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Abstract. This paper studies various modeling approaches to design resilient Supply Networks (SN) for the location-transportation problem under uncertainty. The future environment of the SN is shaped by random demands, and by disruptions perturbing depots capacity and ship-to-point demand processes. The paper proposes several stochastic programming models incorporating alternative resilience seeking formulations. A generic approach to model SN disruptions, and to elaborate and evaluate SN designs is also proposed. Experiments are made to compare the SN design models formulated, and recommendations are drawn on the approach to use to design effective and robust supply networks.

Keywords. Supply Network Design, Uncertainty, Resilience, Scenario Planning, Network Disruptions, Multihazard, Stochastic Programming

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1. Introduction

A Supply Network (SN) is a configuration of supply facilities geographically deployed in order to serve a customer base. Strategic SN design decisions involve the determination of the number, location and mission of these facilities. At that level, the main objective of the firm is to design a SN maximising shareholders' value during an appropriate planning horizon. However, at design time the future environment under which the SN will evolve is unknown which complicates the strategic evaluation of potential designs. Traditional SN design approaches assume that the environment is deterministic, which give rise to classical location models (Klose and Drexl, 2005). Typical extensions of these models take into account random factors using stochastic programming (Birge and Louveaux, 1997) or depot failures using robust optimisation (Kouvelis and Yu, 1997). A recent review of supply chain networks design problems under uncertainty is found in Klibi *et al.*, (2010a). However, to the best of our knowledge, no comprehensive SN design methodology considering both business-as-usual random events and high impact disruptions is currently available.

A major preoccupation of contemporary businesses is the consideration of risks when designing SNs. In addition to the random variables associated to business-as-usual factors, several catastrophic events can disrupt supply chains. Rice and Caniato (2003) and Christopher and Peck (2004) investigate network vulnerability to extreme unforeseen events such as natural disasters and strikes, and Sheffi (2005) examines the case of several companies who suffered from fires, earthquakes, floods, intentional attacks, etc. SNs are geographically dispersed across large regions which increase their exposure to extreme events and, in order to design robust SNs, the impact of such events must be considered. To this end, this paper examines the business-as-usual and the extreme random events that shape future SN environments. The challenge is to elaborate a SN design methodology taking all these event types into account while remaining sufficiently synthetic to be practical. The approach proposed is based on the modeling of *multihazards*: it involves the definition of SN vulnerability sources and exposure levels, the estimation of multihazard arrival processes, and the assessment of multihazard consequences. The risk modeling concepts applied are based on Haimes (2004), Grossi and Kunreuther (2005), Kleindorfer and Saad (2005) and Banks (2006).

In these circumstances, responsiveness and resilience become key elements to enhance the robustness of SN designs. Response policies are operational rules adopted to deal with random demands as well as disruptions when they occur. Resilience is the ability of a SN to bounce back from disruptions (Sheffi, 2005). Resilience strategies are predispositions of network resources favouring risk avoidance and mitigation. Currently most SNs have difficulty coping with emergencies (Lee, 2004), and they do not develop plans to protect against high-impact low likelihood events (Chopra and Sodhi, 2004). Recently, SN risk management strategies incorporating redundancy and flexibility have been proposed in the literature (Chopra and Sodhi, 2004; Sheffi, 2005; Tang 2006; Snyder *et al.*, 2006; Tang and Tomlin, 2008; Pettit *et al.*, 2010). However, these strategies were not explicitly incorporated in SN design models. Using the capacitated location-transportation problem under uncertainty as a typical SN design problem, this paper proposes an approach to model the impact of multihazards on SN capacity and demand, it proposes alternative SN design models to anticipate response policies, to consider the risk-attitude of decision-makers and to elaborate risk avoidance and resilience strategies, and it investigates the performance of the SN designs provided by these models.

In the supply context considered, random customer orders are fulfilled from pre-assigned depots using different transportation means, and customers expect next day deliveries. When a disruption occurs, some depots may lose part of their capacity, and some orders may have to be reassigned based on the response policy of the company. It is through this order fulfilment process that sale revenues and operational costs are generated. When designing the SN, these revenues and costs must be anticipated to determine if the investments required to open depots will generate value. In classical location-allocation models, investment costs are associated to binary location variables, operational revenues and costs are anticipated by introducing aggregate depot-to-customer flow variables, and risks are neglected. This is a very crude approximation. A better anticipation can be obtained by modeling transportation decisions more closely, risk can be taken into account by modeling demands and multihazards as stochastic processes and a more resilient SN can be designed by introducing risk mitigation constructs in the design model. The stochastic programming models thus obtained are clearly more difficult to solve than classical location-allocation models. Is this extra effort worthwhile? This is the question we address in this paper.

To answer this question, we need to elaborate several original stochastic SN design models and to evaluate them using an approach which replicates the network users supply operations as closely as possible. Some of the design models proposed are stochastic reformulations of classical location-allocation models, and others extend the location-transportation model proposed by Klibi *et al.*, (2010) to take into account depots capacity, disruptions and resilience strategies. These models are solved using the Sample Average Approximation (SAA) method, as is commonly done in stochastic programming (Shapiro, 2003). The plausible future scenario samples required to formulate the stochastic design models, and to evaluate the designs provided by these models, are generated using Monte-Carlo methods. Moreover, the design evaluation process is not based solely on an expected value criterion but also on design robustness measures for low-risk, high-risk and worst-case scenarios. The methodology adopted to address our research question is summarized in **Figure 1**.

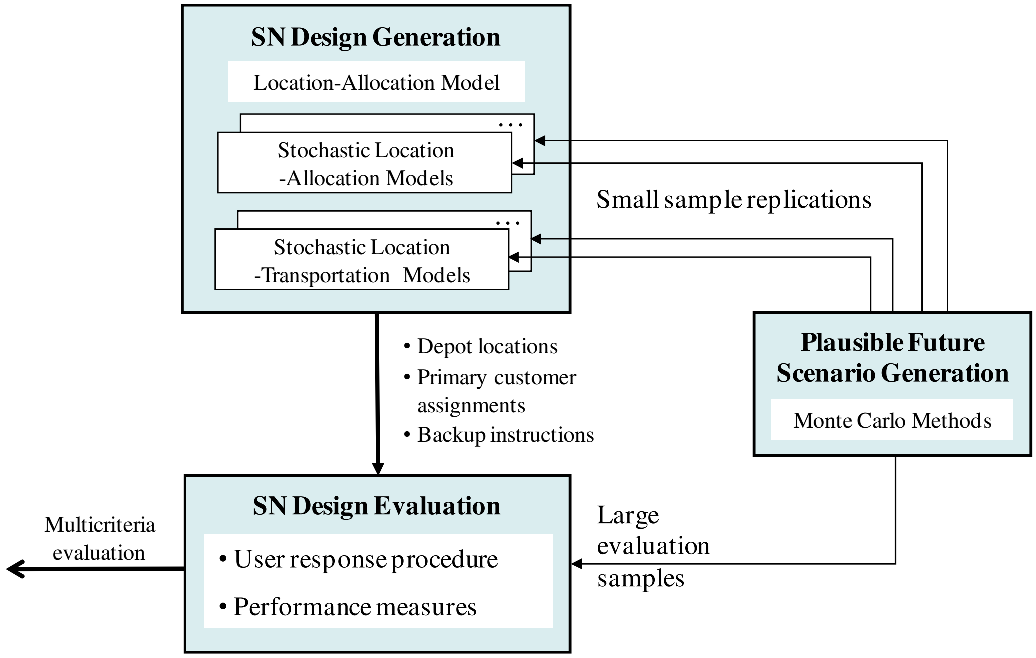

Figure 1- Research Methodology

The rest of the paper is organized as follows. Section 2 describes the location-transportation problem (LTP) under uncertainty and characterizes underlying demand and hazard processes. Section 3 presents a scenario-based SN design approach and proposes two design models using stochastic programming. Section 4 discusses risk avoidance and SN resilience, and it proposes three design models to improve SN resilience. Section 5 proposes a generic approach to generate and to evaluate SN designs. Computational results are presented and analysed in section 6. Finally, section 7 concludes the paper.

2. The Location-Transportation Problem under Uncertainty

Problem Context

The company considered purchases a product family from a number of vendors. This product is sold to customers located in a large geographical area and hence it must be shipped to a large number of ship-to-points. In order to serve its customers, the company must implement a number of capacitated depots with similar processes and technology. For a given day, the capacity of a depot reflects the maximum throughput sustained by its resources. In addition to its regular capacity level, we assume that under normal business conditions, the depot can provide an additional capacity per day using local recourses (ex. overtime).

Customers order a varying quantity of product and the company wants to provide next day delivery from a single depot using common or contract carriers. To this end, several transportation options with different unit costs are available, namely:

- Full truckloads (FTL), i.e. using TL transportation for a customer order that requires all the truck capacity;
- Single customer partial truckloads (STL), i.e. using TL transportation for a customer order that requires only a fraction of the truck capacity;
- Multi-drop truckloads (MTL) i.e. using TL transportation to serve several customers on a route to be determined;
- Less than truckload (LTL) transportation.

On a given day, when all the orders are in, the company plans its transportation for the next day and it requests from its carriers the trucks required to deliver products to ship-to-points. However, the network's depots are under the threat of disruptions and, consequently, their capacity to respond adequately to ship-to-point's orders can be perturbed. Therefore, in order to complete the orders received for a given day, the company relies first on its regular capacity, and second on a local recourse such as overtime. If this is not sufficient, external resources can be used to satisfy all its customers.

Let L be the set of potential depots considered to perform distribution operations, P the set of all ship-to-points and P_l the subset of ship-to-points that could be served by depot $l \in L$. Also, let a_l be the capacity of depot *l*, i.e. the quantity of products it could ship during a day. At design time, strategic decisions are made on the subset of depots $\underline{L} \subset L$ to use during the planning horizon, and on their mission $\underline{P}_l \subset P_l$, $l \in \underline{L}$. These decisions are denoted by the vector $\mathbf{x}$. However, at design time, these decisions are taken under uncertainty and they must consider plausible future scenarios over a discrete planning horizon T. To model the daily ordering process of customers adequately, we assume that the periods $\tau \in T$ of this planning horizon are days.

Demand and Disruption Processes

Based on the information available at design time, two types of events shaping the business environment can be distinguished, namely business-as-usual random events and low-probability high-impact disruptions. In this paper, customer demands and network disruptions are modeled as compound stochastic processes. We assume that the demand of the SN ship-to-points $p \in P$ follows a compound process with a random order inter-arrival time q_p and a random order size o_p. The cumulative distribution functions of inter-arrival times and order sizes are denoted respectively by $F_p^q(.)$ and $F_p^o(.)$. Disruptions are first amalgamated into meta-events with generic impacts called *multihazards* (Scawthorn *et al.*, 2006), and SN vulnerability sources are identified. Second, a compound stochastic process is defined to describe how multihazards occur in space and in time, and to specify incident's intensity and duration. Third the impact of hits on the SN is modelled using recovery functions.

In our context, the depots L and ship-to-points P define a set of network locations $N = P \cup L$. We assume that depots and ship-to points have different incident profiles in terms of impact and time to recovery and thus constitute two distinct *vulnerability sources* denoted, respectively, by s_L and s_P. The notation $s(n)$ is used to identify the vulnerability source of location $n \in N$. To map potential threats, the geographical territory in which the SN operates is partitioned into a set of *hazard zones* Z delineating areas with similar exposure characteristics. Using an exposure measure, each hazard zone $z \in Z$ is assigned to a discrete exposure level $g(z)$, $g \in G$. Based on its geographical position each network location $n \in N$ is positioned in a hazard zone $z(n) \in Z$ and it has an exposure level $g(n) = g(z(n))$. We assume that multihazards occur independently in zones $z \in Z$, and that the time between their successive occurrences is a random variable λ_z characterized by a stochastic arrival process with cumulative distribution function $F_z^{\lambda}(.)$.

When zone $z \in Z$ is hit by a multihazard, the severity of the incident is characterized by two correlated random variables, expressed in terms of metrics depending on the vulnerability source $s \in \{s_L, s_P\}$, namely the *impact intensity* β_{zs}, with cumulative distribution function $F_{g(z)s}^{\beta}(.)$, and the *time to recovery* θ_{zs}. The time to recovery is related to the impact intensity through an *impact-duration function* $\theta_{zs} = f_s(\beta_{zs}) + \varepsilon_s$, where ε_s is a random error term with probability distribution function $F_s^{\varepsilon}(.)$. **Figure 2** illustrates this function for depots, in %-capacity loss, and for ship-to-points, in %-demand variation. Note that, following a hit on ship-to-points, first necessity products would see their demand raising but luxury products would see their demand dropping. Since a single product family is considered here, the impact intensity provides a net effect for the entire product family. We could have a demand surge for some zones and a drop for others, however.

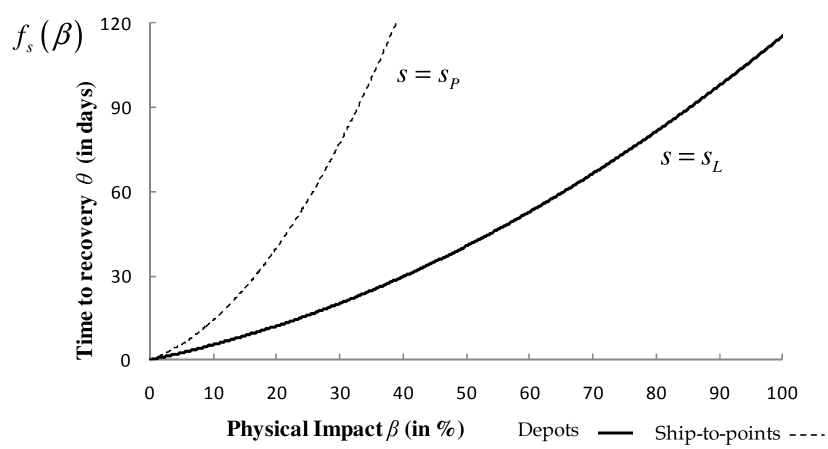

Figure 2- Impact-Duration Functions for Depots and Ship-to-Points in a Zone

The occurrence of an incident in a hazard zone $z \in Z$ does not necessarily result in a hit of all the SN locations in that zone. Conditional attenuation probabilities α_n, $n \in N_z$, are defined to reflect location hit likelihood. When a location $n \in N_z$ associated to vulnerability source *s* is hit, the *impact intensity* and the *time to recovery* are provided respectively by $\beta_n = \beta_{z(n)s(n)}$ and $\theta_n = \theta_{z(n)s(n)}$. Consider a multihazard hitting location $n \in N_z$ at the beginning of period $\tau' \in T$, and, to simplify the presentation, let the indexes $i = q, o, a$ be associated respectively to the order arrival (*q*), the customer order size (*o*), and the depot capacity (*a*) processes. The impact of a hit is not necessarily felt uniformly during the time to recovery: several phases can be observed, depending on the nature of the multihazard and of the vulnerability source (Sheffi, 2005). Such phase-dependent impacts can be characterized by defining discrete *recovery functions* $\rho^i_{n\tau} = \phi^i(\beta_n, \theta_n, \rho^i_{n\tau})$, $\tau = \tau', ..., \tau' + \theta_n - 1$, where $\rho^i_{n\tau}$ is a capacity/demand amplification percentage for process *i* at location *n* in period τ. The $\rho^i_{n\tau}$ value used as an argument in the function reflects amplification percentages before the hit and the function returns percentages after the hit. As illustrated in **Figure 3**, if the periods affected by the multihazard are not still recuperating from a previous incident, then the a priori percentages are $\rho^i_{n\tau}$=100%, $\forall i, n,\ \tau = \tau', ..., \tau' + \theta_n - 1$. The amplitude of the amplification depends on the multihazard impact intensity β_n. Using these recovery functions, the capacity and the demand can be calculated for specific periods and locations. For the order inter-arrival times and sizes, this gives rise to the perturbed random variables $q_{p\tau} = \rho^q_{p\tau} q_p$ and $o_{p\tau} = \rho^o_{p\tau} o_p$, $\tau = \tau', ..., \tau' + \theta_p - 1$, and to their associated distributions functions $F^q_{p\tau}(.)$ and $F^o_{p\tau}(.)$. For the depots, this yield perturbed capacity levels $a_{l\tau} = a_l \rho^a_{l\tau}$, $\tau = \tau', ..., \tau' + \theta_l - 1$.

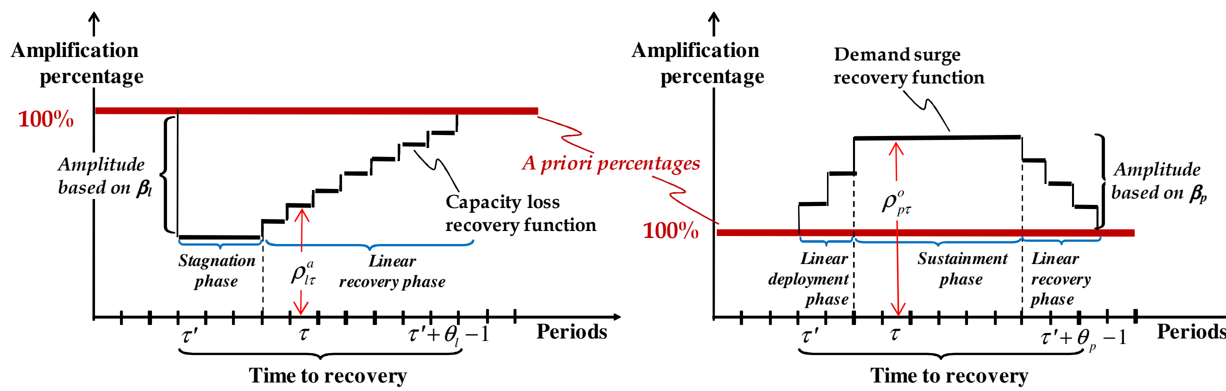

Figure 3- Recovery Function Examples for Depot *l* and Ship-to-Point *p*

Plausible Future Scenarios

The instantiation of the demand and multihazard processes described previously over all the possible values of the random variables involved yields a set Ω of plausible future scenarios with associated probabilities $\pi(\omega)$, $\omega \in \Omega$. The Monte Carlo procedure in **Figure 4** can be used to generate a scenario instance $\omega \in \Omega$. The procedure uses independent pseudorandom numbers

u, uniformly distributed on the interval $[0,1]$, first to generate multihazard arrivals, second to generate recovery functions, and third to generate daily ship-to-point demands $\mathbf{d}(\omega)=[d_{p\tau}(\omega)]_{p\in P,\ \tau\in T}$ and depot capacities $\mathbf{a}(\omega)=[a_{l\tau}(\omega)]_{l\in L,\ \tau\in T}$. The input parameters $\mathbf{a}$ and $\boldsymbol{\alpha}$ are capacity and attenuation probability vectors, respectively, and $\mathbf{F}$ denotes the set of all the previously defined probability distributions.

MonteCarlo $\left((N_z, z\in Z), T, \mathbf{a}, \boldsymbol{\alpha}, \mathbf{F}, (\phi^i, i=q,o,a); \mathbf{d}(\omega), \mathbf{a}(\omega)\right)$

1) For all $z\in Z$, do:
 Using the cumulative distribution of λ_z, generate multihazard arrival moments $\mathcal{T}_z \subseteq T$
 Set $\rho^i_{n\tau}=1$ for all $n\in N_z$, $\tau\in T$, $i=q,o,a$
 For all $\tau'\in\mathcal{T}_z$, do:
  Compute $\beta_{zs}=F^{\beta^{-1}}_{g(z),s}(u)$ and $\theta_{zs}=\left\lceil f_s\left(\beta_{zs}\right)+F^{\varepsilon^{-1}}_s\left(u\right)\right\rceil$, $s\in\{s_L, s_P\}$
  For all $n\in N_z \,|\, u\le\alpha_n$, do:
   $\rho^i_{n\tau}=\phi^i(\beta_{z(n)s(n)},\theta_{z(n)s(n)},\rho^i_{n\tau})$, $\tau=\tau',...,\tau'+\theta_n-1$, $i=q,o,a$
  End For
 End For
2) For all $p\in P$, do:
 $\eta=0$; $d_{p\tau}(\omega)=0$, $\tau\in T$; $\tau=1$; $F^q_{p\tau}=F^q_p$
 While $\eta\le|T|$, do:
  Compute the next order arrival time $\eta=\eta+F^{q^{-1}}_{p\tau}(u)$ and $\tau=\lceil\eta\rceil$
  For $i=q,o$: Derive $F^i_{p\tau}$ from F^i_p and $\rho^i_{p\tau}$
  Compute the daily demand $d_{p\tau}(\omega)=d_{p\tau}(\omega)+F^{o^{-1}}_{p\tau}(u)$
 End While
 End For
 For all $l\in L$ and $\tau\in T$: Compute the daily capacity $a_{l\tau}(\omega)=a_l\rho^a_{l\tau}$

Figure 4- Scenario ω Generation Procedure

Some of the plausible future scenarios in Ω may involve only a few multihazard over the planning horizon but others may be much more chaotic. An intuitive measure to assess the risk associated to a scenario $\omega\in\Omega$ is the number of product-days of depot capacity $\gamma(\omega)$ lost during the planning horizon. For a given hit, this is given by the area under the full capacity line in the capacity lost function represented in **Figure 3**. A simpler measure which may be sufficient in some contexts is the number of hits during the horizon. The later does not take the duration and intensity of hits into account explicitly, but our numerical experiments have shown that it is strongly correlated with the former. **Figure 5** illustrates the two measures for a large sample of scenarios, generated using procedure **MonteCarlo** with exponential multihazard inter-arrival times. In order to distinguish between the scenarios a decision-maker would consider as acceptable, in term of the risks involved, and those that would raise a serious concern, we define a hazard tolerance level κ. This level is the maximum number of product-days lost (hits) the decision-maker can tolerate without serious concern. This tolerance level is used to partition Ω in two subsets: a set of low-risk scenarios $\Omega^L = \{\omega | \gamma(\omega) \leq \kappa\}$ with associated low-risk probability $\pi^L = \Sigma_{\omega \in \Omega^L} \pi(\omega)$ and conditional scenario probabilities $\pi^L(\omega)$, $\omega \in \Omega^L$, and a set of high-risk scenarios $\Omega^H = \Omega \backslash \Omega^L$ with associated high-risk probability $\pi^H = \Sigma_{\omega \in \Omega^H} \pi(\omega)$ and conditional scenario probabilities $\pi^H(\omega)$, $\omega \in \Omega^H$. The risk associated to a SN depends on the size of the territory it covers and on its density in terms of number of depots/points per unit area (Craighead *et al.*, 2007). For this reason, the distribution of product-days lost (hits) is increasingly skewed towards the right as the size of the network increases. Consequently, the value κ selected also tends to increase with the size of the network considered. Also, the worst-case scenarios in the tail of the distribution are of particular interest to us. They will be useful to assess the robustness of the SN designs considered.

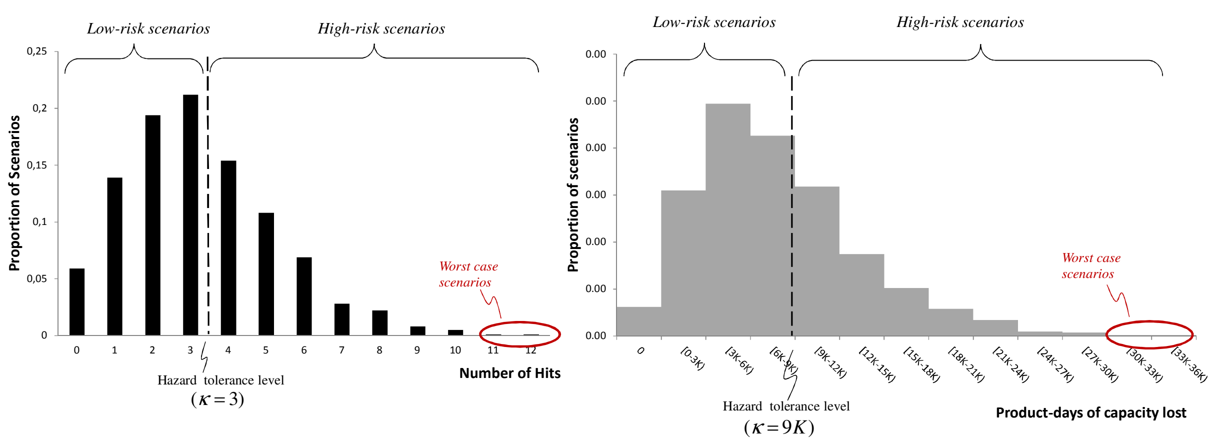

Figure 5- Distribution of the Product-days Lost (Hits) for a Large Scenario Sample

Finally, note that for a given scenario ω, on day τ, the depots capacity $a_{l\tau}(\omega)$, $l \in L$, is known and the set of depots L can be partitioned into operational depots $L_\tau(\omega) = \{l | a_{l\tau}(\omega) = a_l\}$ and partially operational depots $L_\tau^d(\omega) = \{l | a_{l\tau}(\omega) < a_l\}$. Similarly, the ship-to-points demand $d_{p\tau}(\omega), p \in P,$ is known and the set of ordering points $P_\tau(\omega) = \{p | d_{p\tau}(\omega) > 0\}$ can be specified.

Response Policy and Demand Fulfillment Procedures

Now let us examine more closely the daily operations of the SN under a given design **x**. In a single-sourcing delivery context, one would like to serve each ship-to-point p from a unique depot denoted $l(p)$. Due to hazards, short-term variations in capacity and demand occur and recourse actions are necessary on a daily basis to provide an adequate response to customers. To respond in time, the company must decide on a daily basis if the primary mission of depots is maintained or adapted. In the latter case, the company makes order reassignment decisions and, ultimately, resort to external supply sources. Tomlin (2006) and Schutz and Tomasgard (2009) discuss the value of delivery flexibility in supply chain operations. Once the order reassignment is done, the company makes shipping decisions, at each depot, with the objective to maximise sales net revenues. To implement this, based on the response policy of the company, *order assignment* and *transportation* procedures must be specified.

For a given day τ under scenario ω, the set of ordering ship-to-points $P_\tau(\omega)$, their demand $d_{p\tau}(\omega), p \in P_\tau(\omega)$, and the depots daily capacity $a_{l\tau}(\omega), l \in \underline{L}$, are known. Thus, based on depots primary missions, revised daily assignment decisions $\underline{P}_{l\tau}(\omega), l \in \underline{L}$, must be made to ensure that the depot capacity constraints $\sum_{p \in \underline{P}_{l\tau}(\omega)} d_{p\tau}(\omega) \le a_{l\tau}(\omega), l \in \underline{L}$, are respected. Additionally, internal recourses (ex: overtime) or external resources can be used to satisfy unfulfilled orders. When a depot l is operational on day τ under scenario ω, additional capacity $\zeta_l a_l$, is available for use, where ζ_l is a fixed proportion of regular daily capacity. As illustrated in **Figure 6**, the stochastic demand level and depot capacity on each day dictate the kind of response decisions to take. When depot l is operational it serves its primary ship-to-point orders and it can process reassigned orders from other depots. When depot l is partially operational, however, it serves only a subset of its primary ship-to-point orders and the remaining ones are transferred.

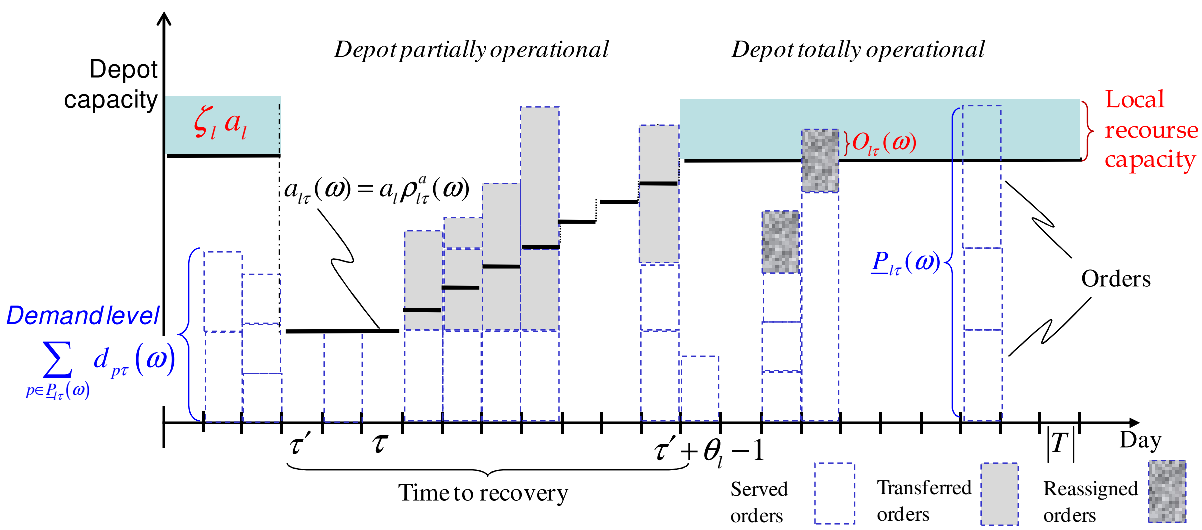

Figure 6- Demand Level at a Given Depot *l* under Scenario ω

Let $\mathcal{P}$ be a priority list ranking ship-to-points $p \in P$ in decreasing order of their importance for the company. When all the SN orders have been received on a given day, we assume that list $\mathcal{P}$ is used to assign the most important customers to their primary depot $l(p)$ and to transfer the remaining orders to an alternative depot $l' \in \underline{L}_p \setminus \{l(p)\}$ or to an external supply source (identified using index $l = 0$). We assume that the reassignment policy used is based on a proximity rule (default policy), or on explicit or implicit instructions provided by the design decisions. More specifically, we assume that the default reassignment policy is to supply ship-to-point p orders not shipped by $l(p)$ from the nearest depot in $\underline{L}_p \setminus \{l(p)\}$, that is to set

$l'(p) = \arg\min_{l' \in \underline{L}_p \setminus \{l(p)\}} m_{l'p}$, where m_{lp} is the distance in miles between depot l and ship-to point p. If depot $l'(p)$ cannot ship the order, the company asks the external supply source[1] ($l = 0$) to make a direct shipment to ship-to-point *p*. On day τ, the ship-to-points supplied from the external depot are thus $P_{0\tau}(\omega) = P_\tau(\omega) \setminus \cup_{l \in \underline{L}} \underline{P}_{l\tau}(\omega)$.

Once reassignments have been made, the set $\underline{P}_{l\tau}(\omega)$ and the loads $d_{p\tau}(\omega)$, $p \in \underline{P}_{l\tau}(\omega)$, to deliver on day τ are known for each depot $l \in \underline{L} \cup \{0\}$. Then, the company can plan its transportation for the next day, and requests the trucks required for each depot from its carriers. These shipping decisions are made by the SN depots in two steps. First, for loads that are larger than a truckload, a decision is made to ship as much as possible in FTL. We assume that a single type of vehicle is available to make full truckload shipments to point *p*. Let,

- $y_{p\tau}^{FTL}(\omega)$: The number of truckloads shipped to point *p* on day τ
- b^F: The capacity of vehicles used for FTL shipments
- w_{lp}: The cost of a FTL shipment to point *p* from depot *l*

To determine the FTL shipments to make to point $p \in \underline{P}_{l\tau}(\omega)$, problem (1) below is solved by inspection. Then the residual loads to be inserted in the STL, MTL or LTL shipments are given by (2).

$$y_{p\tau}^{FTL}(\omega) = \arg\left(\max_{y=0,1,\ldots} b^F y \middle| b^F y \le d_{p\tau}(\omega) \right) \tag{1}$$

$$\bar{d}_{p\tau}(\omega) = d_{p\tau}(\omega) - b^F y_{p\tau}^{FTL}(\omega) \tag{2}$$

Next, the best delivery routes must be constructed. Let,

- P_k: Ordered set of ship-to-points in route *k*
- $K_{l\tau}(\omega)$: Set of non-dominated feasible STL, MTL or LTL routes (i.e. such that $P_k \subset \underline{P}_{l\tau}(\omega)$ and $\sum_{p \in P_k} \bar{d}_{p\tau}(\omega) \le b_k$, $k \in K_{l\tau}(\omega)$, where b_k is the capacity of route *k* vehicles) from depot *l*, on day τ, under scenario ω
- w_k: Transportation cost of route $k \in K_{l\tau}(\omega)$
- δ_{kp}: Binary coefficient taking the value 1 if ship-to-point *p* is covered by route *k*, and 0 otherwise
- y_k: Binary decision variable equal to 1 if route *k* is used for the depot, day and scenario considered, and 0 otherwise

For scenario ω, the best routes are obtained at depot *l* on day τ by solving the following transportation sub-problem:

[1] This external recourse is specified to ensure that all orders can be shipped on a given day, but it is required very rarely. When the need occurs, one could always reassign outstanding orders to the second nearest depot. Modeling this explicitly would complicate things significantly, however, and it would not remove the need to include an external recourse. For this reason, we assume here that a single backup depot is available.

$$C_{l\tau}^{u}(\omega) = \min_{\mathbf{y}} \sum_{k \in K_{l\tau}(\omega)} w_k y_k \tag{3}$$

subject to

$$\sum_{k \in K_{l\tau}(\omega)} \delta_{kp}\, y_k = 1 \qquad p \in \underline{P}_{l\tau}(\omega) \tag{4}$$

$$y_k \in \{0,1\} \qquad k \in K_{l\tau}(\omega) \tag{5}$$

where $C_{l\tau}^{u}(\omega)$ is the cost of the optimal shipments made by depot l on day τ under scenario ω. In this paper, a heuristic proposed by Klibi *et al.* (2010), based on perturbed Clarke and Wright savings and 2-opt improvements, is used to solve this problem. In practice, the solution approach implemented by the company should however be used. The cost of the transportation solution thus obtained is denoted by $\hat{C}_{l\tau}^{u}(\omega)$.

The shipments made on a daily basis generate sales revenues, and additional costs are incurred for recourse actions. Let,

- v_l: Unit cost of products shipped from depot l (taking into account the product production/procurement costs, inbound shipment costs, warehousing costs and inventory holding costs under normal operations)
- u_p: Unit price of products sold to ship-to-point p
- $O_{l\tau}(\omega)$: Recourse capacity (ex: overtime) needed at depot l on day τ ($O_{l\tau}(\omega) = \sum_{p \in \underline{P}_{l\tau}(\omega)} d_{p\tau}(\omega) - a_l$ if $l \in L_{\tau}(\omega)$ and 0 otherwise)
- c_l^O: Additional unit cost incurred when recourse capacity is needed to ship products from depot l, with the consequence that the unit cost of the products shipped becomes $v_l^O = v_l + c_l^O$
- v^e: Unit cost of products supplied from the external emergency source using STL, MTL or LTL shipments
- v^h: Unit cost of products supplied from the external source using FTL shipments

Since the emergency supply source would be used only in extreme cases, the costs v^e and v^h are assumed to be the same for all ship-to-points. These costs are external recourse penalties with values higher than product values and prices, i.e. such that $v_l < v_l^O < v^h < v^e$ for all l and $u_p < v^h < v^e$ for all p. From this, unit external supply loss can be defined as: $c_p^e = v^e - u_p$ and $c_p^h = v^h - u_p$, for all p. Under scenario ω, for day τ, the net revenues $\hat{R}_{l\tau}(\omega)$ generated by depot $l \in \underline{L}$ and the network loss for external recourses $\hat{C}_{0\tau}(\omega)$ can be calculated as follows:

$$\hat{R}_{l\tau}(\omega) = \sum_{p \in \underline{P}_{l\tau}(\omega)} \left(\left[(u_p - v_l)\, d_{p\tau}(\omega) \right] - c_l^O O_{l\tau}(\omega) - w_{lp}\, y_{p\tau}^{FTL}(\omega) \right) - \hat{C}_{l\tau}^{u}(\omega) \tag{6}$$

$$\hat{C}_{0\tau}(\omega) = \sum_{p \in \underline{P}_{0\tau}(\omega)} \left(c_p^e \bar{d}_{p\tau}(\omega) + c_p^h b^F\, y_{p\tau}^{FTL}(\omega) \right) \tag{7}$$

The **UserResponse** procedure described above is summarized in **Figure 7**, and it can be used to calculate the net revenues $\hat{R}^u(\mathbf{x},\omega)$ generated over the planning horizon for a given SN design $\mathbf{x}$, under a scenarios ω. The next section proposes a design approach for the LTP under uncertainty based on the risk modeling, scenario generation and response procedures presented previously.

UserResponse $(\mathbf{x},(l'(p),p\in P),\mathbf{a}(\omega),\mathbf{d}(\omega),(L_\tau(\omega),P_\tau(\omega),\tau\in T),\mathcal{P};\hat{R}^u(\mathbf{x},\omega))$

For all $\tau\in T$, do:

Set $\underline{P}_{l\tau}(\omega)=\varnothing$, $l\in\underline{L}\cup\{0\}$; $a_{l\tau}(\omega)=(1+\zeta_l)a_l$, $l\in L_\tau(\omega)\cap\underline{L}$

Assign orders to depots

For all $p\in P_\tau(\omega)$ in order of the priority in $\mathcal{P}$, do

If $a_{l(p)\tau}(\omega)\geq d_{p\tau}(\omega)$ then

Set $\underline{P}_{l(p)\tau}(\omega)=\underline{P}_{l(p)\tau}(\omega)\cup\{p\}$ and $a_{l(p)\tau}(\omega)=a_{l(p)\tau}(\omega)-d_{p\tau}(\omega)$

Else If $a_{l'(p)\tau}(\omega)\geq d_{p\tau}(\omega)$ then

Set $\underline{P}_{l'(p)\tau}(\omega)=\underline{P}_{l'(p)\tau}(\omega)\cup\{p\}$ and $a_{l'(p)\tau}(\omega)=a_{l'(p)\tau}(\omega)-d_{p\tau}(\omega)$

Else: $P_{0\tau}(\omega)=P_{0\tau}(\omega)\cup\{p\}$

End do

Compute depot revenues and network loss

For all $l\in\underline{L}$, do

Solve the transportation problem (3)-(5) with Klibi *et al.* heuristic (2010)

Compute depot *l* net revenues $\hat{R}_{l\tau}(\omega)$ with (6)

End do,

Compute the network loss for external recourses $\hat{C}_{0\tau}(\omega)$ with (7)

End do

Compute the SN net revenues $\hat{R}^u(\mathbf{x},\omega)=\sum_{\tau\in T}\left(\sum_{l\in\underline{L}}\hat{R}_{l\tau}(\omega)-\hat{C}_{0\tau}(\omega)\right)$

Figure 7- Response Procedure for Design $\mathbf{x}$ under Scenario ω

3. Scenario-Based SN Design Approach

The LTP under uncertainty is a hierarchical decision problem due to the temporal hierarchy between the location decisions and the transportation decisions. At design time depot location and mission decisions $\mathbf{x}$ must be made. However, these strategic decisions impose resource constraints on the network user's transportation decisions and they may restrict recourse actions in response to customer demands and network disruptions. On day τ, for scenario ω, based on the response policy of the company, depot l users make shipping decisions, denoted by $y_{kl\tau}(\omega), k\in K_{l\tau}(\omega)$, to fulfil the orders received from ship-to-points. These decisions must be anticipated at design time since it is through them that net sales revenues are generated. In addition, operations can be perturbed by demand and/or depots disruptions. The recourse actions then employed to provide efficient service must also be anticipated to design a resilient network structure. This can be done by formulating the SN design problem as a two stage stochastic program with recourse. When based on exact anticipations, these SN design models are extremely complex and their solvability is an issue. To produce good designs, adequate precision-solvability trade-offs must thus be made. To this end, in what follows, we examine two approximate anticipations of operational responses involving scenario and period sampling, and transportation decisions aggregation.

Risk-Neutral Design Model

The strategic decisions to make involve the selection of a subset of depots $\underline{L}^* \subset L$ to operate during the planning horizon T, and the assignment of ship-to-points $\underline{P}_l^* \subset P_l$, $l \in \underline{L}^*$ to these depots, to maximize total expected profits. An important aspect of the problem is that the mission of the selected depots, defined by their customer sets, $\underline{P}_l^*$, $l \in \underline{L}^*$ must remain the same for each day $\tau \in T$ of the planning horizon. To anticipate operational revenues and expenditures, transportation sub-problem (3)-(5) as well as the revenues and loss functions (6) and (7) must be incorporated in the design model. Let,

x_l:	Binary variable equal to 1 if depot *l* is opened, and 0 otherwise
x_{lp}:	Binary variable equal to 1 if ship-to point *p* is assigned to depot *l*, and 0 otherwise
$e_{lp\tau}(\omega)$:	Binary variable equal to 1 under scenario ω if the ship-to point *p* order for day τ is shipped from the emergency source instead of its primary depot l, and 0 otherwise
$h_{lp\tau}(\omega)$:	Integer variable giving the number of FTL shipments made to ship-to point *p* from the emergency source, instead of depot *l*, for day τ under scenario ω
A_l:	Fixed operating cost incurred when depot $l \in L$ is used

Also, the additional unit costs incurred when products are shipped from the emergency supply source instead of from supply depot *l* are denoted by $c_l^e = v^e - v_l$ for STL, MTL or LTL shipments and by $c_l^h = v^h - v_l$ for FTL shipments.

For a risk-neutral decision-maker, a SN design maximizing expected net revenues is obtained by solving the following two-stage stochastic program with recourse:

$$R = \max_{\mathbf{x}} \sum_{S=L,H} \pi^S \sum_{\omega \in \Omega^S} \pi^S(\omega) R^{du}(\mathbf{x}, \omega) - \sum_{l \in L} A_l x_l \tag{8}$$

subject to

$$\sum_{l \in L_p} x_{lp} = 1 \qquad p \in P \tag{9}$$

$$x_{lp} \le x_l \qquad l \in L,\ p \in P_l \tag{10}$$

$$x_l, x_{lp} \in \{0,1\} \qquad l \in L,\ p \in P_l \tag{11}$$

Based on (3)-(6), the optimal value $R^{du}(\mathbf{x},\omega)$ of the second stage program for design $\mathbf{x}$ and scenario ω is given by:

$$R^{du}(\mathbf{x},\omega) = \sum_{\tau\in T}\left[\sum_{l\in L}\sum_{p\in P_\tau(\omega)\cap P_l}\left(\left[(u_p - v_l)d_{p\tau}(\omega)\right]x_{lp} - w_{lp}\left[y_{p\tau}^{FTL}(\omega)x_{lp} - h_{lp\tau}(\omega)\right]\right) - C_\tau^{du}(\mathbf{x},\omega)\right] \tag{12}$$

with, for a given day $\tau \in T$,

$$C_\tau^{du}(\mathbf{x},\omega) = \min\sum_{l\in L}\left[\sum_{k\in K_{l\tau}(\omega)} w_k y_{kl\tau}(\omega) + c_l^O O_{l\tau}(\omega) + \sum_{p\in P_\tau(\omega)\cap P_l}\left[c_l^e \bar{d}_{p\tau}(\omega)e_{lp\tau}(\omega) + c_l^h b^F h_{lp\tau}(\omega)\right]\right] \tag{13}$$

subject to

$$\sum_{k\in K_{l\tau}(\omega)} \delta_{kp}\, y_{kl\tau}(\omega) + e_{lp\tau}(\omega) = x_{lp} \qquad l\in L,\ p\in P_\tau(\omega)\cap P_l \tag{14}$$

$$h_{lp\tau}(\omega) \le y_{p\tau}^{FTL}(\omega)x_{lp} \qquad l\in L,\ p\in P_\tau(\omega)\cap P_l \tag{15}$$

$$\begin{aligned}\sum_{k\in K_{l\tau}(\omega)}\Big(\Big[\sum_{p\in P_\tau(\omega)\cap P_l}\delta_{kp}\bar{d}_{p\tau}(\omega)\Big]y_{kl\tau}(\omega)\Big) + \sum_{p\in P_\tau(\omega)\cap P_l} b^F y_{p\tau}^{FTL}(\omega)x_{lp} \\ \le a_{l\tau}(\omega) + O_{l\tau}(\omega) + \sum_{p\in P_\tau(\omega)\cap P_l} b^F h_{lp\tau}(\omega) \qquad l\in L\end{aligned} \tag{16}$$

$$0 \le O_{l\tau}(\omega) \le \zeta_l a_l,\ l\in L_\tau(\omega); \qquad O_{l\tau}(\omega) = 0,\ l\in L_\tau^d(\omega) \tag{17}$$

$$y_{kl\tau}(\omega), e_{lp\tau}(\omega) \in \{0,1\}, \quad h_{lp\tau}(\omega)\ integer \qquad k\in K_{l\tau}(\omega),\ l\in L,\ p\in P_\tau(\omega)\cap P_l \tag{18}$$

In the first term of objective function (8) the expected net revenues are calculated and in the second term the depot fixed costs are subtracted to get expected profits. Constraints (9) in the first stage program enforce single depot assignments for ship-to-points and constraints (10) limit ship-to-point assignments to opened depots. For design **x**, under scenario ω, expression (12) estimates the anticipated net revenues based on allocation decisions, depots FTL shipment costs and other transportation and recourse costs obtained by solving the second stage program (13)-(18). The objective function (13) computes STL, MTL or LTL shipment costs, depot overcapacity costs and emergency supply costs. Constraints (14) are route coverage and also coupling relations ensuring that daily routes selection respects depots mission decisions for the second stage. Constraints (15) insure that FTL recourses are employed for day τ only if ship-to-point *p* is assigned to depot *l*. Constraints (16) ensure that each depot *l* capacity is respected given the demand on day τ for the assigned ship-to-points. Constraints (17) limit local recourse proportionally to the depot capacity.

Design Models Based on Approximate Anticipations

The stochastic program (8)-(18) is intractable due to the infinite number of plausible future scenarios and the extremely large number of possible transportation routes. Thus, approximate anticipations must be used to obtain solvable SN design models. Two approximate anticipations providing good quality-solvability trade-offs are considered in this paper. The first one is obtained through scenario and period sampling, and the second one through transportation decision aggregations transforming the stochastic LTP into a stochastic location-allocation problem.

The stochastic complexity of the problem can be reduced by solving program (8)-(18) for a random sample of scenarios generated with the **MonteCarlo** procedure presented in **Figure 4**. This Sample Average Approximation (SAA) method (Shapiro, 2003) has been successfully applied to solve several SN design problems (Santoso *et al.*, 2005; Vila *et al.*, 2007; Schutz *et al.*, 2009; Klibi *et al.*, 2010). Given our partition of the plausible future scenarios in two subsets Ω^L and Ω^H, the idea in our context is first to generate a large independent sample of M equiprobable scenarios $\Omega^M \subset \Omega$ using procedure **MonteCarlo**, and then to partition it into a subset Ω^{M_L} of M_L low-risk scenarios and a subset Ω^{M_H} of M_H high-risk scenarios. An estimate of the probabilities π^L and π^H is then given by $\bar{\pi}_L = M_L / M$ and $\bar{\pi}_H = 1 - \bar{\pi}_L$. Second, a small sample Ω^{m_L} of m_L scenarios is randomly selected in Ω^{M_L} and a small sample Ω^{m_H} of m_H scenarios is randomly selected in Ω^{M_H} to get $\Omega^m = \Omega^{m_L} \cup \Omega^{m_H}$. These sets of equiprobable scenarios can then be used to formulate a SAA model. Unfortunately, the multi-period SAA model obtained is still extremely difficult to solve with current solvers. Since the ship-to-point demand processes are stationary, the problem can be further simplified through period sampling, i.e. by considering only a subset $\hat{T} \subset T$ of daily periods (for example, one randomly selected day per week) with associated ship-to-point demands $d_{pt}(\omega), p \in P_t(\omega), \omega \in \Omega^m, t \in \hat{T}$, and depot capacities $a_{lt}(\omega), l \in L, \omega \in \Omega^m, t \in \hat{T}$. The net revenues must then be multiplied by the period shrinking factor $|T|/|\hat{T}|$ to obtain an adequate approximation of the total expected profits[2]. Also, since generating all possible routes yield extremely large models, the approximation proposed is based on adequately generated subsets of routes $\hat{K}_{lt}(\omega) \subset K_{lt}(\omega), l \in L, \omega \in \Omega^m, t \in \hat{T}$. Klibi *et al.* (2010b) showed that this period and route sampling approach gives very good results for the unbounded capacity LTP without disruptions.

Objective function (8), based on the probabilities π^L and π^H, provide the total expected profit, which is an adequate performance measure for a risk-neutral decision-maker. However, if the decision-maker is risk-averse, these probabilities need to be replaced by weights $\hat{\pi}_H > \pi^H$ and $\hat{\pi}_L = 1 - \hat{\pi}_H$ to give more importance to high-risk scenarios. The value of these weights are based on the estimated probability $\bar{\pi}_H$. Given these elements, program (8)-(18) is transformed into the following approximate *Location-Transportation* model (**LT**):

[2] Note that reducing the problem size by aggregating periods, instead of sampling periods, does not provide a good approximation because the capacity available at the end of a week, for example, can then be used to fulfill orders received at the beginning of the week, which clearly overestimates the capacity available.

$$\hat{R}_{\mathrm{LT}} = \max \frac{|T|}{|\hat{T}|} \sum_{S=L,H} \frac{\hat{\pi}_S}{m_S} \left\{ \sum_{\omega\in\Omega^{m_s}} \sum_{t\in\hat{T}} \sum_{l\in L} \left[\sum_{p\in P_t(\omega)\cap P_l} \left(\left[\left(u_p - v_l\right) d_{pt}(\omega) \right] x_{lp} - w_{lp} \left[y_{pt}^{FTL}(\omega) x_{lp} - h_{lpt}(\omega) \right] \right) \right. \right.$$
$$\left. \left. - \sum_{k\in\hat{K}_{lt}(\omega)} w_k y_{klt}(\omega) - c_l^O O_{lt}(\omega) - \sum_{p\in P_t(\omega)\cap P_l} \left[c_l^e \overline{d}_{pt}(\omega) e_{lpt}(\omega) + c_l^h b^F h_{lpt}(\omega) \right] \right] \right\} - \sum_{l\in L} A_l x_l \tag{19}$$

subject to

$$\sum_{k\in\hat{K}_{lt}(\omega)} \delta_{kp}\, y_{klt}(\omega) + e_{lpt}(\omega) = x_{lp} \qquad l\in L,\, p\in P_t(\omega)\cap P_l,\, \omega\in\Omega^m,\, t\in\hat{T} \tag{20}$$

$$h_{lpt}(\omega) \le y_{pt}^{FTL}(\omega) x_{lp} \qquad l\in L,\, p\in P_t(\omega)\cap P_l,\, \omega\in\Omega^m,\, t\in\hat{T} \tag{21}$$

$$\sum_{k\in\hat{K}_{lt}(\omega)} ([\sum_{p\in P_t(\omega)\cap P_l} \delta_{kp}\overline{d}_{pt}(\omega)] y_{klt}(\omega)) + \sum_{p\in P_t(\omega)\cap P_l} b^F y_{pt}^{FTL}(\omega) x_{lp}$$
$$\le a_{lt}(\omega) + O_{lt}(\omega) + \sum_{p\in P_t(\omega)\cap P_l} b^F h_{lpt}(\omega) \qquad l\in L,\, \omega\in\Omega^m,\, t\in\hat{T} \tag{22}$$

$$0 \le O_{lt}(\omega) \le \zeta_l a_l,\ l\in L_t(\omega); \quad O_{lt}(\omega) = 0,\ l\in L_t^d(\omega) \qquad \omega\in\Omega^m,\ t\in\hat{T} \tag{23}$$

$$y_{klt}(\omega), e_{lpt}(\omega) \in \{0,1\};\ h_{lpt}(\omega)\ integer \qquad l\in L,\, p\in P_t(\omega)\cap P_l,\, k\in\hat{K}_{lt}(\omega),\, \omega\in\Omega^m,\, t\in\hat{T} \tag{24}$$

and to the location-allocation constraints (9)-(11).

Model **LT** is much simpler than the original model but it is still difficult to solve when the set of plausible future scenarios Ω^m is large. Additional simplifications are possible when the transportation sub-problems are replaced by flow variables between depots and ship-to-points. When this is done, routing costs are replaced by unit flow costs $\hat{w}_{lp}$ between depots $l\in L$ and ship-to-points $p\in P_l$. These costs are estimated by regression using daily historical data (Klibi *et al.*, 2010b). This introduces second-stage binary variable $x_{lpt}(\omega)$ which takes value 1 if ship-to-point p is served by depot l in period t under scenario ω, and 0 otherwise. Also, under scenario ω, the model needs a binary recourse variable $e_{pt}(\omega)$ to specify if products are supplied to ship-to-point p by the emergency source in period t. An average unit net revenue $\left(u_p - \overline{v}^e\right)$, with $\overline{v}^e = \left(v^e + v^h\right)/2$, is associated to these flows. The SAA formulation of the stochastic *Location-Allocation* model thus obtained has the following form (**LA**):

$$\hat{R}_{\mathrm{LA}} = \max \frac{|T|}{|\hat{T}|} \sum_{S=L,H} \frac{\hat{\pi}_S}{m_S} \left\{ \sum_{\omega\in\Omega^{m_s}} \sum_{t\in\hat{T}} \left[\sum_{l\in L} \left(\sum_{p\in P_t(\omega)\cap P_l} \left[\left(u_p - v_l - \hat{w}_{lp}\right) d_{pt}(\omega) \right] x_{lpt}(\omega) - c_l^O O_{lt}(\omega) \right) \right. \right.$$
$$\left. \left. + \left(u_p - \overline{v}^e\right) \sum_{p\in P_t(\omega)} d_{pt}(\omega) e_{pt}(\omega) \right] \right\} - \sum_{l\in L} A_l x_l \tag{25}$$

subject to

$$\sum_{l\in L_p} x_{lpt}(\omega) + e_{pt}(\omega) = 1 \qquad p\in P_t(\omega),\, \omega\in\Omega^m,\, t\in\hat{T} \tag{26}$$

$$x_{lpt}(\omega) \le x_{lp} \qquad l\in L,\, p\in P_t(\omega)\cap P_l,\, \omega\in\Omega^m,\, t\in\hat{T} \tag{27}$$

$$\sum_{p\in P_t(\omega)\cap P_l} d_{pt}(\omega) x_{lpt}(\omega) \le a_{lt}(\omega) + O_{lt}(\omega) \qquad l\in L,\, \omega\in\Omega^m,\, t\in\hat{T} \tag{28}$$

$$0 \le O_{lt}(\omega) \le \zeta_l a_l,\ l \in L_t(\omega); \quad O_{lt}(\omega) = 0,\ l \in L_t^d(\omega) \qquad \omega \in \Omega^m, t \in \hat{T} \tag{29}$$

$$x_{lpt}(\omega), e_{pt}(\omega) \in \{0,1\} \qquad l \in L,\ p \in P_t(\omega) \cap P_l, \omega \in \Omega^m, t \in \hat{T} \tag{30}$$

and to the location-allocation constraints (9)-(11).

Since **LA** is solved relatively easily, larger scenario samples can be used. Note that a gross anticipation could also be obtained by neglecting uncertainty, and by using the familiar deterministic capacitated location-allocation model found in the literature. This requires the calculation of annual average demands $\hat{D}_p$ from the demand process parameters, and the estimation of average unit transportation costs $\hat{w}_{lp}$ by regression. The depots capacity can be set to $|T|a_l$, $l \in L$, which is clearly an overestimation. The following formulation results (**DLA**):

$$\hat{R}_{\text{DLA}} = \max_{\mathbf{x}} \sum_{l \in L} \sum_{p \in P_l} \left\{ \left(u_p - v_l - \hat{w}_{lp} \right) \hat{D}_p \right\} x_{lp} - \sum_{l \in L} A_l x_l \tag{31}$$

subject to $\sum_{p \in P_l} \hat{D}_p x_{lp} \le |T| a_l \qquad l \in L \qquad (32)$

and to location-allocation constraints (9)-(11).

4. Resilience Strategy Formulations

As mentioned, due to demand randomness and disruptions, SN operations can be perturbed and response actions must be tailored to occurring events to maintain business continuity. However, response policies are rarely anticipated in SN design models. The stochastic programming models proposed in the previous section anticipate response policies through the use of the second stage variables $y_{klt}(\omega)$, $O_{lt}(\omega)$, $e_{lpt}(\omega)$ and $h_{lpt}(\omega)$. Due to the costs associated to these variables, the models position the depots and specify their mission to avoid risks as much as possible. Moreover, by considering the risk attitude of the decision-maker, they can be more or less drastic in their effort to avoid disruptions. For these reasons, models **LT** and **LA** are appropriate mainly when pursuing a *risk avoidance* (**ra**) strategy. A discussion of risk avoidance in supply chain management is found in Manuj and Mentzer (2008).

Despite efforts to avoid risks as much as possible, it is clear that the SN designed will be hit by occasional disruptions. Given this, the question then becomes: what kind of risk mitigation constructs could be incorporated in the design models to obtain more robust SN designs? In other words, how can we design the network to make sure that it will bounce back quickly when hit? This can be done by investing in flexible and/or redundant network structures (Sheffi, 2005), and by elaborating better resilience policies. This is the domain of *resilience strategies*. An additional concern is to design robust SN not only to hedge against major disruptions but also to improve business-as-usual operations. Despite the growing number of papers on the need to design resilient networks, little has been done to incorporate this concept in SN design models. However, several formulations proposed in the literature to model coverage (Church and Revelle, 1974), vector assignment (Weaver and Church, 1985) and reliability (Snyder *et al.*, 2006; Tomlin, 2006; Murray and Grubesic, 2007) can help to foster resilience.

Our aim in this section is to propose three LTP network design models to improve resilience under the response policy described earlier. These models distinguish themselves from **LT** and **LA** by the fact that they strive to provide either explicit or implicit instructions on the back-up depot $l'(p)$ to use when the response procedure in **Figure 7** is applied. The first model finds the optimal primary and back-up depots to use for each ship-to-point. The second model allows multiple sourcing for each ship-to-point in order to reduce the risk of disruption when the primary depot is hit. The idea behind the third model is to offer a better network coverage by ensuring that at least two depots are geographically located within desired distances of each ship-to-point. The next subsections extent model **LT** for each of these approaches. Analogous extensions for model **LA** are provided in *Appendix A*.

Optimal Back-up Formulation **(bu)**

The aim of this formulation is to specify the backup depot to use when the primary depot assigned to a ship-to-point cannot supply its orders. It is inspired by the work of Weaver and Church (1985) and Snyder *et al.* (2006) on the vector assignment problem and on the reliable fixed charge location problem, respectively. Binary variables are defined to specify primary allocation decisions x_{lp}^1 and backup allocation decisions x_{lp}^2. The variables x_{lp}^r, $r=1,2$, take the value 1 if ship-to-point p is allocated to depot l as level r (primary or back-up) supply facility, and 0 otherwise. In the model, these mission specification variables are first stage variables and thus they remain the same for all the periods of the planning horizon. However, for each period t of scenario $\omega \in \Omega^m$, a second stage binary variable $s_{lpt}(\omega)$ needs to be introduced: it takes the value 1 if ship-to-point p is supplied by depot l in period t under scenario ω, and 0 otherwise. As before, an external emergency source takes over when both the primary and backup depots are unable to supply a given ship-to-point. This leads to the transformation of model **LT** into the following SAA model (**LTbu**):

$$\hat{R}_{LTbu} = \max \frac{|T|}{|\hat{T}|} \sum_{S=L,H} \frac{\hat{\pi}_S}{m_S} \left\{ \sum_{\omega\in\Omega^{m_s}} \sum_{t\in\hat{T}} \sum_{l\in L} \left[\sum_{p\in P_t(\omega)\cap P_l} \left(\left[(u_p - v_l) d_{pt}(\omega) \right] s_{lpt}(\omega) - w_{lp} \left[y_{pt}^{FTL}(\omega) s_{lpt}(\omega) - h_{lpt}(\omega) \right] \right) \right. \right.$$
$$\left. \left. - \sum_{k\in\hat{K}_{lt}(\omega)} w_k y_{klt}(\omega) - c_l^O O_{lt}(\omega) - \sum_{p\in P_t(\omega)\cap P_l} \left[c_l^e \bar{d}_{pt}(\omega) e_{lpt}(\omega) - c_l^h b^F h_{lpt}(\omega) \right] \right] \right\} - \sum_{l\in L} A_l x_l \qquad (33)$$

subject to

$$\sum_{l\in L_p} x_{lp}^r = 1 \qquad p\in P,\ r=1,2 \qquad (34)$$

$$\sum_{r=1,2} x_{lp}^r \le x_l \qquad p\in P_l,\ l\in L \qquad (35)$$

$$s_{lpt}(\omega) \leq \sum_{r=1,2} x_{lp}^{r} \qquad l \in L,\, p \in P_t(\omega) \cap P_l,\ \omega \in \Omega^m,\ t \in \hat{T} \tag{36}$$

$$h_{lpt}(\omega) \leq y_{pt}^{FTL}(\omega)\, s_{lpt}(\omega) \qquad l \in L,\, p \in P_t(\omega) \cap P_l, \omega \in \Omega^m, t \in \hat{T} \tag{37}$$

$$\sum_{k \in \hat{K}_{lt}(\omega)} ([\sum_{p \in P_t(\omega) \cap P_l} \delta_{kp} \bar{d}_{pt}(\omega)] y_{klt}(\omega)) + \sum_{p \in P_t(\omega) \cap P_l} b^F y_{pt}^{FTL}(\omega) s_{lpt}(\omega) \leq a_{lt}(\omega) + O_{lt}(\omega) + \sum_{p \in P_t(\omega) \cap P_l} b^F h_{lpt}(\omega) \qquad l \in L, \omega \in \Omega^m, t \in \hat{T} \tag{38}$$

$$\sum_{k \in \hat{K}_{lt}(\omega)} \delta_{kp}\, y_{klt}(\omega) + e_{lpt}(\omega) = s_{lpt}(\omega) \qquad l \in L,\, p \in P_t(\omega) \cap P_l, \omega \in \Omega^m, t \in \hat{T} \tag{39}$$

$$\sum_{l \in L_p} s_{lpt}(\omega) = 1 \qquad p \in P_t(\omega),\ \omega \in \Omega^m,\ t \in \hat{T} \tag{40}$$

$$x_l, x_{lp}^r, s_{lpt}(\omega) \in \{0,1\} \qquad l \in L,\, p \in P_t(\omega) \cap P_l, \omega \in \Omega^m, t \in \hat{T}\ \ r = 1,2 \tag{41}$$

and to constraints (23)-(24).

Constraints (34) and (35) ensure that each ship-to point has distinct primary and backup depots. Constraints (36) guarantee that shipments are made only from primary or backup depots. Constraints (40) guarantee that the single sourcing rule is respected for every period. Note that this model includes a large number of binary variables, which complicates its resolution. The instruction provided by the model to the response procedure is the following:

$$l(p) = l \,\big|\, x_{lp}^1 = 1 \text{ and } l'(p) = l \,\big|\, x_{lp}^2 = 1,\ p \in P$$

Multiple Sourcing Formulation (ms)

The idea behind this formulation is to allow multiple sourcing for each ship-to-point to identify natural supply depots backups, as opposed to forced backups as in the previous section. For formulation **LT**, this is done essentially by removing constraints (9) from the first-stage program, and by introducing second-stage assignment variables $s_{lpt}(\omega)$ with value 1 when depot l supplies ship-to-point p in period t under scenario ω. The following model results (**LTms**):

$$\hat{R}_{\mathrm{LTms}} = \max\ \frac{|T|}{|\hat{T}|} \sum_{S=L,H} \frac{\hat{\pi}_S}{m_S} \Bigg\{ \sum_{\omega \in \Omega^{m_S}} \sum_{t \in \hat{T}} \sum_{l \in L} \Bigg[\sum_{p \in P_t(\omega) \cap P_l} \Big(\big[(u_p - v_l) d_{pt}(\omega) \big] s_{lpt}(\omega) - w_{lp} \big[y_{pt}^{FTL}(\omega) s_{lpt}(\omega) - h_{lpt}(\omega) \big] \Big) - \sum_{k \in \hat{K}_{lt}(\omega)} w_k\, y_{klt}(\omega) - c_l^O O_{lt}(\omega) - \sum_{p \in P_t(\omega) \cap P_l} \big[c_l^e \bar{d}_{pt}(\omega) e_{lpt}(\omega) + c_l^h b^F h_{lpt}(\omega) \big] \Bigg] \Bigg\} - \sum_{l \in L} A_l x_l \tag{42}$$

subject to

$$s_{lpt}(\omega) \leq x_{lp} \qquad l \in L,\, p \in P_t(\omega) \cap P_l, \omega \in \Omega^m, t \in \hat{T} \tag{43}$$

$$s_{lpt}(\omega) \in \{0,1\} \qquad l \in L,\, p \in P_t(\omega) \cap P_l, \omega \in \Omega^m, t \in \hat{T} \tag{44}$$

and to constraints (10)-(11), (23)-(24) and (37)-(40).

When this model is solved, several depots may be used to serve a given ship-to-point during the planning horizon. The primary depot for a ship-to-point is then defined as the one shipping the largest quantity of products to this point during the planning horizon. The depot with the second largest shipment quantity is specified as the backup depot. More specifically, we define:

$$l(p) = \arg\max_{l\in \underline{L}_p} \frac{1}{m}\sum_{\omega}\sum_{t} d_{pt}(\omega)\, s_{lpt}(\omega) \text{ and } l'(p) = \arg\max_{l\in \underline{L}_p\setminus\{l(p)\}} \frac{1}{m}\sum_{\omega}\sum_{t} d_{pt}(\omega)\, s_{lpt}(\omega)$$

Note that the backup depot is specified only for ship-to-points supplied by more than one depot during the planning horizon.

Coverage Formulation (co)

As in classical covering problems (Church and Revelle, 1974), the idea behind the third formulation is to offer better network coverage by using proximity criteria specifying maximum primary/backup depot to ship-to-point distances. Let $L_p^2 \subseteq L_p$ be the set of depots located within the backup distance specified for ship-to-point *p*, and $L_p^1 \subset L_p^2$ be the set of depots located within the primary distance specified. This formulation imposes that for each ship-to-point $p \in P$, at least one depot is in L_p^1 and at least 2 depots are in L_p^2. This tends to increase the number of opened depots and to spread them more evenly on the territory. This leads to the transformation of model **LT** into the following SAA model (**LTco**):

$$\begin{aligned}\hat{R}_{\mathrm{Ltco}} = \max\ \frac{|T|}{|\hat{T}|}\sum_{S=L,H}\frac{\hat{\pi}_S}{m_S}\Bigg\{&\sum_{\omega\in\Omega^{m_S}}\sum_{t\in\hat{T}}\sum_{l\in L}\Bigg[\sum_{p\in P_t(\omega)\cap P_l}\Big(\big[(u_p-v_l)\,d_{pt}(\omega)\big]x_{lp}-w_{lp}\big[y_{pt}^{FTL}(\omega)\,x_{lp}-h_{lpt}(\omega)\big]\Big)\\ &-\sum_{k\in\hat{K}_{lt}(\omega)} w_k\, y_{klt}(\omega) - c_l^O O_{lt}(\omega) - \sum_{p\in P_t(\omega)\cap P_l}\Big[c_l^e\,\overline{d}_{pt}(\omega)\,e_{lpt}(\omega)+c_l^h b^F h_{lpt}(\omega)\Big]\Bigg]\Bigg\} - \sum_{l\in L} A_l x_l\end{aligned} \tag{45}$$

subject to

$$\sum_{l\in L_p^1} x_l \geq 1 \qquad p\in P \tag{46}$$

$$\sum_{l\in L_p^2} x_l \geq 2 \qquad p\in P \tag{47}$$

and to constraints (9)-(11) and (20)-(24).

This model is easier to solve than the previous ones. The instruction it provides to the response procedure is $l(p) = l \,|\, x_{lp} = 1$. $l'(p)$ is not specified and the default backup policy is used.

5. SN Design Models Solution and Evaluation Approach

The approach used to solve the SAA models formulated previously, and to evaluate their performance in terms of value creation and robustness was introduced in **Figure 1**. It involves three phases: scenarios generation, design generation and design evaluation. It can be seen as an adaptation of the SAA method used to solve stochastic programs (Shapiro, 2003). The scenario generation is done using procedure **MonteCarlo** presented in **Figure 4** and it provides scenarios to the design generation and evaluation phases. A large sample of scenarios Ω^{M} is generated and partitioned into low-risk scenarios Ω^{M_L} and high-risk scenarios Ω^{M_H}, based on the hazard tolerance level κ. From these samples, two subsets of scenarios are randomly selected to perform the designs evaluation: a subset $\Omega^{M_L^e} \subset \Omega^{M_L}$ of M_L^e low-risk scenarios, and a subset $\Omega^{M_H^e} \subset \Omega^{M_H}$ of M_H^e high-risk scenarios. A subset $\Omega^{M_W^e} \subset \Omega^{M_H}$ of M_W^e worst-case scenarios is also selected in the tail of the distribution of the number of hits (**Figure 5**).

The design generation phase involves the solution of the SAA models. As normally done with the SAA method, each model is used to generate several designs using N replications of small scenario samples. To do this, another large sample of scenarios $\Omega^{M'}$ is independently generated and partitioned into low and high risk subsets $\Omega^{M'_L}$ and $\Omega^{M'_H}$. From these samples, the probabilities $\bar{\pi}_L$ and $\bar{\pi}_H$ are estimated, and I replications of small scenario samples $\Omega_i^{m_L}, \Omega_i^{m_H}$, $i = 1,...,I$, are randomly selected to construct the SAA models. Based on $\bar{\pi}_L$ and $\bar{\pi}_H$ the risk aversion weights $\hat{\pi}_L$ and $\hat{\pi}_H$ are also specified. The MIPs obtained are then solved for each sample replication, using a commercial solver such as CPLEX-11 or a specialized solution algorithm. Given the different models proposed and the I replications made, the design generation phase produces a set of alternative SN designs $\mathbf{x}^j, j = 1,...,J$. Each SN design obtained specifies the set of depots $\underline{L}$ to open and their primary mission $l(p), p \in P$. For some models, an instruction $l'(p), p \in P$, on the backup depot to use is also provided. When no backup depot is specified, the default reassignment policy applies.

In the evaluation phase, the alternative designs $\mathbf{x}^j, j = 1,...,J$ are compared using the plausible future scenario sample $\Omega^{M^e} = \Omega^{M_L^e} \cup \Omega^{M_H^e} \cup \Omega^{M_W^e}$. Since the models proposed incorporate approximate anticipations of operational responses, it is not adequate to use their objective function to determine the best design. Moreover, these objective functions do not cover all the value creation and robustness dimensions that decision-makers may want to explore. For these reasons, the evaluation of the SN designs is based on a set of measures related to the net revenues $\hat{R}^u(\mathbf{x}^j, \omega)$, $\omega \in \Omega^{M^e}, j = 1,...,J$, provided by the **UserResponse** procedure. More specifically, for a given design $\mathbf{x}^j$, these performance measures are based on the value added during the planning horizon under the scenarios considered, that is:

$$\hat{R}(\mathbf{x}^j, \omega) = \hat{R}^u(\mathbf{x}^j, \omega) - \sum_{l \in L} A_l(x_l^j), \ \omega \in \Omega^{M^e} \tag{48}$$

An adequate SN design evaluation must be based on expected value and robustness measures, and it must take the decision-makers risk attitude into account. The expected return $\bar{R}(\mathbf{x}^j)$ of a design $\mathbf{x}^j$ is provided by:

$$\bar{R}(\mathbf{x}^j) = \sum\nolimits_{S=L,H} \bar{\pi}_S \bar{R}_S(\mathbf{x}^j); \qquad \bar{R}_S(\mathbf{x}^j) = \frac{1}{M_S^e} \sum\nolimits_{\omega \in \Omega^{M_S^e}} \hat{R}(\mathbf{x}^j, \omega), \ S = L, H \tag{49}$$

where $\bar{R}_L(\mathbf{x}^j)$ and $\bar{R}_H(\mathbf{x}^j)$ are conditional expected returns for low and high risk scenarios, respectively. Robustness is related to the variability of the returns obtained under different scenarios. Since downside deviations from mean returns are undesirable, an adequate variability measure to assess a design $\mathbf{x}^j$ is the mean-semideviation $MSD(\mathbf{x}^j)$ given by:

$$\begin{aligned} MSD(\mathbf{x}^j) &= \sum\nolimits_{S=L,H} \bar{\pi}_S MSD_S(\mathbf{x}^j) \\ MSD_S(\mathbf{x}^j) &= \frac{1}{M_S^e} \sum\nolimits_{\omega\in\Omega^{M_S^e}} \max\left[\left(\bar{R}_S(\mathbf{x}^j) - \hat{R}(\mathbf{x}^j,\omega)\right);0\right],\ S = L,H \end{aligned} \tag{50}$$

where $MSD_L(\mathbf{x}^j)$ and $MSD_H(\mathbf{x}^j)$ are conditional mean-semideviations for low and high risk scenarios, respectively. The mean-semideviation is a coherent risk measure (Shapiro, 2007).

Decision-makers are also interested by the behaviour of the designs under extreme conditions. Using worst-case scenarios, this is often evaluated with the absolute robustness criteria proposed by Kouvelis and Yu (1997). For design $\mathbf{x}^j$ this measures the minimum return $\bar{R}_W(\mathbf{x}^j)$ under all worst case scenarios, calculated as follows:

$$\bar{R}_W(\mathbf{x}^j) = \min_{\omega\in\Omega^{M_W^e}} \left\{\hat{R}(\mathbf{x}^j,\omega)\right\} \tag{51}$$

Measures (49)-(51) provide the basis for a multi-criteria evaluation of the designs considered. Note that these measures can be used to compare the designs provided by the I replications of a given SAA model, as well as the best designs obtained from the different SAA models.

The previous performance measures can also be used to construct a compound return measure reflecting the decision-makers aversion to variability and to extreme events. Such a measure is provided by the following expression:

$$\mathcal{R} = (1-\psi) \sum_{S=L,H} \hat{\pi}_S \left(\bar{R}_S(\mathbf{x}^j) + \varphi_S MSD_S(\mathbf{x}^j)\right) + \psi \bar{R}_W(\mathbf{x}^j) \tag{52}$$

where $\varphi_S \in [0,1]$, $S = L,H$, are variability aversion weights for low and high risk scenarios, and where $\psi \in [0,1]$ is an extreme event aversion weight. Note that if we set $\varphi_L = \varphi_H = \psi = 0$, the return function obtained corresponds to the objective function of the SAA models used in the design generation phase.

6. Computational Results

This section presents the experiments made to compare the models proposed in the previous sections and it analyses the results obtained. In addition to the eight SAA formulations elaborated (**LT**, **LTbu**, **LTms**, **LTco**, **LA**, **LAbu**, **LAms**, **LAco**), two models where included to provide conventional SN design approaches benchmarks. One of them is a deterministic location-allocation model (**DLA**) which completely neglects uncertainty. The second one is a variant of

LT neglecting hazards: it's a SAA model considering only the randomness in demand and denoted by **LTD**. Also, for the eight SAA formulations considering hazards, two types of decision-makers (DM) are considered: a risk neural DM and a risk averse DM. Consequently, in what follows, 18 distinct SN design models are compared.

Plan of experiments

In order to test the SN design models proposed, several problem instances were generated based on the following four dimensions: the SN breadth, the cost structure, the demand characteristics and the depots size. First, two problem instances are considered with different number of potential depots and ship-to points, scattered over different geographical areas, as specified in **Table 1**. The distances between the network nodes are based on existing road networks and they are calculated with PC*MILER (www.alk.com). A one year planning horizon including $|T|$=240 working days is used. The next day delivery requirement is implemented through a 400 miles limit on the distance between depots and ship-to points. Exceptionally, when the number of incident lanes of a ship-to-point is less than three, depots with distances larger than 400 miles are also considered.

Problem instance	Geographical Area	Potential depots	Number of ship-to-points
P_1	North-Eastern US States	7	206
P_2	North-Eastern & Midwest US States	15	706

Table 1- Test Problem Instances

Based on the cost structure of a real case, instances with high level fixed costs (*hfc*) and low level fixed costs (*lfc*) were defined. The fixed cost for each depot A_l is randomly generated in $[180K, 200K]$ for *hfc* and in $[60K, 80K]$ for *lfc*. Also, the unit product values, v_l, are selected randomly in $[19,21]$ and the product prices, u_p, are fixed to 23 for all ship-to-points. The values v^h and v^e of the products coming from the external supplier are fixed to 25 and 24, respectively, and the local capacity unit recourse cost c_l^z is fixed to 1 for all the depots.

We assumed that the distribution $F_p^q(.)$ of order inter-arrival times is exponential with an expected time between orders η_p. Also a log-normal distribution with mean μ_p and standard deviation σ_p was used for the order quantity distribution $F_p^o(.)$. Three ship-to-points size (Large, Medium and Small) were defined to generate two types of network: larger ship-to-points networks (*LN*) dominated by large and medium size customers and smaller ship-to-points networks (*SN*) dominated by small customers. The proportion of ship-to-points of different size in each network type is given in **Table 2**. The table also provides the probability distribution parameters used to generate orders for each ship-to-point size. For model **DLA** the annual average demand of ship-to-points is given by $\hat{D}_p = |T| \mu_p / \eta_p$.

Ship-to-point size:	Large	Medium	Small
Larger ship-to-points Network (*LN*)	15%	65%	20%
Smaller ship-to-points Network (*SN*)	10%	30%	60%
μ (cwt)	[480,580]	[300,400]	[120,220]
σ (% μ)	7%	10%	16%
η (days)	[2.5,4.5]	[5.5,15.5]	[20.5,35.5]

Table 2- Ship-to-Point Demand Structure

Finally, problems with large capacity depots (*LD*) and tight capacity depots (*TD*) were tested. Each depot l has a capacity level given by $a_l = \upsilon \Sigma_{p \in P} \mu_p / \eta_p$, where υ is a factor randomly generated in the intervals given in **Table 3**. The additional capacity, available for local recourse, is fixed to 25 % of the regular capacity level. Note also that all the vehicles capacity (b^F and $b_k, k \in K$) are fixed to 400 cwt. The combination of these four dimensions yields 16 problem instances. Each instance is denoted by the quadruplet (i, j, k, l); $i \in \{P_1, P_2\}$, $j \in \{hfc, lfc\}$, $k \in \{LN, SN\}$, $l \in \{TD, LD\}$.

Capacity factor υ range	*LN*	*SN*
Tight depots structure (*TD*)	$[0.75;1]$	$[1;1.25]$
Large depots structure (*LD*)	$[1.25;1.5]$	

Table 3- Test Problems Depots Capacity Structure

The procedures required to support the research methodology described in **Figure 1**, were programmed in VB.Net 2005, and the experiments reported in this section were performed on a 64 bits server with a 2.5 GHz Intel XEON processor and 16 GB of RAM. All the models were generated with OPL Studio 6.1 and solved with CPLEX-11. The design models were solved to optimality with the following CPLEX parameters: *MIP Emphasis* = Optimality, *Aggressive Cuts*, *MIP Relative Tolerance* = 0.005 and *Time Limit* = 10 hours. The calibration and estimation of the various sampling, aggregation and risk modeling parameters required to generate the models solved are discussed in *Appendix B*.

Numerical Results

Given the 16 problem instances specified previously, this section discusses the solvability of the SN design models proposed, and it studies the quality of the design they provide using performance measures (49)-(52). **Table 4** provides the design model characteristics for problem instances $(\text{-}, hfc, LN, TD)$, the mean solution time (MST in seconds) and the solution time standard deviation (STSD in seconds) for P_1 and P_2. Models based on **LT** have a very tight LP relaxation which helps reduce solution times significantly even with the large number of binary variables (> 1 000 000) involved. The solution times are smaller for P_2 than P_1 for **LTbu** and **LTms** because a smaller number of scenarios and routes are used. Even if the models based on **LA** include a mix of binary and continuous variables, their solution times are sometimes longer than for the corresponding **LT**-model because the scenarios sample used is five times larger. The deterministic model **DLA** is trivial to solve. Note that there is a lot of variability in solution times from one instance to another. This is explained partly by the fact that CPLEX-11 incorporates a number of heuristics to reduce solution times, and that these heuristics do not work as well on all problem structures and instances.

		LT	LTbu	LTms	LTco	LA	LAbu	LAms	LAco	LTD	DLA
P_1	Variables	1060375	1429062	1269822	1304038	416031	397725	393549	416031	1325259	1453
	Constraints	128811	207914	199431	128006	433030	413690	459742	433442	125708	1658
	MST (s)	786	4696	4448	241	5093	1852	5620	6288	437	< 1
	STSD (s)	1224	8870	4722	127	8259	2099	3201	8057	631	0
P_2	Variables	1801570	1952566	1947790	1801570	883766	888542	883766	883766	1796446	10609
	Constraints	276312	444865	443453	277724	906057	906763	905351	907469	271238	11314
	MST (s)	2108	2509	3102	612	1297	939	5200	343	1463	< 1
	STSD (s)	3943	2366	3357	643	1108	1075	3561	209	1283	0

Table 4- Model Characteristics and Average Solution Times for P_1 and P_2

An important issue examined in this paper is the impact of the precision of the operational response anticipation incorporated in the design model on the quality of the designs obtained. The location-transportation formulations (**LT-**models) proposed include a relatively accurate anticipation, but the location-allocation formulations (**LA-**models) are less precise. Our results show that the two formulations never produce the same design, although in most cases, the location decisions are similar. The difference comes mainly from the assignment of ship-to-points to depots. **LT**-models provide different designs for each scenario sample replications which is to be expected because the sample size is relatively small. **LA**-models often produce at least two similar designs among the four replications solved which, again, is normal since larger scenario samples are used. For the smaller problem P_1, **LA** performs extremely well since it gives the best design half of the time. This is explained by the fact that we were able to use large samples of scenarios in this case (50 scenarios, comparatively to 10 for **LT**-models). However, for P_2, the scenario sample size had to be reduced and **LT**-models almost always perform better than **LA**-models. For some instances, the difference in expected design value reaches 7%. This shows that significant gains may be made by using more precise anticipations and larger scenario samples.

We stressed earlier that our models avoid risky depot locations as much as possible. Let $\gamma(\mathbf{x}^j, \omega)$ be the total number of hits on depots under scenario ω when design $\mathbf{x}^j$ is implemented. For a given problem instance, the average number of hits on design $\mathbf{x}^j$ for the scenarios evaluated is given by $\bar{\gamma}(\mathbf{x}^j) = \sum_{\omega\in\Omega^{M^e}} \gamma(\mathbf{x}^j, \omega) \Big/ |\Omega^{M^e}|$. **Table 5** reports the mean of these values by problem type ($\bar{\gamma}_{P_1}(\mathbf{x})$ and $\bar{\gamma}_{P_2}(\mathbf{x})$) for the optimal design provided by each model. The lowest average number of hits is obtained with **LT**, which is congruent with the fact that **LT** is a risk avoidance model. The classical deterministic and stochastic models **DLA** and **LTD** also perform very well from this point of view. This indicates that for the regions of the USA covered by P_1

and P_2, the optimal solutions of **DLA** and **LTD** provide a natural cover against disruptions. This would certainly not be the case in general. Note that the designs provided by the resilience-seeking models are hit more often. This is happening, because there is a demand surge for some ship-to-points when they are hit. Resilient networks can then make additional profits by providing a good support to the victims of extreme events. Finally, it can be seen that **LA**-based models yield a higher average number of hits than **LT**-based models.

	LT	LTbu	LTms	LTco	LA	LAbu	LAms	LAco	LTD	DLA
$\overline{\gamma}_{P_1}(\mathbf{x})$	0.89	1.19	0.90	1.33	1.12	1.31	1.24	1.35	0.90	0.94
$\overline{\gamma}_{P_2}(\mathbf{x})$	1.62	2.19	1.72	2.14	2.18	2.44	2.41	2.44	1.65	1.80

Table 5- Average Number of Depot Hits by Scenario

The behaviour of the designs when hit can be analysed further by examining the average value of a design (calculated from $\hat{R}(\mathbf{x}^j, \omega)$, $\omega \in \Omega^{M^e}$) for scenarios including $\gamma = 0, 1, 2, \ldots$ hits. **Figure 8** presents a value-hit graph of non-dominated risk-mitigation formulations (**ms** and **co**) for $(\text{-}, hfc, LN, TD)$ problem instances. In terms of value creation under a given number of hits, **ra**, **bu**, **DLA** and **LTD** models are dominated either by **ms** or **co** formulations. For low-risk scenarios, **ms**-models tend to give better designs than **co**-models. However, for high-risk scenarios, **co**-models create more value. A similar behaviour is observed for other problem instances. Note however that **ra**-models are often almost as good as **ms**-models.

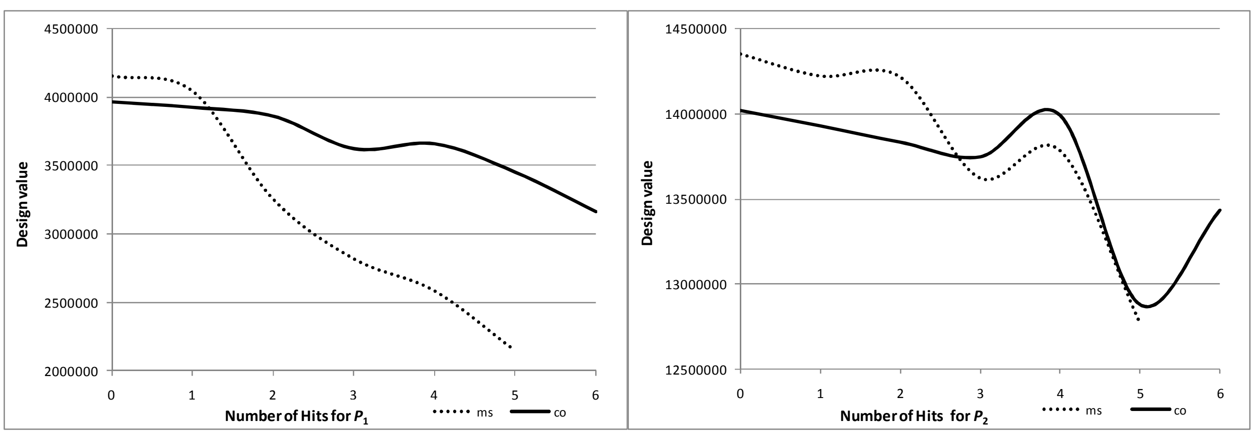

Figure 8- Design Value Behavior by Hit Level for Non-Dominated Models

Robust SN designs should also exhibit low value mean-semideviations (MSD)**. Figure 9** provides value-MSD tradeoffs graphs for P_1 and P_2. The points shown on the graphs for a given risk mitigation formulation are average values over all the models solved. The graphs show that from a value variability point-of-view, **bu**-models are very conservative: they provide lower value but with lower variability. At the other extreme, **ra**, **ms**, **DLA** and **LTD** models are more aggressive: they provide more value but with more variability. **co**-models provide a compromise between these two extremes. Note however that the MSD is relatively low, in comparison with expected values, for all models. In other words, for the cases considered, variability does not stand out as a strong discriminating factor.

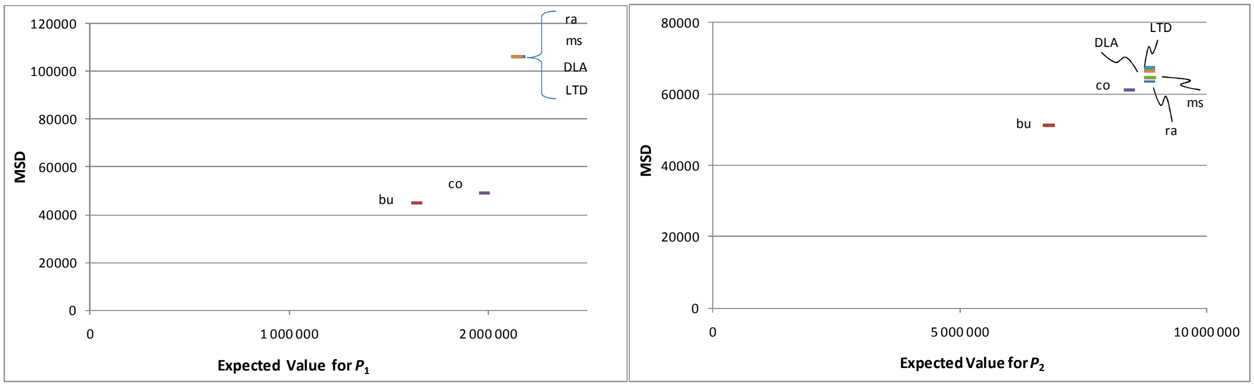

Figure 9- Expected Value – Mean-Semideviation Tradeoffs for P_1 and P_2

All our models were solved with risk-neutral and risk-averse DM weights (see **Table 8** in *Appendix B*). **Table 6** presents the percentage of identical design decisions obtained with each risk attitude weights. The table shows that sensitivity to risk attitude depends very much on the model. **bu**-models are much more sensitive to risk attitude than the other models. In general, however, it is clear that the weights selected have an impact on the SN design obtained.

	LT	LTbu	LTms	LTco	LA	LAbu	LAms	LAco
P_1	69%	0%	56%	44%	63%	0%	41%	59%
P_2	19%	0%	19%	22%	88%	3%	47%	81%

Table 6- Percentage of Identical Designs for Risk-Neutral and Risk-Averse DM

Table 7 provides more detailed results on the performance of the models for the sixteen problem instances generated. The results are expressed in terms of %-deviation from the return of the best design obtained. When the models are solved with risk-neutral DM weights, the return is evaluated with performance measure (49). When the models are solved with risk-averse DM weights, the results are compared using compound return measure (52), with the weights in **Table 8**. Note that evaluations using higher weights were also made, but it did not have a significant impact on the models ranking. For the SAA models, the table provides the %-deviation of the best design obtained with the 4 scenario sample replications generated. The best design value for each problem instance is highlighted.

P_1	*hfc,LN,TD*	*hfc,LN,LD*	*lfc,LN,TD*	*lfc,LN,LD*	*hfc,SN,TD*	*hfc,SN,LD*	*lfc,SN,TD*	*lfc,SN,LD*	**All**
Expected Return $(\overline{R})$ for Risk Neutral Models									
LT	-2,25%	-0,94%	-1,06%	-1,87%	-0,30%	-0,68%	-1,10%	-0,17%	-0,38%
LTbu	-12,66%	-21,74%	-8,14%	-8,30%	-20,16%	-24,84%	-11,31%	-12,04%	-12,86%
LTms	0,00%	-0,94%	-1,42%	-1,90%	-0,14%	-0,72%	0,00%	0,00%	0,00%
LTco	-4,51%	-4,47%	-3,21%	-0,56%	-8,72%	-9,35%	-4,73%	-2,74%	-3,25%
LA	-2,69%	0,00%	-3,05%	-1,86%	-2,07%	0,00%	-4,39%	-3,88%	-1,43%
LAbu	-15,61%	-14,29%	-8,68%	-8,67%	-19,95%	-30,23%	-19,48%	-19,57%	-14,27%
LAms	-1,39%	-1,07%	0,00%	0,00%	0,00%	-0,48%	-2,29%	-2,89%	-0,05%
LAco	-4,43%	-3,26%	-2,92%	-1,86%	-8,57%	-8,37%	-4,63%	-3,88%	-3,27%
LTD	-4,25%	-1,05%	-0,92%	-1,89%	-2,44%	-0,82%	-0,96%	-0,53%	-0,85%
DLA	-4,31%	-1,91%	-0,96%	-1,87%	-2,74%	-0,73%	-0,30%	-0,47%	-0,93%
Compound Return Measure $(\mathcal{R})$ for Risk Averse Models									
LT	-2,18%	-4,25%	-0,18%	-5,47%	-3,08%	-4,86%	-1,31%	-0,23%	-2,04%
LTbu	-11,69%	-24,77%	-8,41%	-7,71%	-15,65%	-24,39%	-11,33%	-14,23%	-13,02%
LTms	0,00%	-4,25%	0,00%	-5,51%	-0,13%	-4,89%	0,00%	0,00%	-1,33%
LTco	-6,17%	-6,26%	-4,24%	-3,52%	-11,10%	-12,54%	-4,97%	-3,47%	-5,14%
LA	-3,62%	-3,05%	-2,07%	-2,42%	-3,12%	-1,83%	-4,41%	-4,93%	-2,36%
LAbu	-11,24%	-11,20%	-8,30%	-9,67%	-21,01%	-30,96%	-15,85%	-15,10%	-12,80%
LAms	-1,26%	0,00%	-0,17%	0,00%	0,00%	0,00%	-2,09%	-3,12%	0,00%
LAco	-6,04%	-3,26%	-2,15%	-3,57%	-10,73%	-9,25%	-6,23%	-4,93%	-4,29%
LTD	-10,85%	-4,36%	-0,19%	-5,50%	-9,35%	-5,04%	-0,97%	-0,85%	-3,82%
DLA	-10,96%	-4,26%	-0,24%	-5,47%	-9,79%	-4,92%	-0,33%	-0,76%	-3,78%

P_2	*hfc,LN,TD*	*hfc,LN,LD*	*lfc,LN,TD*	*lfc,LN,LD*	*hfc,SN,TD*	*hfc,SN,LD*	*lfc,SN,TD*	*lfc,SN,LD*	**All**
Expected Return $(\overline{R})$ for Risk Neutral Models									
LT	-0,12%	-0,09%	-0,28%	0,00%	-0,70%	-0,11%	-0,36%	-0,04%	-0,10%
LTbu	-19,35%	-21,40%	-14,55%	-14,91%	-25,78%	-23,14%	-19,12%	-17,44%	-18,73%
LTms	0,00%	0,00%	-0,37%	-0,14%	0,00%	0,00%	0,00%	0,00%	0,00%
LTco	-2,22%	-2,03%	0,00%	-0,60%	-6,33%	-4,76%	-1,76%	-0,70%	-1,85%
LA	-2,39%	-1,90%	-0,70%	-1,20%	-2,05%	-0,14%	-1,74%	-0,51%	-1,27%
LAbu	-25,19%	-25,58%	-19,92%	-18,31%	-28,56%	-25,52%	-26,97%	-23,22%	-23,44%
LAms	-1,93%	-1,83%	-0,70%	-1,19%	-7,09%	-4,27%	-1,65%	-1,11%	-2,05%
LAco	-3,61%	-3,04%	-0,67%	-1,19%	-7,69%	-4,73%	-2,01%	-0,94%	-2,58%
LTD	-0,20%	-0,08%	-0,23%	-0,03%	-0,68%	-0,07%	-0,87%	-0,03%	-0,15%
DLA	-0,82%	-0,08%	-0,27%	-0,62%	-1,95%	-0,08%	-1,41%	-0,33%	-0,54%
Compound Return Measure $(\mathcal{R})$ for Risk Averse Models									
LT	-0,16%	-0,43%	-0,69%	-0,15%	0,00%	-0,18%	-0,47%	-0,18%	-0,18%
LTbu	-17,42%	-20,76%	-14,74%	-13,13%	-25,70%	-26,00%	-16,79%	-18,47%	-18,18%
LTms	0,00%	0,00%	-0,41%	0,00%	-0,68%	0,00%	0,00%	0,00%	0,00%
LTco	-1,87%	-2,20%	0,00%	-0,74%	-5,23%	-4,71%	-1,50%	-0,84%	-1,69%
LA	-1,48%	-2,10%	-0,85%	-1,21%	-0,87%	-0,09%	-2,18%	-0,62%	-1,10%
LAbu	-25,03%	-25,70%	-19,37%	-17,89%	-26,59%	-24,09%	-24,05%	-22,86%	-22,62%
LAms	-0,63%	-2,00%	-0,90%	-1,21%	-6,12%	-3,55%	-2,06%	-0,62%	-1,73%
LAco	-2,29%	-3,34%	-0,64%	-1,21%	-6,56%	-4,52%	-2,31%	-1,06%	-2,31%
LTD	-0,31%	-0,40%	-0,60%	-0,19%	-0,69%	-0,12%	-1,72%	-0,17%	-0,36%
DLA	-0,72%	-0,38%	-0,60%	-1,01%	-2,57%	-0,10%	-2,24%	-0,54%	-0,80%

Table 7- Models Performance in Terms of Deviation from the Best Design

Several observations can be drawn from this table. Note first that the different models never give the same SN design. Model **LTms** performs extremely well for larger problems (P_2): it usually provides the best design and, when it does not, it is very close to the best. For smaller problems (P_1), when it is not the best, it is usually outranked by **LAms**. This confirms that risk-mitigation formulation **ms** outperforms other formulations. Our results also show clearly that **bu**-models do not perform very well. Although this risk-mitigation approach is conceptually appealing (Snyder *et al*., 2006), it is too conservative and it provides poor returns. Since **co**-models tend to open more depots, they do not provide the best expected returns but, as shown in **Figure 8**, they may be attractive to decision-makers wishing to obtain reasonable returns under high disruptions, mainly when depots fixed costs are low.

For P_2, the risk-avoidance model **LT** gives the best result for two problem instances, and it is generally not too far from the best model. Models **LTD** and **DLA** are dominated, but they also provide surprisingly good results for models not considering disruptions explicitly. This can be explained as follows. First, in our evaluation process, we use a default depot reassignment policy based on a proximity rule. It turns out that this response policy is excellent and that it copes well with disruptions even for designs not optimized for resilience. Second, our results show that the designs obtained are very sensitive to the problem size and topology (P_1 vs P_2). For some problem topology, the designs provided by models not seeking resilience explicitly (**LT**, **LA**, **LTD** and **DLA**) are naturally resilient. Note finally that, except for **co**-formulations which are sensitive to cost structures, the other models are relatively insensitive to cost variations, customer size and depot size.

7. Conclusions

This paper studies a SN design approach under uncertainty. In the context of the multi-period location-transportation problem, it proposes design models incorporating resilience-seeking formulations. A generic solution approach is also proposed to produce effective and resilient SN designs. The models formulated are based on approximate anticipations of the operational response procedure implemented by the SN users. Our results show that the quality of the user response anticipation incorporated in a design model is a critical issue: significant gains can be made by using more precise representations of delivery decisions (routes vs flows) and larger scenario samples. Given the computational power currently available, trade-offs are however necessary. The best approach seems to be to seek an adequate equilibrium between all the dimensions involved (ex: route set cardinality vs scenario sample size) instead of neglecting some dimensions (ex: using a deterministic model to be able to incorporate more routes).

Our results also show that more robust designs are obtained by modeling disruptions explicitly. This is particularly important when additional revenues can be generated by providing the demand surge goods required by customers under extreme events. The models proposed to cope with disruptions try to avoid risk and to provide resilient network structures. However, the incorporation of resilience-seeking constructs in the design models may induces biases that are not necessarily congruent with decision-makers objectives, mainly when the user response procedures are only roughly anticipated. The **bu**-models proposed to select backup depots are good examples of this phenomenon. This also stresses the importance of evaluating potential SN designs with response procedures that are as close as possible to those used by the company considered. By doing this, we found that a good approach to design effective and robust SN is given by **ms**-models which assume that customers can be served from multiple depots. These models provide resilient networks, even when the user operates under a single-sourcing policy. For decision-makers averse to disruptions, the **co**-models provide an interesting alternative, particularly when the depots fixed costs are low. In some context, models seeking simply to avoid risks (**LT** and **LA**) are also a good alternative.

This paper sheds some light on some important SN design issues, but it also raises several questions to address in future research. Our analysis was based on a single product two-echelon location-transportation problem centered on location decisions at the design level and transportation decisions at the user level. More complex multi-product multi-echelon problems incorporating sourcing, capacity and market selection decisions at the design level, as well as inventory and production decisions at the user level, should be studied. Also, our experiments were based on two realistic SN with similar topologies in the north-east of the USA. Experiments should be made with more varied network topologies and disruption processes in different parts of the world. Also, other modeling approaches to get resilient networks can certainly be investigated. Finally, as the design problems considered become more complex, the models formulated become more difficult to solve. This raises the need for the elaboration of heuristic methods to solve the problems, and the issue of the trade-offs between the model precision and solvability.

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APPENDIX A

This Appendix provides the resilience seeking models derived from the location-allocation formulation (**LA)** for the three modeling approaches proposed:

1) *Backup Optimization Model* (**LAbu**)

The following model is obtained for this case:

$$\hat{R}_{\text{LAbu}} = \max \frac{|T|}{|\hat{T}|} \sum_{S=L,H} \frac{\hat{\pi}_S}{m_S} \left\{ \sum_{\omega\in\Omega^{m_S}} \sum_{t\in\hat{T}} \left[\sum_{l\in L} \left(\sum_{p\in P_{lt}(\omega)} \left[\left(u_p - v_l - \hat{w}_{lp}\right) d_{pt}(\omega) \right] s_{lpt}(\omega) - c_l^O O_{lt}(\omega) \right) \right.\right.$$
$$\left.\left. + \left(u_p - \overline{v}^e\right) \sum_{p\in P_t(\omega)} d_{pt}(\omega) e_{pt}(\omega) \right] \right\} - \sum_{l\in L} A_l x_l \tag{53}$$

subject to

$$\sum_{l\in L_p} s_{lpt}(\omega) + e_{pt}(\omega) = 1 \qquad p\in P_t(\omega), \omega\in\Omega^m, t\in\hat{T} \tag{54}$$

$$\sum_{p\in P_t(\omega)\cap P_l} d_{pt}(\omega) s_{lpt}(\omega) \le a_{lt}(\omega) + O_{lt}(\omega) \qquad l\in L, \omega\in\Omega^m, t\in\hat{T} \tag{55}$$

$$0 \le O_{lt}(\omega) \le \zeta_l a_l,\ l\in L_t(\omega); \quad O_{lt}(\omega) = 0,\ l\in L_t^d(\omega) \qquad \omega\in\Omega^m, t\in\hat{T} \tag{56}$$

$$e_{pt}(\omega) \in \{0,1\} \qquad p\in P_t(\omega)\cap P_l, \omega\in\Omega^m, t\in\hat{T} \tag{57}$$

and to constraints (34)-(36) and (41).

2) *Multiple Sourcing Formulation* (**LAms**)

For this case, the second-stage program is defined in terms of continuous flow variables instead of binary assignment variables, namely: $X_{lpt}(\omega)$ the quantity of product supplied by depot l to ship-to point p in period t under scenario ω, and $E_{pt}(\omega)$ the quantity supplied to p by the emergency supply source. This leads to the following model:

$$\hat{R}_{\text{LAms}} = \max \frac{|T|}{|\hat{T}|} \sum_{S=L,H} \frac{\hat{\pi}_S}{m_S} \left\{ \sum_{\omega\in\Omega^{m_S}} \sum_{t\in\hat{T}} \left[\sum_{l\in L} \left(\sum_{p\in P_t(\omega)\cap P_l} \left[u_p - v_l - \hat{w}_{lp} \right] X_{lpt}(\omega) - c_l^O O_{lt}(\omega) \right) \right.\right.$$
$$\left.\left. + \left(u_p - \overline{v}^e\right) \sum_{p\in P_t(\omega)} E_{pt}(\omega) \right] \right\} - \sum_{l\in L} A_l x_l \tag{58}$$

subject to

$$\sum_{l\in L} X_{lpt}(\omega) + E_{pt}(\omega) = d_{pt}(\omega) \qquad p\in P_t(\omega), t\in\hat{T}, \omega\in\Omega^m \tag{59}$$

$$X_{lpt}(\omega) \le d_{pt}(\omega) x_{lp} \qquad l\in L, p\in P_t(\omega)\cap P_l, t\in\hat{T}, \omega\in\Omega^m \tag{60}$$

$$\sum_{p\in P_{lt}(\omega)} X_{lpt}(\omega) \le a_{lt}(\omega) + O_{lt}(\omega) \qquad l\in L, t\in\hat{T}, \omega\in\Omega^m \tag{61}$$

$$X_{lpt}(\omega) \ge 0, E_{pt}(\omega) \ge 0 \qquad l\in L, p\in P_t(\omega)\cap P_l, t\in\hat{T}, \omega\in\Omega^m \tag{62}$$

and to constraints (10)-(11) and (29).

The instruction provided by the model is:

$$l(p) = \arg\max_{l \in \underline{L}_p} \frac{1}{m} \sum_{\omega} \sum_{t} X_{lpt}(\omega) \text{ and } l'(p) = \arg\max_{l \in \underline{L}_p \setminus \{l(p)\}} \frac{1}{m} \sum_{\omega} \sum_{t} X_{lpt}(\omega)$$

3) *Coverage Formulation* (**LAco**)

This approach leads to the transformation of model **LA** into the following model:

$$\hat{R}_{\text{LAco}} = \max \frac{|T|}{|\hat{T}|} \sum_{S=L,H} \frac{\hat{\pi}_S}{m_S} \left\{ \sum_{\omega \in \Omega^{m_S}} \sum_{t \in \hat{T}} \left[\sum_{l \in L} \left(\sum_{p \in P_t(\omega) \cap P_l} \left[\left(u_p - v_l - \hat{w}_{lp} \right) d_{pt}(\omega) \right] s_{lpt}(\omega) - c_l^O O_{lt}(\omega) \right) + \left(u_p - \overline{v}^e \right) \sum_{p \in P_t(\omega)} d_{pt}(\omega) e_{pt}(\omega) \right] \right\} - \sum_{l \in L} A_l x_l \tag{63}$$

subject to constraints (9)-(11), (54)-(57), (44) and (46)-(47).

APPENDIX B

This Appendix discusses the calibration and estimation of SN design model's parameters, taking into account the solvability of the resulting MIPs. Recall that the SAA models are formulated using a subset of $m = m_L + m_H$ scenarios, a subset of planning periods $\hat{T}$, as well as route subsets $\hat{K}_{lt}(\omega), l \in L, \omega \in \Omega^m, t \in \hat{T}$, for the location-transportation models. The size of these subsets is therefore an important issue and, clearly, the models become much more difficult to solve as the size increases. Solvability is consequently a major concern, and the subsets were selected as large as possible to remain solvable with CPLEX-11. For all the instances solved, one day per week was sampled in the planning horizon yielding $|\hat{T}| = 48$. For the models based on **LT**, using the set of all routes and a sample of 10 scenarios (with $m_L = m_H = 5$) produced SAA design models that could be solved to optimality for P_1. For P_2, the largest SAA models we were able to solve to optimality were obtained with 6 scenarios samples (with $m_L = m_H = 3$) using a reduced subset of non-dominated routes. Based on preliminary results, the total number of routes was limited to 1 500 000, including all routes with at most one drop and a subset of interesting routes with at least two drops. Note that since the demand and hazard processes are stationary, and since the planning horizon includes 48 days, for each scenario the operational response submodel is replicated 48 times for demands and disruptions generated from the same processes. For this reason, what appears here to be a very small number of scenarios gives reasonable results. Also, for each SAA model, $I = 4$ replications were solved with different scenario samples. These subsets size and replication parameters were used for models **LT**, **LTbu**, **LTms**, **LTco** and **LTD**. The absence of binary route selection variables in the location-allocation models based on **LA** makes them more tractable, and the number of scenarios can be significantly increased to improve the statistical optimality gap. After several preliminary tests on various sample size, m was fixed to 50 for P_1 and to 30 for P_2 (with $m_L = m_H = m/2$). This sample size was applied to models **LA**, **LAbu**, **LAms** and **LAco**, with $|\hat{T}| = 48$ and I = 4. For the evaluation phase, the scenarios sample is larger to provide a precise evaluation of the SN designs performance, and $|\Omega^{M^e}|$ was fixed to 103, including 100 Monte-Carlo scenarios and the 3 worst case scenarios with the larger number of hits.

Samples of M = 1000 scenarios were generated for each problem instance to obtain the smaller design generation and evaluation scenario samples required, and to estimate the probability $\bar{\pi}_L$ and $\bar{\pi}_H$ of low and high risk scenarios. Histograms for one of these samples are presented in **Figure 5**. These samples and probabilities were used to calibrate the hazard tolerance level κ and the weight of high-risk scenarios $\hat{\pi}_H$ for risk averse decision-makers. The values retained for P_1 and P_2 are given in **Table 8**. The table also provides the weights applied to evaluate the designs with (52), for risk-neutral and risk-averse decision-makers.

	Risk Neutral DM				Risk Averse DM			
	κ	$\bar{\pi}_H$	φ_L and φ_H	ψ	κ	$\hat{\pi}_H$	φ_L and φ_H	ψ
P_1	2	0.28	0	0	2	0.35	0.2	0.2
P_2	3	0.38			3	0.45		

Table 8- Decision-Makers Risk Attitude Parameters

The models based on **LA** and **DLA** require transportation cost approximation functions estimated by regression. Using a representative sample of daily routes generated with procedure **UserResponse**, a linear regression function $\hat{w}_{lp} = \hat{\varsigma}_0 + \hat{\varsigma}_1 m_{lp}$ was estimated for each model type. The regression parameter values obtained for P_1 and P_2 are provided in **Table 9**. In addition, for models **LTco** and **LAco**, the cover-radius to use to define the sets L_p^1 and L_p^2 need to fixed a priori. Several values were tested and the inner and outer radiuses retained for P_1 were 200 and 400 miles respectively. For P_2, the values used were 300 and 400 miles.

	P_1		P_2	
	LA	**DLA**	**LA**	**DLA**
$\hat{\varsigma}_0$	0,1091	0,1009	0,2714	0,1009
$\hat{\varsigma}_1$	0,0057	0,0044	0,0058	0.005

Table 9- Regression Parameters by Problem Instance for LA-models and DLA

The disruption model parameters also needed to be estimated. The US states in the regions covered by problems P_1 and P_2 were used as hazard zones. The exposure levels and the multi-hazard arrival process for each zone (state) was estimated from historical data on major disasters provided by FEMA (www.fema.gov). We assumed that the inter-arrival time distribution $F_z^\lambda(.)$

is exponential, and the mean inter-arrival times $\bar{\lambda}_z$ estimated from the FEMA data for each zone $z \in Z$, are provided in **Table 10**. The table also gives the state exposure level $g(z)$ estimated on a scale from 1 to 4. We assumed that the impact intensity distribution $F^{\beta}_{g(z)s}(.)$ is Uniform for all z and s. For depots it is expressed in terms of capacity loss with lower and upper bounds [0,0.25), [0.25,0.5), [0.5,0.75), [0.75,1] for exposure levels g =1,2,3,4, respectively. For ship-to-points it is expressed in terms of demand surge/reduction with amplitude parameters [0,0.1), [0.1,0.2), [0.2,0.3), [0.3,0.4] for exposure levels g =1,2,3,4, respectively. The sign of the amplitude was randomly selected to get a 0.5 surge proportion. For all nodes, the attenuation probability α_n is randomly generated in an interval [0.1,0.2), [0.2,0.3), [0.3,0.4) or [0.4,0.5] depending on the relative area of state $z(n)$. The impact duration functions (**Figure 2**) used were $\theta_l = 0.007\beta_l^2 + 0.4709\beta_l + \varepsilon_L$ for depots (s_L) and $\theta_p = 0.8419\beta_p + \varepsilon_P$ for ship-to points (s_P). The recovery functions used are similar to the ones illustrated in **Figure 3**, with a stagnation phase of $\lceil 0.25\theta_l \rceil$ periods for depots, and instantaneous deployment and recovery phases and a sustainment phase of θ_p periods for ship-to-points.

State (z)	**VT**	**DE**	**DC**	**MA**	**NY**	**NJ**	**WV**	**KY**	**OH**	**IN**
$\bar{\lambda}_z$ (in days)	537	430	567	578	293	609	391	358	344	466
$g(z)$	4	4	4	4	4	4	4	4	4	3
State (z)	**ME**	**NH**	**RI**	**CT**	**PA**	**VA**	**IL**	**MD**	**WI**	**MI**
$\bar{\lambda}_z$ (in days)	405	577	757	703	355	340	371	607	412	611
$g(z)$	3	3	3	3	3	3	3	2	2	1

Table 10- Multihazard Exposure Levels and Mean Inter-Arrival Times