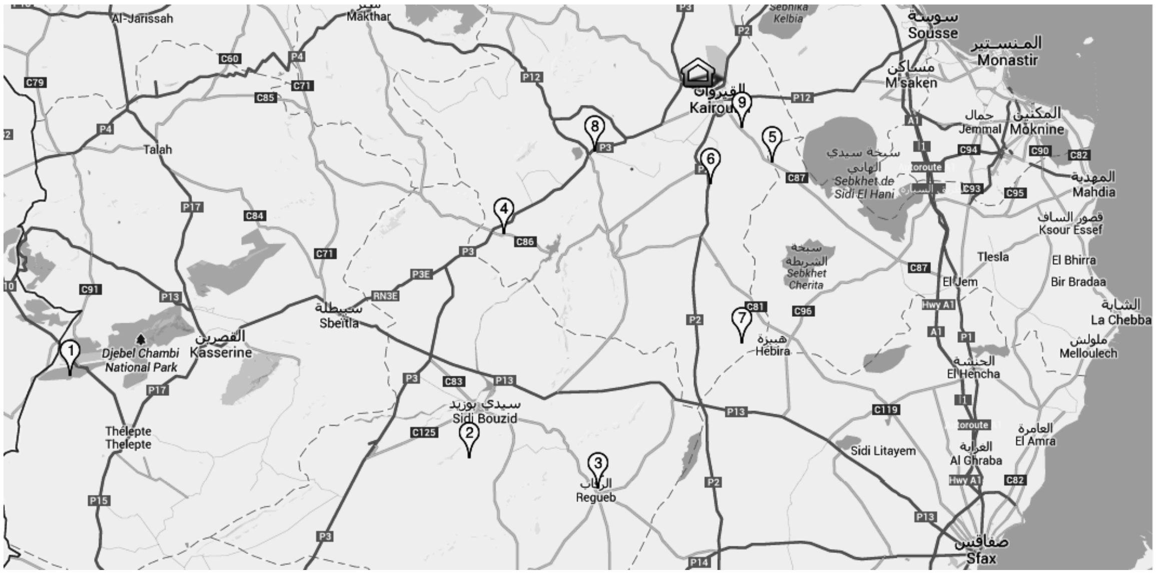
The industrial partners of the regions Sfax and Kairouan do not currently dispose of the same fleet composition. The first office controls a heterogeneous limited size fleet of single-

**Figure 2:** Geographical locations of the producers around the region of Sfax (Source: Google Maps, accessed March 2014)




compartment vehicles having a capacity of 10 tons, and double-compartment vehicles in which each compartment has a capacity of five tons. These two configurations will be denoted by type I and type II, respectively. The regional office of Kairouan, which presently uses single-compartment vehicles only, will revise its short-term procurement policy after we will have identified the cost savings achieved by using multi-compartment vehicles. Some drivers are outsourced, which enables the service providers to perform tours and to exploit the available fleet over six periods spread out during day-time and in the evening. The data exploitation results in five original instances, with an asterisk prepended to their names. To cover the different scenarios under a different fleet composition, we generate 10 instances identical to the original ones, but with a different set of vehicles. The names of the test instances highlight the factors that may affect the results. These factors include the first four letters of the regional office they refer to, the total number of requests, and the number of vehicles of type I and type II, respectively. Since the original instances may contain some products with a supply of more than five tons, some restrictions had to be made while generating the fleet composition. Each collection route requires at least one single-compartment vehicle for the Kairouan region, and two single-compartment vehicles for the Sfax region. Table 2 summarizes the characteristics of these instances. Regarding the objective function, we have used the following parameters after a tuning phase and

**Figure 3:** Geographical locations of the producers around the region of Kairouan (Source: Google Maps, accessed March 2014)




discussions with our industrial partner:

- $\alpha_{ij} =$ €1 per driving kilometer between vertices $i$ and $j$;
- $\beta^k =$ €15 per vehicle $k$ used per period $t$ regardless of the vehicle type;
- $\gamma =$ €10 per cleaning activity.

## 4.2 Computational results

We have run the proposed algorithm over the data sets shown on Table 2. Table 3 summarizes the performance of the algorithm. We assess the performance of the routing and assignment cuts by comparing the solutions obtained for different configurations. The implementation with constraints (1)−(16) is used as a reference point. Specifically, for each instance test we present the number of nodes explored in the branch-and-cut tree, the ratio of the lower bound at the root node between the configuration with cuts and the basic configuration (1)−(16), and finally the running time in seconds.

**Table 1:** Producers locations in Figures 2 and 3

| Producer | Location | Producer | Location |
|---|---|---|---|
| 1 | Fériana, Kasserine | A | Bir El Hfey, Sidi Bouzid |
| 2 | Jelma, Sidi Bouzid | B | Sened, Gafsa |
| 3 | Regueb, Sidi Bouzid | C | Sidi Ali Ben Aoun, Sidi Bouzid |
| 4 | Hajeb El Ayoun, Kairouan | D | North of Sfax |
| 5 | El Khazaziya, Kairouan | E | Road Gremda, Sfax |
| 6 | Boussari, Kairouan | F | Road Mahdia, Sfax |
| 7 | Cherarda, Kairouan | G | Sidi Bouzid |
| 8 | El Houareb, Kairouan | | |
| 9 | South of Kairouan | | |

**Table 2:** Configurations of the real-world instances

| Instance | # Producers | # Requests | Fleet composition | |
|---|---|---|---|---|
| | | | Type I | Type II |
| *Kair_27_3_0 | 6 | 27 | 3 | 0 |
| Kair_27_1_2 | 6 | 27 | 1 | 2 |
| Kair_27_2_1 | 6 | 27 | 2 | 1 |
| *Kair_33_3_0 | 5 | 33 | 3 | 0 |
| Kair_33_1_2 | 5 | 33 | 1 | 2 |
| Kair_33_2_1 | 5 | 33 | 2 | 1 |
| *Kair_34_3_0 | 6 | 34 | 3 | 0 |
| Kair_34_1_2 | 6 | 34 | 1 | 2 |
| Kair_34_2_1 | 6 | 34 | 2 | 1 |
| *Kair_45_3_0 | 7 | 45 | 3 | 0 |
| Kair_45_1_2 | 7 | 45 | 1 | 2 |
| Kair_45_2_1 | 7 | 45 | 2 | 1 |
| *Sfax_39_2_2 | 7 | 39 | 2 | 2 |
| Sfax_39_3_1 | 7 | 39 | 3 | 1 |
| Sfax_39_4_0 | 7 | 39 | 4 | 0 |

The algorithm proves optimality over all the 15 instances within very short computing times. For most of the instances of the region of Kairouan with three vehicles and up to seven producers, the algorithm takes less than one second to reach optimality. However, it requires more computational effort for the instances of Sfax, especially when the proposed cuts are disabled. This may be explained by the fact that increasing the number of vehicles generates much more symmetry.

A deeper analysis of the tested configurations shows that the introduction of valid inequalities significantly improves the performance of the algorithm. The average running time is reduced from 50.58 to 2.77 seconds and the average number of explored nodes goes down from 101,184 to 384 when the full model is implemented. The short computational time results from the fact that the model provides a high quality lower bound at the root node of the search tree. On average, the lower bound of the model with all cuts is almost equal to 1.5 times the initial lower bound value of the basic formulation (1)−(16). The full set of proposed cuts is essential to achieve the best algorithmic performance. In particular, the assignment cuts (20)−(22) have a more positive impact on these instances than the routing cuts (17)−(19). Without the assignment cuts, the model requires more iterations and explores more nodes to identify the best solution. On average, the algorithm explores respectively 660 and 2,253 nodes when using separately assignment cuts and routing cuts. Under these two configurations, we have obtained almost the same improvement in the lower bound value (1.31 against 1.20) by imposing the assignment cuts.

When this study was undertaken, our industrial partner was trying to find ways to minimize the total associated logistic costs, i.e., the fixed and variable costs related to transport and cleaning related costs. In particular, it was paying close attention to the fleet composition component and its impact on the overall costs. Using our methodology, we can provide alternative solutions to the managers by generating 10 instances while the overall capacity remains unchanged and the number of vehicles of types I and II varies. Table 4 summarizes these results. We note that substantial savings are achieved if both types of vehicles are used. We observe that a combination of vehicles yields better quality solu-

**Table 3:** Summary of computational results of the test instances with respect to several configurations

| Model | Instance | # Nodes | Lower bound increase | Time(s) | Instance | # Nodes | Lower bound increase | Time(s) |
|---|---|---|---|---|---|---|---|---|
| (1)-(16) | *Kair_27_3_0 | 1250 | 1 | 4.32 | Kair_34_2_1 | 219 | 1 | 3.36 |
| (1)-(19) | | 98 | 1.28 | 1.19 | | 114 | 1.25 | 2.45 |
| (1)-(16), (20)-(22) | | 125 | 1.45 | 1.16 | | 88 | 1.29 | 1.89 |
| (1)-(22) | | 1 | 1.54 | 0.30 | | 32 | 1.40 | 1.46 |
| (1)-(16) | Kair_27_1_2 | 3242 | 1 | 19.92 | *Kair_45_3_0 | 801 | 1 | 7.11 |
| (1)-(19) | | 212 | 1.28 | 3.49 | | 94 | 1.20 | 2.9 |
| (1)-(16), (20)-(22) | | 237 | 1.13 | 2.51 | | 17 | 1.18 | 1.32 |
| (1)-(22) | | 119 | 1.54 | 2.07 | | 1 | 1.39 | 0.54 |
| (1)-(16) | Kair_27_2_1 | 50 | 1 | 1.98 | Kair_45_1_2 | 230 | 1 | 5.52 |
| (1)-(19) | | 14 | 1.25 | 0.73 | | 256 | 1.21 | 5.61 |
| (1)-(16), (20)-(22) | | 7 | 1.29 | 0.82 | | 78 | 1.08 | 3.3 |
| (1)-(22) | | 1 | 1.39 | 0.26 | | 93 | 1.26 | 2.89 |
| (1)-(16) | *Kair_33_3_0 | 85 | 0.89 | 0 | Kair_45_2_1 | 88 | 1 | 3.86 |
| (1)-(19) | | 22 | 1.57 | 0.68 | | 39 | 1.19 | 2.18 |
| (1)-(16), (20)-(22) | | 4 | 1.36 | 0.22 | | 76 | 1.12 | 2.63 |
| (1)-(22) | | 1 | 1.80 | 0.16 | | 5 | 1.31 | 1.20 |
| (1)-(16) | Kair_33_1_2 | 34 | 1 | 2.2 | *Sfax_39_2_2 | 18008 | 1 | 98.51 |
| (1)-(19) | | 112 | 1.40 | 0.64 | | 6056 | 1.37 | 31.61 |
| (1)-(16), (20)-(22) | | 74 | 1.13 | 1.73 | | 5314 | 1.10 | 26.5 |
| (1)-(22) | | 28 | 1.43 | 0.57 | | 1090 | 1.85 | 7.06 |
| (1)-(16) | Kair_33_2_1 | 105 | 1 | 2.2 | Sfax_39_3_1 | 19090 | 1 | 99.01 |
| (1)-(19) | | 76 | 1.51 | 0.96 | | 9373 | 1.29 | 48.64 |
| (1)-(16), (20)-(22) | | 28 | 1.25 | 1.31 | | 732 | 1.04 | 7.34 |
| (1)-(22) | | 21 | 1.68 | 0.70 | | 1801 | 1.77 | 10.66 |
| (1)-(16) | *Kair_34_3_0 | 678 | 1 | 3.11 | Sfax_39_4_0 | 108657 | 1 | 504.11 |
| (1)-(19) | | 54 | 1.28 | 1.68 | | 16831 | 1.31 | 76.38 |
| (1)-(16), (20)-(22) | | 108 | 1.43 | 1.75 | | 2943 | 1.04 | 14.92 |
| (1)-(22) | | 6 | 1.53 | 0.60 | | 2504 | 1.79 | 12.15 |
| (1)-(16) | Kair_34_1_2 | 216 | 1 | 2.56 | | | | |
| (1)-(19) | | 82 | 1.28 | 1.24 | | | | |
| (1)-(16), (20)-(22) | | 72 | 1.13 | 1.47 | | | | |
| (1)-(22) | | 60 | 1.29 | 0.87 | | | | |

tions for all Kairouan related instances, with improvements ranging from 1.9% to 21.7%, and averaging 11.7%. For the instances related to the office of Sfax, the best solutions are obtained with the current fleet composition, i.e., by using two vehicles of each type. One possible explanation is that compartments equipped with debit meters enable the collection of small quantities of the same product and the segregation of non-mixable products.

Finally, we compare our results to the solution currently applied by our industrial partner. Table 4 indicates that the proposed method provides an improvement over the current solution designed manually by the dispatcher since it reduces the overall costs for all the data sets. Our solutions minimize the distance traveled with an improvement of up to 7% and optimize the products assignment process to avoid unnecessary cleaning costs. However, the same number of vehicles is needed to cover all the producers' locations. We have run further test with an hierarchical objective function, which first minimizes the required number of vehicles, and then the routing and cleaning costs. These tests reveal that the current number of vehicles used by our industrial partner is in fact optimal. For the managers, optimizing both the collection routes and the fleet composition is important but difficult to achieve through manual methods. They were asked to evaluate the solutions obtained and declared themselves very satisfied with the results.

# 5 Conclusions

We have tackled a real-world and rich multi-compartment vehicle routing problem arising in the olive oil collection industry. This practical application concerns the pick-up of one or more commodities from a set of geographically scattered producers in the center of Tunisia. We have presented a mathematical model including known and new valid inequalities, as well as a branch-and-cut algorithm for its solution. We have tested the algorithm on several sets of realistic instances. Our experimental results show the effectiveness of the proposed algorithm. Extensive tests have enabled us to generate solutions that can help

**Table 4:** Relative savings compared to existing situation

| Instance | Optimized solution | | | Manual solution | | |
|---|---|---|---|---|---|---|
| | # Km | # Vehicles | # Cleanings | # Km | # Vehicles | # Cleanings |
| *Kair_27_3_0 | 2361 | 17 | 1 | 2435 | 17 | 0 |
| Kair_27_1_2 | 1848 | 15 | 0 | - | - | - |
| Kair_27_2_1 | 1848 | 15 | 1 | - | - | - |
| *Kair_33_3_0 | 2173 | 16 | 0 | 2339 | 16 | 0 |
| Kair_33_1_2 | 1990 | 16 | 0 | - | - | - |
| Kair_33_2_1 | 2104 | 16 | 0 | - | - | - |
| *Kair_34_3_0 | 2737 | 17 | 1 | 2829 | 17 | 1 |
| Kair_34_1_2 | 2322 | 16 | 1 | - | - | - |
| Kair_34_2_1 | 2424 | 17 | 1 | - | - | - |
| *Kair_45_3_0 | 4477 | 18 | 0 | 4721 | 18 | 0 |
| Kair_45_1_2 | 4008 | 17 | 3 | - | - | - |
| Kair_45_2_1 | 4388 | 17 | 0 | - | - | - |
| *Sfax_39_2_2 | 2755 | 21 | 0 | 2866 | 21 | 0 |
| Sfax_39_3_1 | 2787 | 21 | 0 | - | - | - |
| Sfax_39_4_0 | 2811 | 21 | 0 | - | - | - |

support decision making at the tactical level, i.e., the purchase of new vehicles, and at the operational level, i.e., the redesign of vehicle loading and of collection routes.

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