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**October 2009**

**CIRRELT-2009-43**

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# The Dynamic Multi-Period Vehicle Routing Problem

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**Abstract***.* This paper considers the *Dynamic Multi-Period Vehicle Routing Problem* which deals with the distribution of orders from a depot to a set of customers over a multi-period time horizon. Customer orders and their feasible service periods are dynamically revealed over time. The objectives are to minimize total travel costs and customer waiting, and to balance the daily workload over the planning horizon. This problem originates from a large distributor operating in Sweden. It is modeled as a mixed integer linear program, and solved by means of a three-phase heuristic that works over a rolling planning horizon. The multi-objective aspect of the problem is handled through a scalar technique approach. Computational results show that the proposed approach can yield high quality solutions within reasonable running times.

**Keywords**. Dynamic, multi-period, multi-objective, vehicle routing, variable neighborhood search.

**Acknowledgements.** This work was partly supported by the Natural Sciences and Engineering Research Council of Canada under grants 227837-04 and 39682-05. This support is gratefully acknowledged. We thank the anonymous referees for their valuable comments.

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## 1 Introduction

The purpose of this paper is to model and solve the *Dynamic Multi-Period Vehicle Routing Problem* (DMPVRP). Our study is motivated by the case of Lantmännen, a large distributor operating in Sweden, but our contribution is of general applicability. In the DMPVRP, customers place orders dynamically over a planning horizon consisting of several periods (or days). Each request specifies a demand quantity, a delivery location and a set of consecutive periods during which delivery can take place. The distributor must plan its delivery routes over several days so as to minimize the routing cost and customer waiting, and to balance the daily workload over the planning horizon.

Lantmännen is one of the largest groups within the food, energy and agricultural industries in the Nordic countries. The company is owned by 42,000 Swedish farmers, hires 13,000 employees, and generates sales of SEK 36 billion per year. One of its activities is the distribution of fodder to the farmers at their request from one of several terminals which usually operate independently of each other, except in periods of intense activity. Here we consider a single terminal, Västerås, located in southern Sweden. It is the busiest terminal in terms of number of vehicles and orders. Figure 1 shows the locations of the customers and of the terminal. The distribution problem is very complicated in practice and involves many special restrictions, for example, rules regarding the use of compartments in the vehicle and special loading restrictions. In this work, we consider a simplified problem in which customers place orders over time and the distribution schedule of a given day is constructed for several vehicles at the beginning of that day. It serves some of the unfulfilled orders and typically leaves some for the following days. A fair amount of foresight is required so as not to create infeasible situations in the future while creating efficient routes. Unfulfilled orders after the schedule has been built and new orders accumulated during the day are considered for scheduling the following day. Because the drivers do not interact with the customers when delivering, no time windows need to be specified.

The literature on the DMPVRP is rather scarce. To our knowledge, the closest work was done by Angelelli et al. (2007, 2009). Angelelli et al. (2007) have considered a special case of the DMPVRP in which each order has two consecutive feasible visit days after its arrival and only one uncapacitated vehicle is available each day. This problem has been extended in a paper by Angelelli et al. (2009) in which a fixed fleet of vehicles is available and on-line requests are considered by re-optimizing the plan during the day. A request is called on-line if it arrives during the day when

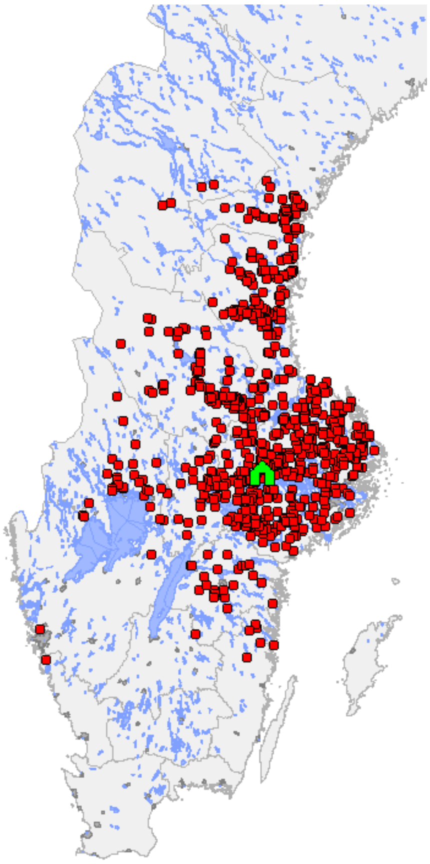

**Figure 1.** Locations of customers and depot (represented by a house) in the Lantmännen case study

the vehicles are already moving in the area. An on-line request can be either postponable or unpostponable, which means it must be served on the day that it arrives. It is assumed in the paper that unpostponable requests can only arrive before a fixed time in order to ensure the feasibility of the solution.

The DMPVRP is closely related to the *Periodic Vehicle Routing Problem* (PVRP) in which all information is available at the beginning of the planning horizon. In the PVRP, customers specify a service frequency and sets of allowable combinations of visit days. For example, if a customer specifies a frequency of 2 and the combinations {1, 3} and {2, 4}, then the customer wishes to be visited twice, on days 1 and 3, or on days 2 and 4. In the DMPVRP, visit frequencies are equal to 1 and visit combinations are made up of consecutive days. The PVRP is usually solved heuristically.

The best known algorithms for this problem are those of Cordeau, Gendreau and Laporte (1997) and of Hemmelmayr, Doerner and Hartl (2009). Francis, Smilowitz and Tzur (2008) have solved a variant of the PVRP in which service frequency is a decision variable. Mourgaya and Vanderbeck (2007) have solved another variant that includes routing cost minimization and daily workload balance.

Other routing problems with a dynamic component are often encountered in the context of dynamic pickup and delivery problems (Psaraftis, 1988; Mitrović-Minić, Krishnamurti and Laporte, 2004; Branke et al. 2005; Hvattum, Løkketangen and Laporte, 2006, 2007; Pureza and Laporte, 2008), but these papers do not consider a multi-period horizon. For recent literature reviews, see Larsen, Madsen and Solomon (2008), and Berbeglia, Cordeau and Laporte (2009).

Another strand of literature relevant to our problem is about the *Multi-Objective Vehicle Routing Problem* encountered in school bus routing (Pacheco and Marti, 2006; Alabas-Uslu, 2008), waste collection (Lacomme, Prins and Sevaux, 2006), and hazardous products transportation (Dell'Olmo, Gentili and Scozzari, 2005; Zografos and Androutsopoulos, 2008; Tan, Chew and Lee, 2006). The two main solution strategies for multi-objective problems are the scalar technique, which consists in minimizing a weighted linear combination of the objectives, and the Pareto method which identifies a set of non-dominated solutions. We refer to Jozefowiez, Semet and Talbi (2008) for a recent survey of these methods in the context of vehicle routing.

In this paper we formulate the DMPVRP as a mixed integer linear program using the scalar technique. We then develop a three-phase heuristic for its solution, and we show that our approach can yield high quality solutions within reasonable running times. The remainder of the paper is organized as follows. The mathematical model is described in Section 2. The heuristic is described in Section 3, followed by computational results in Section 4 and by conclusions in Section 5.

## 2 Mathematical Problem Description

We start with a more detailed description of the DMPVRP. To capture the problem more precisely, we also formulate it as a mixed integer linear program.

### 2.1 Problem description and analysis

The DMPVRP is solved over a planning horizon divided into days. Customer orders arrive at any time and must be fulfilled within a set of consecutive service days which can start as early as the day after the order is placed. A set of homogeneous vehicles are available at the depot. These vehicles depart from the depot at 00.00 and return to the depot at the latest at 23.59 on the same day. The objectives are to minimize the total routing cost (proportional to travel time) and customer waiting, and to balance the daily workload over the planning horizon. Each customer must be visited exactly once by one vehicle within its feasible service period, each vehicle must depart from and return to the depot in the same day, and the load of each vehicle cannot exceed its capacity.

This problem is dynamic in the sense that orders are revealed incrementally over time. The daily planning must determine which orders should be fulfilled on that day and in which sequence the vehicles should visit the customers. These decisions are made without the knowledge of future orders. However, even if the problem is dynamic, the routing problem at the beginning of each particular day over the planning horizon can be viewed as a static problem since the routes for that day are planned based on the orders known so far and the routes are fixed before their execution.

Figure 2 illustrates the planning process for a small DMPVRP example consisting of two days. For simplicity, we assume that the demand of each order is one, the capacity of the vehicle is three, and two vehicles are available. Before the first day, as shown in (a), six orders are already logged in the system. Three of these, denoted by triangles, can be fulfilled either on the first day or the second day, while the other three can only be served on the first day. At the beginning of the planning horizon, the planner has to construct the routing plan for the first day, as shown in (b). Before the second day is planned, three new orders have arrived, as shown in (c). The routes for the second day are shown in (d).

This example illustrates that the challenging part of the problem is to decide on the first day whether to serve the triangle orders, or whether to postpone them until the second day without knowing which orders will arrive during the first day. On the one hand, if the new orders are destined for locations close to those of the triangle orders, it may be wise to postpone them so as to minimize the total travel time. On the other hand, if too many orders are postponed, customer

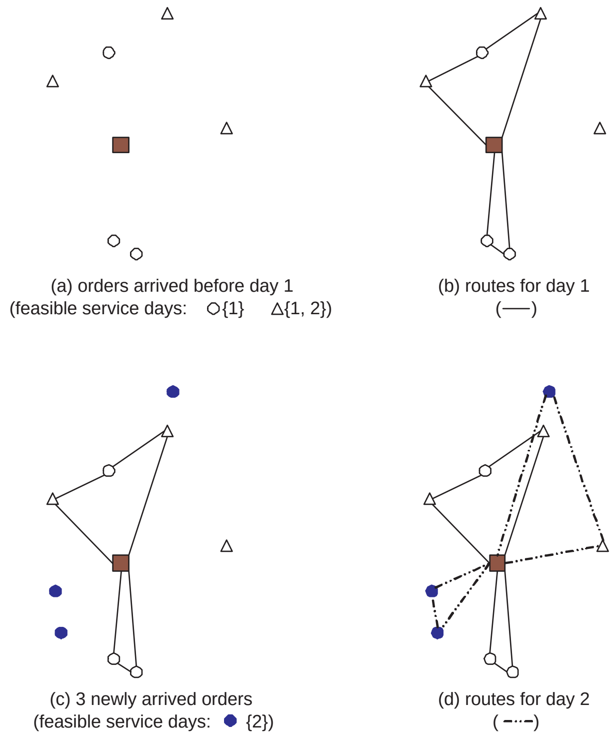


**Figure 2.** A small instance of the DMPVRP

waiting is prolonged and the feasibility of the next day's solution may be jeopardized due to the limited available vehicle capacity.

## 2.2 Mathematical formulation

The planning for each particular day can be regarded as a special case of the PVRP with unit visit frequency and consecutive allowable delivery periods. Without loss of generality, we present the formulation for the planning problem on day $t$ ($t \in T$, where $T = \{1, 2, \ldots, r\}$ denotes the planning horizon). Denote the updated planning horizon on day $t$ by $T' = \{t, t{+}1, \ldots, r\}$, the set of known but unvisited orders by $N = \{1, 2, \ldots, n\}$, and the set of vehicles by $K = \{1, 2, \ldots, m\}$. The depot is located at 0 and the set of all locations is $N_0 = N \cup \{0\}$. The parameter $c_{ij}$ represents the travel time on arc $(i, j) \in A$, where $A$ is the set of arcs between all the locations in $N_0$. Each order $i$ specifies a demand $q_i$ and a service time $d_i$. We denote the original consecutive feasible service days for order $i$ by $\{a_i, \ldots, b_i\}$. Note that the first feasible day has to be adjusted to $a'_i = \max\{t, a_i\}$

when planning on day $t$. Each vehicle has a capacity $Q$ and each route has a duration limit $D$. The binary variables $x^t_{ijkl}$ denote the decisions made on day $t$. They are equal to 1 if and only if vehicle $k$ travels from $i$ to $j$ on day $l$. The constraints are defined as follows:

$$\sum_{l \in \{a_i^{'},\ldots,b_i\}} \sum_{k \in K} \sum_{j:(i,j)\in A} x^t_{ijkl} = 1 \qquad \forall i \in N \tag{1}$$

$$\sum_{i \in N} \sum_{j:(i,j)\in A} q_i x^t_{ijkl} \leq Q \qquad \forall k \in K, l \in T^{'} \tag{2}$$

$$\sum_{i \in N} \sum_{j:(i,j)\in A} (c_{ij} + d_i) x^t_{ijkl} \leq D \qquad \forall k \in K, l \in T^{'} \tag{3}$$

$$\sum_{j \in N} x^t_{0jkl} = 1 \qquad \forall k \in K, l \in T^{'} \tag{4}$$

$$\sum_{i:(i,h)\in A} x^t_{ihkl} - \sum_{j:(h,j)\in A} x^t_{hjkl} = 0 \qquad \forall h \in N, k \in K, l \in T^{'} \tag{5}$$

$$\sum_{i \in N} x^t_{i0kl} = 1 \qquad \forall k \in K, l \in T^{'} \tag{6}$$

$$x^t_{ijkl} \in \{0,1\} \qquad \forall (i,j) \in A, k \in K, l \in T^{'}. \tag{7}$$

Constraints (1) ensure that each customer is visited once by exactly one vehicle within its feasible service days. Constraints (2) guarantee the vehicle capacity limit is not exceeded. The duration limit on each route is ensured by constraints (3). Constraints (4)–(6) state that each vehicle must start and end its route at the depot and that flow is conserved at each customer location. Constraints (7) define the binary variables.

The first objective, minimizing the total travel time of visiting the orders in $N$, can be formulated as

$$f^t_1 = \sum_{l \in T'} \sum_{k \in K} \sum_{(i,j)\in A} c_{ij} x^t_{ijkl}. \tag{8}$$

To minimize the total customer waiting, for each customer having multiple feasible service days, we assign a penalty for not visiting it on the first of its feasible service days. This penalty increases quadratically with customer waiting time, and goes up to 1 if the customer is visited at the end of its feasible service days, as shown in Figure 3. This penalty function favors short waiting times for

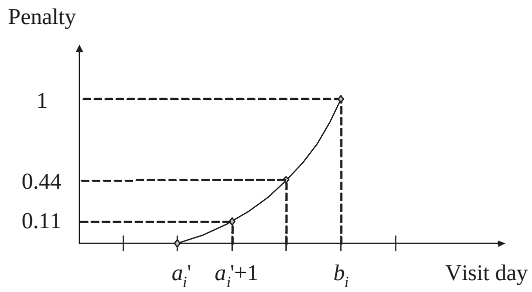


**Figure 3.** Penalty curve

several customers, as opposed to long waiting times for a few. For example, letting three customers wait for one day is preferable to letting one customer wait for three days if all of them have the same time window length. Additionally, we use $a_i'$ instead of $a_i$ to reset the penalty function every day for the unvisited customers and thus treat the unvisited customers and the new customers equally. Let $N'$ denote the set of customers having multiple feasible service days, and let the integer variable $y_i^t$ be the day when customer $i$ is visited. The second objective can be formulated by:

$$f_2^t = \sum_{i \in N'} \left( \frac{y_i^t - a_i'}{b_i - a_i'} \right)^2, \tag{9}$$

where

$$y_i^t = \sum_{l \in \{a_i', \ldots, b_i\}} \sum_{k \in K} \sum_{j:(i,j) \in A} l x_{ijkl}^t \qquad \forall i \in N'. \tag{10}$$

The third objective, balancing the daily workload over the planning horizon, is more difficult to define since future orders are unknown. In a static problem, this objective can be achieved by minimizing the total deviation of daily workload, where a single day's workload deviation is measured by the absolute value of the difference between that day's workload and the average daily workload over the planning horizon. However, in the dynamic case, it is unwise to allocate the known orders evenly to all future days of the planning horizon. Instead, it seems preferable to focus on the workload of the current day, since we have the complete knowledge of the orders accumulated at the beginning of that day. Moreover, since the actual average daily workload cannot be obtained until the end of the planning horizon, we use an estimate of the average daily workload, denoted by $\tilde{w}^t$, based on historical data. The third objective is hence formulated as:

$$f_3^t = \left| \sum_{k \in K} \sum_{(i,j) \in A} c_{ij} x_{ijkt}^t - \tilde{w}^t \right|. \tag{11}$$

As mentioned, scalar techniques and the Pareto method are the two most used strategies for multi-objective optimization. However, in a dynamic context, the Pareto method is inappropriate because even if it were possible to determine a set of Pareto optimal solutions, it would be necessary to implement one of these before the next day's planning, without guidelines on how to make this selection. We have therefore opted to implement the scalar method with weights, 1, $w_2$ and $w_3$, for objective $f_1^t$, $f_2^t$ and $f_3^t$, respectively, and we work with the aggregate objective

$$f^t = f_1^t + w_2 f_2^t + w_3 f_3^t. \tag{12}$$

## 3 A Three-Phase Rolling Horizon Heuristic

We propose a three-phase rolling horizon heuristic to handle the dynamic aspect of the problem. Planning on day $t$ starts with adjusting the set of feasible service days for the yet unvisited customers, including those revealed on day $t-1$. A three-phase heuristic (TPH) is then applied to construct the delivery plan for that day. In order to minimize the total travel time over the planning horizon, instead of only planning the routes for day $t$, the TPH also optimizes the routes for $\tau$ days in the future. Let $T_t = \{t, \ldots, t+\tau\}$ be the planning horizon considered on day $t$. Phase I selects the customers to be visited within $T_t$. The selection is necessary because the feasible service days of the customers may not be entirely included in $T_t$. To this end, we perform a time-space correlation analysis on the known customers. In Phase II, given the customers selected for period $T_t$, routes are constructed by treating the planning problem as a PVRP with a service frequency equal to 1 over the planning horizon $T_t$. This routing problem is solved by means of a variable neighborhood search heuristic. In Phase III, the routes to be executed on day $t$ are postoptimized by means of a tabu search algorithm, and the customers visited on day $t$ are removed from further consideration. This three-phase scheme is summarized in Algorithm 1.

In the TPH, $\tau$ is a user-defined parameter. A small value of $\tau$ results in a planning problem of small size for the subsequent solution phases and hence reduces the computational burden, whereas a

**Algorithm 1** : Rolling horizon framework

Input: the set $N\_new_t$ of customers revealed on each day $t \in T$
Output: the routing plan $\mathcal{R} = \{R_1, \ldots, R_{|T|}\}$ for horizon $T$
$N \leftarrow \emptyset$
**for** $t = 1$ to $|T|$ **do**
  AdjustVisitDays($N$)
  $N \leftarrow N \cup N\_new_{t-1}$
  $N_t \leftarrow$ SelectCustomers($N$) // Phase I
  $\{R_t, \ldots, R_{t+\tau}\} \leftarrow$ RouteCustomer($N_t$, $T_t$) // Phase II
  $R_t \leftarrow$ Optimize($R_t$) // Phase III
  $N \leftarrow N \setminus \{i : i \in R_t\}$
  $\mathcal{R} \leftarrow \mathcal{R} \cup R_t$
**end for**

large value of $\tau$ helps optimize the total routing cost over the planning horizon. A sensitivity analysis on $\tau$ is conducted in Section 4.

### 3.1 Phase I: Customer selection

The customer selection phase attempts to determine a good set of customers to be visited in the future $\tau$ days without relying on routing information. The "good" set is judged only by travel time, since the total travel time is the most important objective of interest and this phase only performs a rough selection of customers in the future $\tau$ days. The other two objectives will be mainly considered in the second phase. The customer selection is achieved by analyzing the time-space correlation between the known customers, as shown in Algorithm 2. More specifically, for each customer $i$ we define a compatibility index $q_{il}$ for each of its allowable service days, where $l \in \{a'_i, \ldots, b_i\}$. A larger value of $q_{il}$ corresponds to a higher visit preference for day $l$. The parameter is determined as follows. First set $q_{il}$ equal to 0 for all customers and feasible service days. Now consider two customers $i$ and $j$ having common allowable service days. If $c_{ij} \leq \rho$ then both $q_{il}$ and $q_{jl}$ are increased by $1/(c_{ij} + \delta)^{\varepsilon}$ ($l \in \{a'_i, \ldots, b_i\} \cap \{a'_j, \ldots, b_j\}$), where $\rho$, $\delta$ and $\varepsilon$ are user-defined parameters. A smaller $c_{ij}$ results in a larger increment (see Figure 4). For each customer $i$, the day with the highest compatibility index is selected as the best service day. The customers whose best service days lie within $T_t$ are selected for visit during that horizon. This procedure is described as Algorithm 2.

**Algorithm 2** : Phase I (Customer selection)

Input: the set of known customers $N$
Output: the set of customers $N_t$ to be visited within period $T_t$
**for** $i = 1$ to $|N|$ **do**
  **for** $l = a'_i$ to $b_i$ **do**
    $q_{il} \leftarrow 0$
  **end for**
**end for**
**for** $i = 1$ to $|N| - 1$ **do**
  **for** $j = i + 1$ to $|N|$ **do**
    **if** $c_{ij} \leq \rho$ and $\{a'_i, \dots, b_i\} \cap \{a'_j, \dots, b_j\} \neq \emptyset$ **then**
      **for** $l \in \{a'_i, \dots, b_i\} \cap \{a'_j, \dots, b_j\}$ **do**
        $q_{il} \leftarrow q_{il} + 1/(c_{ij} + \delta)^{\varepsilon}$
        $q_{jl} \leftarrow q_{jl} + 1/(c_{ij} + \delta)^{\varepsilon}$
      **end for**
    **end if**
  **end for**
**end for**
**for** $i = 1$ to $|N|$ **do**
  $v_i \leftarrow \arg\min_{l \in \{a'_i, \dots, b_i\}} q_{il}$
**end for**
$N_t \leftarrow \{i : v_i \in T_t\}$

### 3.2 Phase II: Variable neighborhood search

The aim of the Phase II is to construct routes for customers on each day of $T_t$. The objective in this phase is to minimize the total travel time, the total customer waiting and balance the daily workload. This problem is treated as a PVRP with frequency 1, where the planning horizon is $\{t, t+1, \dots, t+\tau\}$, and each selected customer $i$ must be served with frequency $1$ between day $\max\{t, a_i\}$ and day $\min\{t+\tau, b_i\}$. The PVRP is solved by means of a variable neighborhood search heuristic (see Algorithm 3), made up of three components: initialization, local search and shaking. An initial solution is first constructed by means of a sweep heuristic. The local search phase is based on a tabu search (TS) algorithm that uses simple insertion moves to transfer customers from their route to another route. For each customer, all possible reinsertion positions are attempted and the one leading to the minimum objective value is selected. If TS fails to find a feasible solution within a preset number of iterations, it is assumed that the customer set selected in Phase I is too large and a customer with the largest number of remaining feasible service days is removed from

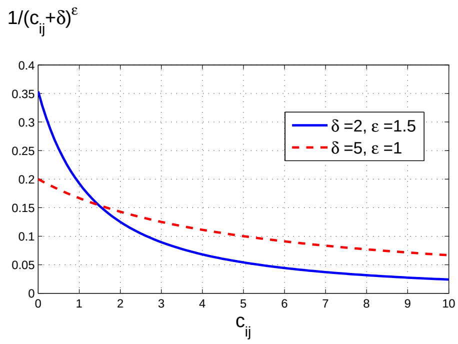


**Figure 4.** The increment value curve for different value of $\delta$ and $\varepsilon$

$N_t$. If TS fails to improve the solution within a preset number $\theta$ of iterations, it restarts from another solution provided by a shaking phase, based on a ruin and recreate approach (RRA) (Schrimpf et al., 2000; Pisinger and Ropke, 2007). This procedure is initiated from the best known solution and attempts to iteratively improve it by removing $\xi$% of the customers that have the largest removal costs, and reinserting them by means of the regret insertion method described in Algorithm 4. If the RRA finds a better solution or fails to improve the best solution after $\kappa$ iterations, TS is reapplied to it. Phase II stops after $\omega$ iterations.

### 3.3 Phase III: Postoptimization

Phase III aims to minimize the total travel time on day $t$. This problem is a *Capacitated Vehicle Routing Problem* which is solved by the TS heuristic of Cordeau, Gendreau and Laporte (1997). In this algorithm, intermediate infeasible solutions are allowed during the search and are controlled by means of a penalized objective $f'(s,t) = c(s,t) + \alpha q(s,t) + \beta d(s,t)$, where $c(s,t)$ is the total travel time by all vehicles on day $t$, and $q(s,t) = \sum_{k \in K}(\sum_{(i,j) \in A} q_i x^t_{ijkt} - Q)^+$ and $d(s,t) = \sum_{k \in K}(\sum_{(i,j) \in A}(c_{ij} + d_i)x^t_{ijkt} - D)^+$ are the total violations of the capacity and duration constraint on day $t$, where $(x)^+ = \max\{0, x\}$. The coefficients $\alpha$ and $\beta$ are positive self-adjusting penalties. It should be noted that the effect of postoptimization on workload balance is negligible compared with that on total travel time because the weight assigned to the travel time in the objective function is much larger, as will be seen later in Section 4. Therefore, the postoptimization of total travel time is also a further optimization in the overall objective stated in equation (12).

**Algorithm 3** : Phase II (Variable neighborhood search)

```
Input: the set of customers N_t to be visited within period T_t
Output: the solution s*
s ← SweepHeuristic(N_t)
s* ← s
iteration ← 0
while iteration < ω do
  counter ← 0
  while counter < θ do
    (s, N_t) ← TabuSearch(s, N_t)
    iteration ← iteration + 1
    if s < s* then
      counter ← 0
      s* ← s
    else
      counter ← counter + 1
    end if
  end while
  counter ← 0
  s ← s*
  while counter < κ do
    s ← RRA(s, ξ)
    iteration ← iteration + 1
    if s < s* then
      s* ← s
      break
    else
      counter ← counter + 1
    end if
  end while
end while
return s*
```

## 4 Computational Results

The heuristic just described was implemented in C and executed on a Linux computer with lx24-amd64 architecture and two Gbytes of RAM. The data and parameters used in our tests are first described. Sensitivity analyses on the parameters used in the heuristic are then performed. Finally we provide the results of our tests on the Lantmännen data, which can be accessed via the Internet at `http://www2.imm.dtu.dk/~mw/dmpvrpData/`.

**Algorithm 4** : Phase II (Ruin and recreate heuristic)

$numToRemove$ is the number of customers to be removed and reinserted
$N_t$ is the set of customers in the solution $s$
Input: current solution s
Output: updated solution $s$
$N_{Rem} \leftarrow \emptyset$
**while** $|N_{Rem}| < numToRemove$ **do**
  **for** $i \in N_t$ **do**
    $RC_i \leftarrow$ CalculateRemovalCost($i$, $s$)
  **end for**
  $i^\star \leftarrow \arg\min_{i \in N_t} RC_i$
  $s \leftarrow$ RemoveCustomer($i^\star$, $s$)
  $N_{Rem} \leftarrow N_{Rem} \cup \{i^\star\}$
  $N_t \leftarrow N_t \setminus \{i^\star\}$
**end while**
**while** $N_{Rem} \neq \emptyset$ **do**
  **for** $i \in N_{Rem}$ **do**
    $bestIC_i \leftarrow$ CalculateBestInsertionCost($i$, $s$)
    $secondIC_i \leftarrow$ CalculateSecondBestInsertionCost($i$, $s$)
  **end for**
  $i^\star \leftarrow \arg\max_{i \in N_{Rem}} (secondIC_i - bestIC_i)$
  $s \leftarrow$ InsertCustomer($i^\star$, $s$)
  $N_{Rem} \leftarrow N_{Rem} \setminus \{i^\star\}$
**end while**

### 4.1 Data and parameters

Real-world data were collected from Lantmännen. There are altogether 11 data sets, five of which involve a 10-day planning horizon and six involve a 15-day planning horizon. On average 80 orders are received every day. The number of feasible service days ranges from one to 15 and is equal to 2.5 on average. Figure 5 shows the distribution of the number of days elapsed between the day at which an order is placed and the first feasible service day. Most customers order two or three days before the start of the service period. The average demand of the orders is 6,306kg, and the vehicles have a capacity of 40,000kg. We use Euclidian distances and assume the vehicle speed is 45km/hour.

Based on preliminary tests, parameters $\rho$, $\delta$ and $\varepsilon$ in Phase I of the TPH were set to 60, 2 and 1.5, respectively. The maximum numbers of non-improving iterations for the TS and the RRA of Phase II, i.e., parameters $\theta$ and $\kappa$, were set to $10^2$ and $10^4$, respectively. In the RRA, between 25% and 35%

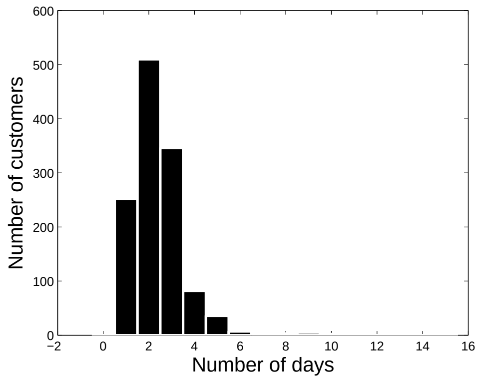


**Figure 5.** Distribution of the number of days before the start of the service period when customers call in

of the customers are removed and reinserted in each iteration. The estimated daily workload for objective function $f_3^t$ in Equation (11) is obtained from the workload of the previous five days and is updated adaptively for each planning day.

### 4.2 Sensitivity analyses

This section describes the sensitivity analyses that were performed to assess the behaviour of the TPH.

***Number of days to plan in TPH***

As mentioned in Section 3, the TPH not only plans the routes for day $t$, but also for $\tau$ days in the future. We tested the TPH with different values of $\tau$ on 11 instances. Figure 6 illustrates the convergence of the TPH for three different values of $\tau$. When $\tau$ equals 1 or 2, Phase I selects approximately 33% or 50% of the customers, respectively. The results show that $\tau = 1$ is not sufficient but $\tau = 2$ works very well. With a short running time (less than four minutes), $\tau = 2$ even provides better results than $\tau = \infty$. This is because within a given short running time, the problem of smaller size can be better optimized due to a more thorough search, and the correlation analysis provides good candidates for the customers that should be visited within the next two days.

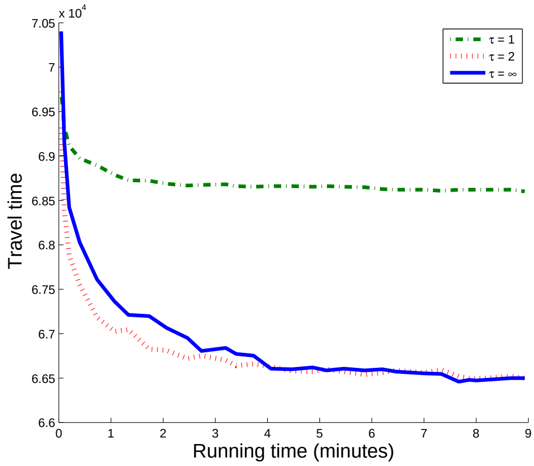


**Figure 6.** Sensitivity analysis of number of planning days ($\tau$)

*Effectiveness of correlation analysis*

To further demonstrate the effectiveness of correlation analysis, we compare the results obtained with correlation analysis to those using a random scheme. In the random scheme, we assume each customer is randomly and uniformly assigned to one of its feasible service days, and customers assigned to the first $\tau$ days are selected. Figure 7 shows the comparison between the two schemes. The horizontal axis is the instance index and the vertical axis gives the total travel time over the planning horizon by using a random selection scheme or correlation analysis. The running time is set at four minutes. For all 11 instances, the solutions provided by the correlation analysis are consistently better than those obtained by the random selection scheme.

*Results for the multi-objective function*

In this experiment, we assess the effectiveness of the TPH to handle the multiple objectives. Table 1 shows the values of the first objective, i.e., total travel time (denoted by $F_1$), and of the second objective, i.e., total customer waiting (denoted by $F_2$), with different values of $w_2$ ranging from 0 to 20. Column $'F_1'$ and $'F_2'$ are the total travel time over the planning horizon and the total number of waiting days for all customers over the planning horizon, respectively. The last row 'Average' shows the average values for the 11 instances. As $w_2$ increases from 0 to 20, the total customer waiting is reduced by half on average, whereas the total travel time increases only slightly, by less

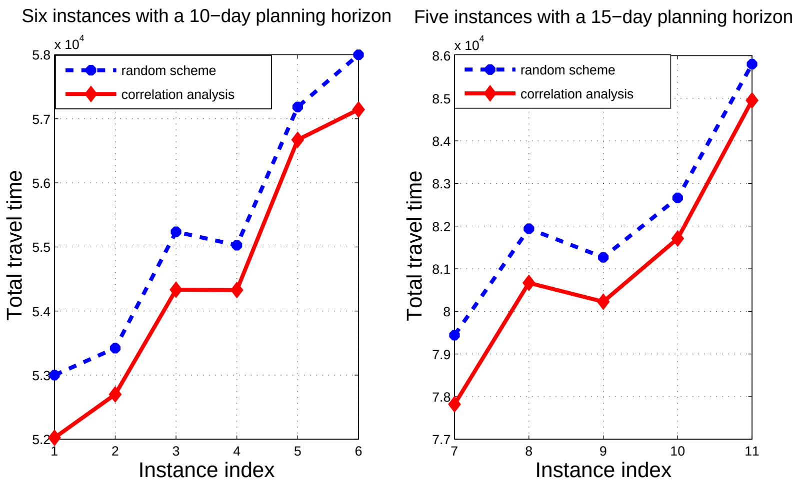


**Figure 7.** Comparison between correlation analysis and random selection scheme

than 1 %. Figure 8 depicts the relative changes of total travel time and total customer waiting as a function of $w_2$.

Similar results are obtained for the total travel time and for the total workload deviations with increasing values of $w_3$, as shown in Table 2 and Figure 9. In Table 2, column $'F_3'$ is the sum of deviations between each day's duration and the average daily duration, over the planning horizon. The last row shows the average values for the 11 instances. As can be seen from the results, when $w_3$ increases from 0 to 0.6, the average workload deviation decreases by more than 70%, whereas the total travel time only increases by approximately 0.5%. We also note that the rate of deviation reduction decreases as $w_3$ increases. In Figure 9, within the interval $0.4 \leq \beta \leq 0.6$, the deviation reduction curve is nearly flat and the deviation reduction is insignificant. This is because the objective function $f_3^t$ used in the TPH minimizes the difference between the workload on day $t$ and an estimation of the average daily workload instead of the actual average workload.

### 4.3 Comparison between TPH solutions and solutions obtained with the company's platform

Lantmännen already works with high quality solutions obtained by running their vehicle routing software for 12 minutes each day on their latest platform. Their software can deal with various practical restrictions. In order to establish a fair comparison, we have used their software to solve

| $w_2$ | 0 | | 2 | | 4 | | 8 | | 12 | | 16 | | 20 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Data set | $F_1$ | $F_2$ | $F_1$ | $F_2$ | $F_1$ | $F_2$ | $F_1$ | $F_2$ | $F_1$ | $F_2$ | $F_1$ | $F_2$ | $F_1$ | $F_2$ |
| 101 | 52024 | 380 | 52271 | 317 | 52060 | 296 | 52295 | 251 | 52306 | 225 | 52386 | 203 | 52546 | 171 |
| 102 | 52700 | 391 | 52786 | 327 | 52919 | 297 | 52974 | 267 | 53001 | 236 | 53039 | 198 | 53222 | 176 |
| 103 | 54333 | 479 | 54322 | 412 | 54401 | 389 | 54439 | 332 | 54605 | 296 | 54802 | 272 | 54939 | 254 |
| 104 | 54328 | 449 | 54222 | 381 | 54305 | 343 | 54471 | 310 | 54798 | 263 | 54641 | 250 | 55038 | 241 |
| 105 | 56672 | 404 | 56522 | 343 | 56591 | 302 | 56607 | 252 | 57012 | 218 | 57063 | 203 | 57164 | 185 |
| 106 | 57144 | 414 | 57279 | 365 | 57346 | 320 | 57381 | 276 | 57684 | 254 | 57582 | 228 | 57872 | 199 |
| 151 | 77820 | 708 | 78103 | 593 | 78057 | 528 | 78079 | 477 | 78341 | 419 | 78377 | 385 | 78564 | 351 |
| 152 | 80669 | 658 | 80628 | 578 | 80485 | 516 | 80671 | 451 | 80825 | 403 | 81077 | 356 | 81232 | 328 |
| 153 | 80226 | 658 | 80213 | 587 | 79985 | 525 | 80373 | 459 | 80339 | 415 | 80690 | 362 | 80631 | 350 |
| 154 | 81710 | 620 | 81809 | 532 | 81879 | 492 | 82159 | 417 | 82248 | 379 | 82203 | 350 | 82431 | 318 |
| 155 | 84950 | 634 | 85015 | 540 | 85031 | 474 | 84948 | 422 | 85142 | 399 | 85420 | 342 | 85465 | 331 |
| **Average** | **66597** | **526** | **66651** | **452** | **66641** | **407** | **66763** | **355** | **66936** | **318** | **67025** | **286** | **67191** | **264** |

**Table 1.** Total travel time and total customer waiting with different values of the wight $w_2$ assigned to customer waiting time

our simplified problem based on Euclidean distances with the same real-life data. We refer to these solutions as the "company's solutions". We have also run our algorithm for 12 minutes on a similar computer, but the improvement obtained after four minutes is insignificant. Comparative results are presented in Table 3. Based on the preliminary tests on the combinations of the parameters $w_2$ and $w_3$, we found that their influence on the total travel time is very little. A good setting is $w_2 = 4$ and $w_3 = 0.15$ in order to keep the total travel time change at an insignificant level while reducing total customer waiting and improving workload balance as much as possible.

Ten random runs for each instance are performed for our algorithm to obtain the average value of the total driving time, total customer waiting, and daily workload deviation. These statistics are provided in the columns 'Average total duration', 'Average total customer waiting' and 'Average total workload deviation', respectively. The best value of the total travel time within the 10 random runs are also presented in column 'Best total duration'. The results provided by the company's platform are for a single run. The average values for all the instances are given in the last row. Regarding the total duration, our average value for 10 runs is slightly better (by 0.2%) than that of the company's solutions, probably because both solutions are very close to optimality. However, the TPH significantly improves customer waiting and workload deviation by up to 24% and 35%, compared with the solutions obtained by using the company's platform. We also found the best

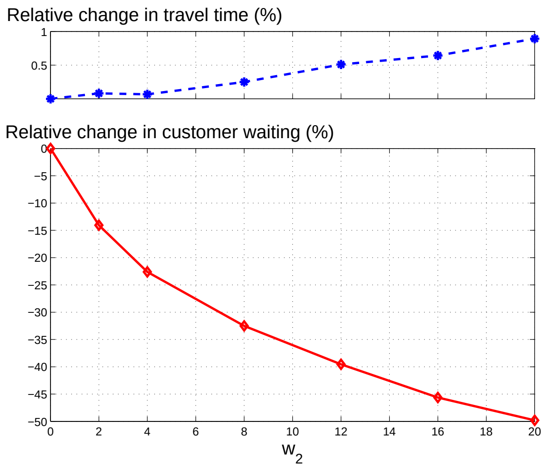


**Figure 8.** Relative changes in total travel time and total customer waiting as a function of the weight $w_2$ assigned to customer waiting time

solutions for all instances. This is a clear sign of the effectiveness of our heuristic. One should bear in mind, however, that customer waiting and daily workload balance may not have been optimized by the company. In addition, the company's solutions used in the comparison are not those that Lantmännen uses in the real life since its daily solutions are generated under several practical restrictions.

## 5 Conclusion

We have considered a real-life dynamic multi-period and multi-objective routing problem encountered by a large distributor operating in Sweden. The planning horizon consists of several periods and the problem considers three objectives, including minimization of the total travel time, minimization of customer waiting, and daily workload balancing over the planning horizon. We have presented a mixed integer linear programming formulation for the problem, and we have proposed a three-phase heuristic embedded within a rolling horizon scheme. The main idea of the heuristic is to wisely select the customers to be visited in the near future, and to route these customers so that the overall travel time can be minimized efficiently. The choice of customers to be routed on a given day is performed rather effectively through a time-space correlation analysis.

| $w_3$ | 0 | | 0.1 | | 0.2 | | 0.3 | | 0.4 | | 0.5 | | 0.6 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Data set | $F_1$ | $F_3$ | $F_1$ | $F_3$ | $F_1$ | $F_3$ | $F_1$ | $F_3$ | $F_1$ | $F_3$ | $F_1$ | $F_3$ | $F_1$ | $F_3$ |
| 101 | 52024 | 1221 | 51972 | 597 | 52284 | 344 | 52174 | 169 | 52303 | 131 | 52447 | 139 | 52247 | 123 |
| 102 | 52700 | 1146 | 52791 | 664 | 52676 | 340 | 52878 | 214 | 52863 | 154 | 52876 | 87 | 52786 | 123 |
| 103 | 54333 | 1285 | 54183 | 572 | 54274 | 430 | 54157 | 266 | 54107 | 239 | 54248 | 207 | 54207 | 190 |
| 104 | 54328 | 1603 | 54463 | 835 | 54566 | 494 | 54851 | 491 | 54812 | 399 | 54877 | 411 | 55063 | 369 |
| 105 | 56672 | 1430 | 56711 | 1043 | 56888 | 599 | 57045 | 527 | 56994 | 461 | 56908 | 405 | 57211 | 488 |
| 106 | 57144 | 1372 | 57310 | 1009 | 57396 | 752 | 57496 | 631 | 57522 | 610 | 57576 | 673 | 57682 | 645 |
| 151 | 77820 | 1218 | 77946 | 808 | 78022 | 638 | 78264 | 603 | 78357 | 552 | 78466 | 555 | 78400 | 500 |
| 152 | 80669 | 1032 | 80718 | 654 | 80832 | 499 | 80596 | 416 | 80747 | 363 | 80603 | 373 | 80701 | 336 |
| 153 | 80226 | 1170 | 80128 | 605 | 80199 | 404 | 80336 | 305 | 80503 | 276 | 80401 | 245 | 80483 | 267 |
| 154 | 81710 | 1526 | 81945 | 848 | 81826 | 726 | 82021 | 551 | 82318 | 510 | 82253 | 512 | 82447 | 494 |
| 155 | 84950 | 1460 | 84848 | 989 | 84971 | 723 | 84921 | 693 | 85378 | 631 | 85393 | 697 | 85386 | 683 |
| **Average** | **66597** | **1314** | **66637** | **784** | **66721** | **540** | **66794** | **442** | **66900** | **393** | **66913** | **391** | **66964** | **383** |

**Table 2.** Total travel time and total workload deviation for the tests with different values of the weight $w_3$ assigned to balance daily workload

The multiple objectives are handled by the scalar technique. The method was implemented and tested on real-life data. Results show that the proposed TPH provides very high quality solutions within a reasonable running time. The results are also compared with the solutions produced by the company's platform. The comparison shows that our method improves upon those solutions in terms of travel time, customer waiting and daily workload balance, with gains of 0.2%, 24% and 35%, respectively. Our method is general and applies to other contexts.

## Acknowledgements

This work was partly supported by the Canadian Natural Sciences and Engineering Research Council under grants 227837-04 and 39682-05. This support is gratefully acknowledged. We thank the anonymous referees for their valuable comments.

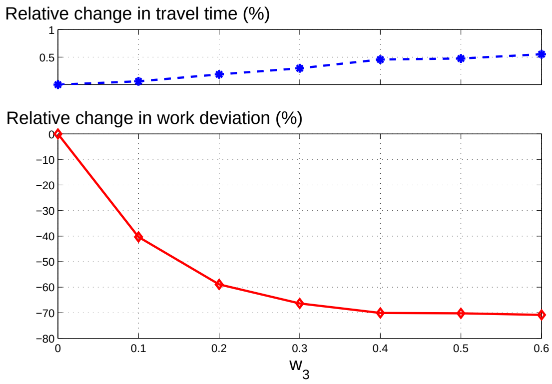


**Figure 9.** Relative changes in total travel time and total workload deviation as a function of the weight $w_3$ assigned to daily workload deviation

| | | Company's solutions | | | TPH solution | | | |
|---|---|---|---|---|---|---|---|---|
| Data set | $\|T\|$ | Total duration | Total customer waiting | Total workload deviation | Average total duration | Best total duration | Average total customer waiting | Average total workload deviation |
| 101 | 10 | 51958 | 382 | 729 | 52064 | 51679 | 330 | 588 |
| 102 | 10 | 52702 | 369 | 844 | 52663 | 52163 | 312 | 693 |
| 103 | 10 | 53727 | 560 | 1231 | 54128 | 53711 | 408 | 530 |
| 104 | 10 | 54299 | 444 | 1117 | 54439 | 54089 | 372 | 721 |
| 105 | 10 | 56476 | 440 | 535 | 56429 | 56009 | 344 | 812 |
| 106 | 10 | 57729 | 411 | 606 | 57211 | 56740 | 352 | 716 |
| 151 | 15 | 77362 | 759 | 1170 | 77706 | 77320 | 632 | 710 |
| 152 | 15 | 81135 | 828 | 978 | 80303 | 79963 | 618 | 627 |
| 153 | 15 | 79600 | 767 | 312 | 79730 | 79342 | 604 | 518 |
| 154 | 15 | 82164 | 713 | 1315 | 81711 | 81321 | 526 | 756 |
| 155 | 15 | 85270 | 685 | 1060 | 84540 | 83992 | 552 | 740 |
| **Average** | | **66584** | **578** | **900** | **66447** | **66029** | **459** | **673** |

**Table 3.** Comparison between the Lantmännen solutions and the TPH solutions

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