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Mitigating Supply Disruption with Backup Supplier under Uncertain Demand : Competition and Cooperation

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Abstract. When supply disruption occurs following major disasters, many supply chains tend to breakdown due to stock-outs or lost sales and take a long time to recover. However, by keeping one or more emergency sources of supply, some supply chains continue to function smoothly, satisfying consumer demand even after a major disaster. In this study, we use the game-theory-based framework to model a supply chain with random and price dependent demand in a competitive environment where suppliers are prone to disruption. To mitigate the negative effect of supply disruption, a backup supplier is incorporated into the proposed model as an emergency source of supply. Further, to enable supply chain coordination, two coordinating mechanisms are addressed. In our study, we investigate how these coordinating contracts work in a supply chain under risk and competitive environment. Finally, we perform a comprehensive numerical study to show the impact of the model parameters on the equilibrium solutions and to signify the performance of the proposed coordination contracts.

Keywords. Supply disruption, non-cooperative game, cooperative game, coordination mechanisms.

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1. Introduction

Nowadays supply chains are increasingly globalized and many unforeseen factors like natural disasters or human intervention (example - bad logistics) can cause unexpected disruptions. Such disruptions hinder the process of normal flow of goods and materials in a supply chain. Supply chain disruption may be broadly classified into three categories: i) Supply related: This occurs when suppliers are unable to fill the orders placed with them. ii) Demand related: This may occur due to sudden drop or sudden rise in customer's orders iii) Miscellaneous risks which include unexpected changes in purchasing costs, interest rates, currency exchange rates, safety regulations by government agencies etc. (OKE and Gopalakrishnana 2009) Over the past few years this supply chain disruption management has received extra attention from both the industrial and the academic points of view. Again, over two decades numerous research efforts have been devoted to enrich supply chain management in different ways. Coordination and competition are two such important directions in which most of the researches have been carried out to maximize total channel profit and to improve supply chain effciency. My present study is related to three different streams of literature, they are - supply disruption, coordination mechanisms and competition within a supply chain. Moreover, our analytical, as well as numerical solutions, are based on the application of game theory.

In today's complex supply chain a manager must account for several supply chain risks when planning suitable mitigation strategies. Supply disruption can be treated as a particular type of supply risks which corresponds to the interruption of the supply of a product. Again, this supply disruption may be modelled as complete disruption where supply halts entirely or as yield uncertainty in which case supplied quantity is different from the or der size placed. For example, in 1994, Kobe earthquake left vast damages to all of the transportation links in Kobe, almost destroyed the world's sixth-largest shipping port and consequently many companies left without parts (Yoshiko 1995). Another example is, in Toyota Company, an estimated production of 20,000 cars equivalent to $ 200 million worth of revenue was lost due to parts shortages (Sheffi 2005).

The research topic of disruption management is a new and fledgling field in the study of supply chain management and has gained significant attention of academicians during the past few years. The initial work on supply disruption was done by Parlar and Berkin (1991) in classical EOQ (economic order quantity) model. Then Berk and Arreola-Risa (1994) proposed a cost function correcting the cost function of Parlar and Berkin (1991), addressing logical errors regarding the occurrence of stock-outs and associated costs. Snyder (2005) proposed a simple but tight approximation for the model introduced by Berk and Arreola-Risa (1994). They provided theoretical as well as numerical bounds on the approximation error in both the cost function and the optimal order quantity. Qi et al. (2009) then extended Snyder (2005)'s model by considering random disruption at both the supplier and the retailer. Tomlin (2006) presented a dual sourcing model in which orders may be placed with either a cheap but unreliable supplier or an expensive but reliable supplier. They discussed three general strategies for coping with supply disruption: inventory control, sourcing and acceptance. In most of the literature developing on supply disruption involves single supplier. But in the modern competitive business environment, stock-outs or lost-sales situation may arise due to yield uncertainty and this will create an opportunity for the other competitors. So in order to manage the supply risks, business houses are now keeping one or more secondary suppliers as an emergency source of supply. Thus, this backup supplier helps to reduce the stock-outs risk and to mitigate the negative effects of supply disruption. For example, in August 2005, when Hurricane Katrina hit the United States Gulf coast, Wal-Mart could able to respond quickly to supply disruption and mitigated the consequences of supply shortage with the use of its backup sourcing strategy (Leonard 2005). Another suitable real-life example is the March 2011 earthquake and tsunami in Japan. A number of automotive products had been affected by the Japan disasters. Specially, Xirallic pigments were among the first automotive inputs to be affected by this, the only plant in the world that makes them. Many of the world's automakers, including Ford, Chrysler, Volkswagen, BMW, Toyota and GM were badly affected by the temporary shutdown of this plant (The Truth About Cars 2011). Moreover, as a result of the Japanese disasters, Toyota, the number 1 automaker in the world, was knocked offline for months because of a lack of dual sourcing strategy (Automotive News 2016) and subsequently Toyota's January-March profit slid to ¥25.4 billion from ¥112.2 billion a year earlier (Associated Press 2011). Recently, due to an explosion at Toyota supplier Aichi Steel Corporation at its Chita plant on 8th January, 2016. As a result of this, Toyota, the world's biggest selling automaker, faces steel shortage. But using the emergency backup sourcing strategy, Toyota was able to recover from its recall crisis in just 7 days (Reuters 2016). Therefore, it is usually valuable for buyers (e.g.: retailer) to have more than one supplier of similar products to reduce the supply risks. In our present article, we adopt this strategy to alleviate the negative effects of supply disruption. Chopra et al. (2007) considered a two-supplier model with dual sourcing under both supply disruptions and yield uncertainty in the context of the single period. Hou et al. (2010) modelled a supply chain with supply disruption where the buyer coordinates with the backup supplier through buyback contract. Chen et al. (2012) studied a periodic-reviewing inventory system with a capacitated backup supplier to mitigate supply disruptions. Hou and Zhao (2012) considered an SC with two suppliers, one (main supplier) prone to disruption and another one (backup supplier) completely reliable, under backup and penalty scheme with the reliable one. Chen and Yang (2014), Chen and Xiao (2014) studied coordination mechanism in an SC model with random demand, where the production of the primary supplier is subject to random yield and the buyer has an emergency backup sourcing with the application of game theory. At the same time, Hishamuddin et al. (2014) proposed a real-time recovery mechanism for a two-stage SC system with one supplier and one retailer.

On the other hand, Coordinating mechanisms play significant role for many successful supply chains. In general, the system of independent profit-maximizing firms (decentralized system) earns lower profit than that of the integrated (centralized) system. In the centralized system, there is a unique decision maker possessing the information on the whole supply chain together with the contractual power to implement every decision. It has been proved that centralized supply chain policy always produces the best result in terms of profit or cost of the whole supply chain and hence supply chain efficiency. In practice, most of the supply chains are decentralized supply chains. In real-life practical situation, decentralized supply chain policy is easy to implement in which each member of the channel is a decision maker, having different objectives of pursuing their own objectives which may be conflicting and may lead to system inefficiency. Hence, only proper coordination mechanisms can modify these incentives of different channel members so that total profit of the supply chain is maximized. For an excellent introduction and summary on coordination management, readers are referred to Lariviere (1999), Tsay et al. (1999) and Cachon (2003). Better coordination can be achieved through contract mechanisms. Price only contract or wholesale price (WP) contract is used as a benchmark to evaluate the expected outcomes of any contract. A WP contract is one in which retailer bears the entire risk for all unsold units. Supply chain model with WP contract has been well studied under deterministic (Jeuland and Shugan 1983, Choi 1991, Choi 1996) as well as stochastic demand settings (Petruzzi and Dada 1999, Pan et al. 2009). It is well known that wholesale price contract cannot coordinate a supply chain (Lariviere 1999). Hence, to establish better coordination among supply chain members, researchers have studied better classes of supply chain contracts.

Over the years, several researchers have proposed various coordinating mechanisms in terms of supply chain contracts in the literature depending on the nature of the parameters associated with the chain. In stochastic market scenario, the main objectives of the contracts are (1) to increase (decrease) the individual as well as supply chain profit (cost) in order to make it closer to that of the centralized supply chain profit (cost) and hence increase the chain efficiency and (2) to share the risk which involves at different stages of the supply chain. In our study, we consider buy-back contract to coordinate our proposed supply chain. A buy-back (return) policy is a commitment of the partners of the supply chain (e.g.: manufacturers, suppliers, retailers, distributors etc.) from the downstream channel members (Padmanabhan and Png 1997) at the end of the selling season. Thus, with this contract supplier (or manufacturer) can encourage buyer (retailer) to place more order. Since several years, buy-back contract has been studied to coordinate the channel members in a supply chain. An earlier investigation of buy-back contracts was carried out by Pasternack (1985) in distribution channels. In Pasternacks

(1985), the market under consideration is composed of one supplier and one retailer, and the newsvendor problem structure of a seasonal product with stochastic demand and underage/overage costs was adopted. In 1997, Padmanabhan examined a return policy for competitive retailers under certain as well as uncertain demand pattern. Mantrala and Raman (1999) and Lau and Lau (2002) proposed buyback contract for newsvendor models under demand uncertainty. Yao et al. (2005) studied buyback contract with information sharing about the demand. Looking at the product property, Hahn et al. (2004) presented the buyback contract for perishable products. Bose and Anand (2007) studied on single-period returns policies by making a clear distinction between models in which transfer price is exogenous and models in which one dominant party unilaterally declares a price. Yao et al. (2008) and Arcelus et al. (2008) exhibited the impact of price-sensitivity factors on characteristics of buyback contracts considering price-dependency stochastic demand in a single-period product supply chain. Then Brown et al. (2008) proposed a multi-item returns policy called pooled (or joint) returns policy. In the same year, Song et al. (2008) studied a buyback contract in a Stackelberg framework with a manufacturer as a Stackelberg leader and the retailer as a Stackelberg follower. Ding and Chen (2008) developed a single period model for three-level supply chain under flexible return policy selling short life cycle products. Chen and Bell (2011) investigated a single-manufacturer single-retailer supply chain with price and customer return dependent stochastic demand and proposed a buyback contract with different buyback prices for unsold inventory and customer returns. With cooperative game theory, Devangan et al. (2013) applied buyback contract in an inventory level dependent stochastic demand scenario. In a duopoly of two manufacturerretailer supply chains, Wu (2013) examined the impact of buyback policy on the retail price, order quantity, and wholesale price. Aiming at identifying the relation between return policy and product quality decision, Yoo (2014) considered a buy-back policy for risk-averse supplier.

Further, with the advancement of the global economy and with the vast application of modern technology in today's business environment, the ways in which the firms compete with each other become competition among the channel members in supply chain management. In the last two decades, this channel competition has drawn increased attention in both management and marketing literature. This channel competition may take in the upstream part of supply the chain (e.g.: competition among the suppliers etc.) and/or in the downstream part of the supply chain (e.g.: competition among the retailers etc.). Jeuland and Shugan (1983) were one of the earliest researchers to incorporate and analyze the concept of channel competition in the supply chain literature. After that, many researchers developed models considering either upstream competition (S.C. Choi 1991, Cachon and Kok 2010, Pan et al. 2010) or downstream competition (Ingene and Parry 1995, Padmanabhan and Png 1997, Cachon and Lariviere 2005). In 1996, S. C. Choi considered a model where upstream as well as downstream competition is considered. Their major finding is that, while (horizontal) product differentiation helps manufacturers, it hurts retailers. Conversely, while (horizontal) store differentiation helps retailers, it hurts manufacturers. All the papers mentioned earlier assumed the customer demand faced by the retailer is price dependent and deterministic in nature. Yao et al. (2008) developed a model comprising one manufacturer and two competing retailers. Recently, Chakraborty et al. (2015) developed a model where two competing suppliers sell their products through a common retail channel.

On the other hand, there are only a few papers studying the impact of supply chain disruption in the competitive environment. Babich (2006) and Babich et al. (2007) developed models to investigate suppliers' competitive pricing decisions with supply disruption. Babich (2006) developed a model with competing hazardous suppliers and single manufacturer and investigated how the supplier default risk and default co-dependence affect the procurement and production decisions of the manufacturer, supplier pricing decisions, and also investigated how the introduction of the deferment option alters supplier competition. Babich et al. (2007) focused on the effect of supplier competition in a market where the retailer is considering diversification as a strategy to reduce supply chain risk. Here, they examined the effects of co- dependence among supplier defaults on the performance of firms and the consequences of the suppliers offering different payment policies.

Now, attention is given to the application of game theory in the supply chain. Game theory can be broadly divided into two categories, namely non-cooperative games and cooperative games. In recent years, there has been a wide variety of research papers that apply non-cooperative game theory to the field of supply chain management. In non-cooperative games, the players choose strategies simultaneously and are thereafter committed to their chosen strategies. For a detailed survey of the existing literature on the applications of non-cooperative games to supply chain management, readers are referred to Cachon and Netessine (2004). On the other hand, the application of cooperative game in supply chain management literature is less prevalent and for a detailed survey of the application of cooperative game in supply chain management literature, we would like readers to refer to Nagarajan and Sosic (2008).

Over the last few years, though considerable amount of research has been done focusing on the joint effect of supply disruption and coordination mechanisms (eg: Tomlin (2006); Hou et al (2010); Cheng & Yang (2014); Chen & Xiao (2014)) in non-competitive market scenario, less attention has been paid to the integrated effect of supply disruption and coordination mechanisms in competitive environments. The initial work of joint effect of competition and coordination mechanisms in supply disruption environment was studied by Li et al. (2010) using newsvendor model structure. In 2010, Li et al. developed a supply chain model with price-independent stochastic demand consisting of one retailer and two competing suppliers under an environment of supply disruption. Here they investigated how the coordination mechanisms of cooperative suppliers are affected in the presence of supply disruption.

Hence, the purpose of this study is to investigate the integrated effect of competition and coordination mechanisms in supply chain disruption and to address some coordinating mechanisms to coordinate channel members. Our present work is closer to Li et al. (2010)'s model where demand is assumed to be stochastic but independent of the effect of price and to coordinate the channel only cooperative game is considered among the channel members. In our present article, we consider stochastic demand which is dependent on the retail price. We further address a modified buy-back contract to coordinate the

supply chain and thus to enhance the supply chain efficiency. To the best of our knowledge, this is the first attempt in disruption management literature where price-dependent stochastic demand is considered. Here we consider that market segment where there is only one retailer in the area. In other words, this market scenario can be interpreted as follows: there is no competition among retailers due to the long distance among them. This may be a strong assumption in some market segment. However, in our present study, through this assumption we focus on competition between the primary suppliers and the impact of supply disruption on the optimal decisions. The research questions addressed in this paper will be (1) How to design the contracts or coordinating schemes among the members in supply chain under risk and competition environments? (2) Is coordination desirable to all parties under disruption and competition concerns? Table 1 represents a comparison study of our proposed models with the existing literature on supply disruption. Our models are suitably fitted to Motor Vehicle supply chain. All through the world, thousands of firms are engaged in motor vehicle parts production, final assembly, and sales. There are a large number of parts suppliers (e.g., Lear, American Axle, Borg Warner) who serve the final assemblers (e.g., General Motors, Honda).
Throughout this article, the retailer is considered as Stackelberg leader (e.g., Wal-Mart, Big Bazaar). Here, we will propose two supply chain models, SC(supply chain model without backup supplier) and SCB (supply chain model with a backup supplier) models. The basic models will be based on wholesale price contract. To improve individual profits as well as supply chain efficiency we will address two further coordinating mechanisms: (i) the first one is between two primary suppliers and (ii) the other one is between each primary supplier and the common retailer. Finally, a comparison study will be carried out between two proposed models.

The remainder of the paper is organized as follows. Section 2 gives the general descriptions of both SC and SCB models. We define SC model and present an equilibrium analysis in Section 3. In this section, we will address two coordinating mechanisms to coordinate the channel members. The model SCB will be developed in Section 4 where we will follow the same approach as in Section 3. In Section 5, a managerial explanation

Table 1: A comparison study of our models with the related literature on supply disruption

Authors	Model structure	Competition	Demand source	Emergency	Coordination	Game theory
Parlar M.and Berkin D. (1991))	EOQ	N	Deterministic	N	N	N
Berk E. and Arreola -Risa A.(1994)	EOQ	N	Deterministic	N	N	N
Snyder L.V. (2005)	EOQ	N	Deterministic	N	N	N
Tomlin, B. (2006)	Newsvendor	N	Deterministic	Back-up supplier	Volume flexibility	N
Babich V. 2006	Newsvendor	Upstream	Deterministic	Back-up supplier	N	N
Babich V. 2007	Newsvendor	Upstream stochastic	Price-independent	N supplier	N	N
Chopra et al. (2007)	Newsvendor	N	Deterministic	Back-up supplier	N	N
Qi et al. (2009)	EOQ	N	Deterministic	N	N	N
Hou et al. (2010)	Newsvendor	N	Price-independent stochastic	Back-up supplier	Buyback policy	N
Li et al. (2010)	Newsvendor	Upstream stochastic	Price-independent	Spot market	Cooperative game	1. Non -cooperative 2. cooperative
Hou J. and Zhao L. (2012)	Newsvendor	N	Price-independent Stochastic	Back-up supplier	N	N
Chen K. and Yang L. (2014)	Newsvendor	N	Price-independent Stochastic	Back-up supplier	1. Subsidy 2. two-part tariff 3. Reinforcement	Non -cooperative
Chen K. and Xiao T. (2014)	Newsvendor	N	Price-independent Stochastic	Back-up supplier	1. Inventory pooling strategy	Non -cooperative
Hishamuddin et al.	EOQ	N	Deterministic	Recovery Mechanism	N	N
Our Models	**Newsvendor**	**Upstream**	**Price-dependent stochastic**	**Backup supplier**	**1. Cooperative game 2. Modified Buyback**	**1. Non -cooperative 2. cooperative**

is given. Finally, we conclude with future research directions. All the proofs are presented in the Appendices.

2. The Model

We consider a single period supply chain consisting of two competing primary suppliers, which we denote with Supplier 1 and Supplier 2 who sell their products through a common retailer, denoted by R (see Figure 1). Here, by competing suppliers we mean independent suppliers. Suppliers (called primary suppliers) are subject to unreliable supply. All the firms are assumed to be risk neutral and pursue expected profit maximization. The retailer buys the same product from the primary suppliers and sells it in a single season. The retailer must make both pricing and order decisions while each supplier also sets the wholesale price.

Depending on the uncertainty in supply the products, the primary suppliers who are subjected to random failure may be in two states: (i) active state and (ii) failure state. If the suppliers are in the active state, the orders placed with it will be delivered in time. If the suppliers are in the failure state, no order can be supplied. Supply failure may be broadly categorized into two types: (a) common cause failure and (b) supplier-specific failure (Li et al. 2010). Both primary suppliers are subjected to common cause failure. For example, seasonal storms, civil war, an earthquake may affect all the suppliers in a region. On the other hand, a supplier may still fail for some supplier-specific reason even if there is no common cause failure. For example, equipment failure, labour strike may affect the one supplier but not the other supplier. In our models, we assume that Supplier 2 is affected by only common cause failure but Supplier 1 may be disrupted due to both types of failure.

To define the models the following notations will be used throughout the paper:

i	:	$i = 1, 2, 3$ for Supplier 1, Supplier 2 and the backup supplier respectively
d	:	price-dependent deterministic demand
D	:	actual demand faced by the retailer
a	:	initial market size of the product, $a > 0$
b	:	price sensitivity coefficient
ϵ	:	random variable denoting the random factor of the customer's demand
	:	faced by the retailer

$f(\cdot)$	:	probability density function (pdf) of the random variable ϵ
$F(\cdot)$	:	cumulative distribution function (cdf) of the random variable ϵ
$A,\ B$	:	lower bound and upper bound of ϵ
c_v	:	the salvage value per unit of residual product by the retailer
c_{v_i}	:	the salvage value per unit of residual product by the ith primary supplier
c_s	:	the goodwill cost of a unit of unmet demand
c_i	:	the delivery cost of a unit of the product of supplier i for $i = 1, 2, 3$
c_r	:	the unit reservation charge at the secondary (backup) supplier
α	:	the probability of common-cause failure not occurring where $0 < \alpha < 1$
β	:	the probability that supplier S_1 does not fail conditional on a common cause failure not occurring, where $0 < \beta < 1$
γ	:	the total proportion of the marginal delivery cost in the event of a failure, where $0 < \gamma < 1$
η	:	the proportion of the marginal cost incurred by the disrupted supplier, where $0 < \eta < 1$

The decision variables are:

P	:	the unit retail price charged by the retailer, a decision variable of the retailer
Q_i	:	the order quantity placed with supplier i, $i = 1, 2$, decision variables of the retailer
I	:	the reserved quantity at the backup supplier i, $i = 1, 2$, a decision variable of the retailer
$w_i(> c_i)$	:	the wholesale price per unit of product offered by suppler S_i, $i = 1, 2$, decision variable of of ith supplier
$w_3(> c_3)$	:	unit wholesale price of the backup supplier

The demand faced by the retailer is not only price-dependent but also stochastic in nature. Let us suppose that ϵ be the parameter representing the randomness in the demand. We investigate the case where randomness of consumer demand is represented in additive form $D(P, \epsilon) = d(P) + \epsilon$ where $d(\cdot)$ is the price-dependent demand function and is given by $d(P) = a - bP$(Mills 1959, Petruzzi and Dada 1999). Here $a(> 0)$, b are the initial market size of demand and price sensitivity coefficient respectively, P is the retail price per unit. ϵ is a random variable with mean μ, defined on the range $[A,\ B]$ with continuous, differentiable, reversible distribution function $F(\cdot)$, which is independent of price vector and with its density function $f(\cdot)$. In order to assure that the positive demand is possible for some range of P, we require that $A > -a$ (Petruzzi and Dada 1999). In this market segment, the distribution of demand is known to both the suppliers and the retailer.

In the absence of disruption, the quantity sold at the market at the retail price is the minimum of the supplied quantity and the actual demand. Due to the stochastic nature of customers demand the retailer may face shortage or leftover. If the actual demand exceeds the supplied quantity, the excess demand is lost and incurs shortage cost. In contrast, if the supplied quantity exceeds the actual demand, then the leftover quantity is sold by the retailer at salvage price at the end of the selling season.

We assume the unlimited capacity for each supplier. Thus, the suppliers are always able to produce the ordered quantity $Q_i,\ i = 1, 2$, in time for the start of the selling season. The lead time for the suppliers is assumed to be negligible. The assumption of negligible lead time is appropriate for those market circumstances where orders are placed to the suppliers quite before the start of the selling season and are expected to be reached to the retailer before or at the start of the selling season. In this paper, our main focus is on the revenues of Supplier 1, Supplier 2 and the common retailer.

When disruption occurs, the ordered quantity produced by the disrupted supplier cannot be sent to the retailer and sold at the salvage price during or at the end of the period and the retailer incurs a shortage cost.

We further extend our SC model by incorporating a backup supplier (called secondary supplier) into the supply chain (SCB model) (see Figure 2). This secondary supplier is assumed to be perfectly reliable with more expensive wholesale price and there is always an upper limit (I in our case) of the quantity that he would be able to supply. The backup supplier is perfectly reliable in the sense that it delivers in time up to a certain quantity reserved earlier. The retailer has to reserve the desired quantity in advance before the start of the selling season. After placing orders of size Q_1 and Q_2 from the primary suppliers, the retailer reserves a quantity of size I from the perfectly reliable secondary supplier at unit reservation cost c_r before the start of the selling season, in addition. After receiving order quantities from the primary supplier (or nothing in case of supply disruption) the retailer has the option of purchasing any amount up to quantity I from the secondary supplier at the unit cost $w_3(> w_i,\ i = 1,\ 2)$ if required in negligible lead time.

In the case of supply disruption a marginal cost $\gamma c_i,\ i = 1, 2$ is incurred. Here this cost is supposed to be assumed jointly by disrupted supplier and the retailer (Li et al. 2010). Suppose the share of this marginal cost of the disrupted supplier is $\eta\gamma c_i,\ i = 1, 2$ and that of the retailer is $(1 - \eta)\gamma c_i,\ i = 1, 2$. This type of distribution of marginal cost is different from most of the existing literature in which only retailer or the disrupted supplier bears the failure penalty in the event of a disruption. But this assumption is not true in general. Because before the realization of supply disruption, both the retailer and the disrupted supplier incur some cost (e.g., fixed set-up costs, variable costs, etc.). Depending on the reliability of the suppliers, it is reasonable to assume that $c_3 > c_2 > c_1 > c_v$. Also, in addition we assume that $P > w_3 > c_3$ and $P > w_i > c_i$ for $i = 1,\ 2$. These inequalities assure that the chain will not produce infinite quantities of the product and each member has a positive profit. In addition, we further assume that $c_v < c_{v_i},\ i = 1, 2$ i.e., the salvage value of the unsold products by suppliers must be higher than that by retailer.

In the next sections, we formulate the suppliers' and retailer's problems for both models: Supply chain model without backup supplier (SC) and Supply chain model with a backup supplier (SCB). In each case, we consider decentralized supply chain in which suppliers are competitive in nature. For these two decentralized models, we assume the complete sharing of information regarding demand function, cost structure, and the decision rules among all the parties concerned. Also, we analyze the equilibrium behavior of the system in a game theoretic setting. Each player of the game is assumed to be rational *i.e.*, each member of the channel is assumed to seek to maximize its own profit. We then address two coordinating mechanisms to coordinate the channel members. We use the following mathematical notation in our paper: a^+ means the positive part of a *i.e.,* $Max\{a, 0\}$.

Insert **Figures 1, 2** here

3. SC model: Supply chain model without backup supplier

Here, the primary suppliers are subject to random disruption and we don't consider any backup supplier. In the decentralized model with competitive suppliers, all players act independently and maximize their individual profit (Li et al. 2010). If Q_i and P be the order quantity and retail price of the retailer for the ith supplier's product, then for given values of wholesale price w_i and depending on the disruption condition, the expected profit of the retailer is given by

$$\begin{aligned}\Pi_{dR}^{WP}(Q_1, Q_2, P|w_1, w_2) \quad = \quad & \alpha\beta E\Big[P\min\Big\{Q_1 + Q_2,\ D(P,\epsilon)\Big\} - w_1Q_1 - w_2Q_2 - c_s\Big\{D(P,\epsilon) - Q_1 - Q_2\Big\}^+ + c_v\Big\{Q_1 + Q_2 - D(P,\epsilon)\Big\}^+\Big] \\ & +\alpha(1-\beta)E\Big[P\min\Big\{Q_2,\ D(P,\epsilon)\Big\} - w_2Q_2 - (1-\eta)\gamma c_1Q_1 - c_s\Big\{D(P,\epsilon) - Q_2\Big\}^+ + c_v\Big\{Q_2 - D(P,\epsilon)\Big\}^+\Big] \\ & +(1-\alpha)E\Big[-c_s\Big\{D(P,\epsilon) - 0\Big\}^+ - (1-\eta)\gamma c_1Q_1 - (1-\eta)\gamma c_2Q_2\Big], \qquad (1)\end{aligned}$$

which captures the revenue for all the three possible cases, namely, no supply disruption case, supplier-specific disruption case and disruption due to common cause case. Here, the subscript and superscript dR and WP stand to denote decentralized retailer under WP contract. Now to facilitate our analysis, it is useful to write $Q_i = a - bP + z_i$ where z_i represents the retailer's amount of safety stock for the ith supplier's product for $i = 1,\ 2$. With this reformulation, the decision variables of the retailer become $z_1,\ z_2,\ P$. If $\Pi_{dR}^{WP}(z_1, z_2, P|w_1, w_2)$ denotes the corresponding expected profit of the retailer, then the optimizing problem of the retailer takes the form

$$\max_{z_1, z_2, P}\ \Pi_{dR}^{WP}\Big(z_1, z_2, P|w_1, w_2\Big). \qquad (2)$$

This is the general newsvendor problem with three decision variables with additive demand uncertainty. Because of the complexity of the objective function (3) it is difficult to find its closed form solutions. However, the following proposition establishes the concavity property of retailer's objective functions with respect to her decision variables. This ensures the existence of the optimal solutions of the decision variables which maximizes the retailer's expected profit.

Proposition 1

There exists an optimal solution $\left(z_1^*, z_2^*, P^*\right)$ *which will maximize the expected profit of the retailer under the condition* **A1:** $\frac{2b(P+c_s-c_v)r(\cdot)}{\overline{F}(\cdot)} \geq 1$, *where* $r(\cdot) = \frac{f(\cdot)}{\overline{F}(\cdot)}$ *is the hazard rate of demand.*

It is to be noted that the assumption **A1** is satisfied for the variety of distributions including uniform distribution, truncated normal distribution, etc. Again, for given z_i and P and depending on the disruption condition, the expected profits of Supplier 1 and Supplier 2 are given by

$$\begin{aligned} \Pi_{dS_1}^{WP}(w_1|z_1,P) &= \left[\alpha\beta w_1 + (1-\alpha\beta)c_{v_1} - (\alpha\beta - \alpha\beta\eta\gamma + \eta\gamma)c_1\right]\left[d(P)+z_1\right], \quad (3) \\ \Pi_{dS_2}^{WP}(w_2|z_2,P) &= \left[\alpha w_2 + (1-\alpha)c_{v_2} - (\alpha - \alpha\eta\gamma + \eta\gamma)c_2\right]\left[d(P)+z_2\right], \quad (4) \end{aligned}$$

respectively. In the decentralized system every member of the channel is assumed to be rational *i.e.*, each member wants to maximize his own profit. The basic model is developed under the assumption that suppliers are competitive. Then to coordinate the suppliers we consider another situation where suppliers cooperate with each other in doing business.

3.1. Equilibrium analysis: Retailer-Stackelberg game

In our decentralized system, wholesale price w_i is the decision variable of ith primary suppliers whereas order quantities and retail price (z_1, z_2, P) are the decision variables of the common retailer. We assume the following two static non-cooperative games among the channel members. The first is a non-cooperative Nash game played between

Supplier S_1 and Supplier S_2 in which they choose their wholesale prices simultaneously. The second is a Stackelberg game which is played between supplier's level and retailer's level. Throughout our article, we have considered retailer as the Stackelberg leader. The sequence of events and consequent decisions of our proposed SC model is given below (see Figure 3):

(1) In this market scenario, Prior to the start of the selling season, as a dominant retailer, the common retailer moves first to announce the retail margin ($m_i = P - w_i,\ i = 1, 2$) for both products to their respective supplier. Then the suppliers respond by choosing the wholesale price w_i. Finally, the common retailer decides the order quantities Q_1 and Q_2 with Supplier 1 and Supplier 2, respectively taking each supplier's reaction functions into consideration.

(2) Then, for given order quantities and retail margin (or retail price), each supplier can be able to realize his own actual wholesale price w_i charged to the retailer for $i = 1,\ 2$ for Retailer-Stackelberg game.

(3) After the start of the selling season, the actual demand of the customer and actual state (*i.e.,* active or failure) of the suppliers are realized.

(4) If there is no supply disruption then ordered quantities are delivered to the retailer at a negligible time. The retailer then sells these to its customer. The excess inventory is salvaged at the end of selling season. Again, for any unmet demand the retailer incurs shortage cost.

(5) Further, in the case of disruption (common cause or supplier specific), the disrupted supplier cannot send the proposed quantity to the retailer and the product is salvaged by the disrupted supplier during or at the end of the selling season. Moreover, for the complete disruption the retailer incurs shortage cost.

Insert **Figure 3 here**

3.1.1. Optimal Strategies when suppliers are competitive

In this section, a competitive market scenario is considered where Supplier 1 and Supplier 2 set individual wholesale price simultaneously to maximize their respective profit before the common retailer places the orders. Again, under Retailer-Stackelberg game where the common retailer holds the greater channel power, the decision sequence is: the retailer moves first to declare the retail margin m_i, the suppliers then respond by choosing the wholesale price w_i. Finally, the retailer places order quantities Q_i and hence safety stock z_i. Following this sequence, the reaction functions of the suppliers are derived as

$$w_1(z_1, P) = \frac{1}{\alpha\beta b}\Big[\alpha\beta(d(P) + z_1) - (1-\alpha\beta)bc_{v_1} + b(\alpha\beta - \alpha\beta\eta\gamma + \eta\gamma)c_1\Big], \tag{5}$$

$$w_2(z_2, P) = \frac{1}{\alpha b}\Big[\alpha(d(P) + z_2) - (1-\alpha)bc_{v_2} + b(\alpha - \alpha\eta\gamma + \eta\gamma)c_2\Big]. \tag{6}$$

Proposition 2

The solutions (5) and (6) are Nash equilibrium between two suppliers.

The above proposition establishes the existence of the Nash equilibrium between two suppliers. If $\alpha\beta - \alpha\beta\eta\gamma + \eta\gamma > 0$ and $\alpha - \alpha\eta\gamma + \eta\gamma > 0$, then solution (5) and (6) indicate that the wholesale price of ith supplier is positively related to its own production cost and the retailer's safety stock of its own product, while it is negatively related to the retailer's margin (or retail price) and salvage value of its own residual product. Substitute w_1 and w_2 by those reaction functions (5) and (6) into the equation (2) and let us denote the corresponding expected profit of the retailer in Retailer-Stackelberg game as $\Pi_{dR}^{WPRS}\Big(z_1, z_2, P/w_1(z_1, P), w_2(z_2, P)\Big)$. Here the superscript *'WPRS'* stands for Retailer-Stackelberg game under wholesale price (WP) contract. In the following proposition, we derive the sufficient conditions for the existence of Retailer-Stackelberg game where the retailer is the Stackelberg leader.

Proposition 3

Under assumption **A1** *and under the parametric restriction* $\frac{1}{3} \leq \beta \leq 1$*, there exists a Stackelberg game where retailer is the Stackelberg leader.*

Thus, through the above proposition, we get the range of supplier-specific failure probability within which the retailer-Stackelberg game exists. Due to the complicated form of the objective function, it is difficult to establish the unimodality of Π_{dR}^{WPRS} even though our extensive computational analysis suggests so. However, the common retailer obtains his optimal retail price (P^{RS^*}) and safety stocks (Z_i^{RS*}) from the objective function Π_{dR}^{WPRS}. Then for these given retail price and safety stocks, each supplier derives his optimal wholesale price for this retailer-Stackelberg game from (5) and (6) for Retailer-Stackelberg game.

3.1.2. Optimal strategies when suppliers are cooperative

Now in order to establish the coordination among the suppliers, let us consider the market scenario where Supplier 1 and Supplier 2 are cooperative among themselves whereas the common retailer is non cooperative with the suppliers as before. In this situation, to find the equilibrium solutions cooperative suppliers will play Nash bargaining game (Nash 1951) among themselves and a non-cooperative Stackelberg game will be played between each supplier and the retailer. When suppliers are cooperative, then for the given retail margin, each supplier will set his wholesale price in order to maximize their total expected profit before the common retailer places the orders. The intuitive assumption is that no supplier will set a lower wholesale price to monopolize the market. In the cooperative game, following the Retailer-Stackelberg game sequence, we will first derive the optimal cooperative wholesale prices which maximize the total expected revenue of the two cooperative suppliers. Then we will turn our attention to divide the optimal profit among those two cooperative suppliers. The sequence of the Retailer-Stackelberg game for cooperative suppliers is depicted at the end of this section. If Π_{dcS} denotes the total revenue of the cooperative suppliers, then for the given safety stocks and retail price, it is given by

$$\begin{aligned}\Pi_{dcS}^{WP}(w_1, w_2/z_1, z_2, P) &= \Big[\alpha\beta w_1 + (1-\alpha\beta)c_{v_1} - (\alpha\beta - \alpha\beta\eta\gamma + \eta\gamma)c_1\Big]\Big[d(P) + z_1\Big] \\ &\quad + \Big[\alpha\beta w_2 + (1-\alpha\beta)c_{v_2} - (\alpha\beta - \alpha\beta\eta\gamma + \eta\gamma)c_2\Big]\Big[d(P) + z_2\Big].\end{aligned}$$

Here, the subscript *'dcs'* stands to denote decentralized supply chain with cooperative suppliers. Hence, the problem of the cooperative suppliers takes the form

$$\begin{aligned}&\max_{w_1, w_2} \; \Pi_{dcS}^{WP}(w_1, w_2/z_1, z_2, P) \\ &\text{s.t.} \;\; w_2 + \frac{(1-\alpha)(1-\eta)\gamma c_2}{\alpha} \geq w_1 + \frac{(1-\alpha\beta)(1-\eta)\gamma c_1}{\alpha\beta}.\end{aligned} \tag{7}$$

It can easily be verified that the hessian matrix of Π_{dcS}^{WP} is negative definite. This ensures that Π_{dcS}^{WP} is jointly concave with respect to $w_1, \; w_2$. Moreover, the solution spaces of $w_1, \; w_2$ are convex. Since the constraint is linear but the objective function is quadratic, it is well known as a quadratic programming problem which can be solved using the built-in software Math lab or Mathematica. Now, we will concentrate on the division approach of the derived optimal profit among two cooperative suppliers. To do so, we will consider the well-known Nash bargaining approach.

Nash bargaining game begins with the identification of a feasible set of payoffs F and a disagreement point d that are predetermined and are independent of the negotiation. Let us define the basic bargaining model in the following manner. Suppose the cooperative suppliers negotiate on their individual expected revenues denoted by $\{\Pi_{dcS_1}^{WP}, \; \Pi_{dcS_2}^{WP}\}$ and their negotiation occurs over the negotiation of some fixed revenue. Here Π_{dcS}^{WPRS*}, the optimal value of the expected profit of the problem (7), is that fixed revenue. Thus, the feasible set of the bargaining is $F = \{\Pi_{dcS_1}^{WP}, \; \Pi_{dcS_2}^{WP} / \Pi_{dcS_1}^{WP} + \Pi_{dcS_2}^{WP} = \Pi_{dcS}^{WPRS*}\}$. Again, let us define the disagreement point d of the two suppliers as the expected profits of the two suppliers in non-cooperative scenario. Then, $d = \{\Pi_{ds_1}^{WPRS*}, \Pi_{ds_2}^{WPRS*}\}$. Hence, Nash bargaining solution of the proposed model is obtained from the following optimization problem which is given by

$$\arg_{(\Pi_{dcS_1}^{WP},\ \Pi_{dcS_2}^{WP})\in F} \max_{(\Pi_{dcS_1}^{WP},\ \Pi_{dcS_2}^{WP})\geq d} \Big(\Pi_{dcS_1}^{WP} - \Pi_{dS_1}^{WPRS*}\Big)\Big(\Pi_{dcS_2}^{WP} - \Pi_{dS_2}^{WPRS*}\Big). \tag{8}$$

From this, the Nash bargaining solutions can be obtained easily as

$$\Big(\Pi_{dcS_1}^{WPRS*}, \Pi_{dcS_2}^{WPRS*}\Big) = \Big(\frac{\Pi_{dcS}^{WPRS*} + \Pi_{dS_1}^{WPRS*} - \Pi_{dS_2}^{WPRS*}}{2}, \frac{\Pi_{dcS}^{WPRS*} + \Pi_{dS_2}^{WPRS*} - \Pi_{dS_1}^{WPRS*}}{2}\Big). \tag{9}$$

Hence, the sequence of the game for cooperative suppliers for Retailer-Stackelberg game is: as a Stackelberg leader, the common retailer first declares the retail margin; the suppliers then respond by choosing the wholesale prices by using the total revenue function Π_{dcS}^{WP}; considering these reaction functions (response) of the cooperative suppliers, the retailer then maximizes his profit by optimizing his decision variables; for these given retailer's decision variables, the cooperative suppliers then maximize their cooperative revenue from (7). Finally, this maximized profit is divided between two cooperative suppliers following the Nash bargaining solutions as given in (9).

3.2. Coordination with modified buyback contract

Here we further consider another coordinating mechanism to coordinate the total channel members. In this paper we consider a modified buyback contract. Under such a contract the suppliers agree to buyback the unsold product from the common retailer at some pre-declared price $c_{b_i},\ i = 1, 2$ at the end of the selling season. Here, the underlying parametric restrictions are (i) $c_i < w_{b_i} < P_{b_i}$ and (ii) $c_v < c_{b_i} < w_{b_i}$. Otherwise, retailer would always prefer to sell the leftover at salvage value (c_v) by herself at the end of the selling season. Other necessary assumption of this contract is that the salvage value of the suppliers must be higher than that of the retailer, which is one of the basic assumptions of our models. In this case also we will consider the common retailer as the Stackelberg leader and the suppliers as the followers. Similar to our previous game sequence, as a Stackelberg leader, the retailer first declares the retail margins; then for these given retail margins the suppliers choose wholesale prices and buyback prices. In practice, retailer's order quantities vary with the market risk that the suppliers would like to bear. Hence, before the selling season, the dominant retailer declares that he will prefer to consider the difference between the wholesale price and the buyback price in order quantity decision (for this reason, we would like to call this contract as modified buyback contract rather than simply buyback contract). Hence, in this case, if $Q_{b_i},\ i = 1, 2$ denotes the moderated order quantity placed with Supplier i, then $Q_{b_i} = d(P_b)+z_{b_i}+q_i$ where $q_i = K-l(w_{b_i}-c_{b_i})$ for $i = 1, 2$ $(K > 0,\ l > 0)$ (Chen and Zhang 2008). Under this contract, let us denote the expected profit of the retailer by $\Pi^{BB}_{dR}(z_{b_1}, z_{b_2}, P_b)$ for given $w_{b_i},\ c_{b_i},\ i = 1, 2$ (see Appendix F). Again, for given retail price and order quantities and hence safety stocks, expected profits of Supplier 1 and Suppler 2 are given by

$$\begin{aligned}\Pi^{BB}_{dS_1}(w_{b_1}, c_{b_1}) &= \Big(\alpha\beta w_{b_1} + (1-\alpha\beta)c_{v_1} - (\alpha\beta - \alpha\beta\eta\gamma + \eta\gamma)c_1\Big)\Big(d(P_b) + z_{b_1} + q_1\Big) \\ &\quad -\alpha\beta(c_{b_1} - c_{v_1})\int_A^{d(P_b)+z_{b_1}+z_{b_2}+q_1+q_2} F(\epsilon)d\epsilon, \qquad (10)\end{aligned}$$

$$\begin{aligned}\Pi^{BB}_{dS_2}(w_{b_2}) &= \Big(\alpha w_{b_2} + (1-\alpha)c_{v_2} - (\alpha - \alpha\eta\gamma + \eta\gamma)c_2\Big)\Big(d(P_b) + z_{b_2} + q_2\Big) \\ &\quad -\alpha\beta(c_{b_2} - c_{v_2})\int_A^{d(P_b)+z_{b_1}+z_{b_2}+q_1+q_2} F(\epsilon)d\epsilon \\ &\quad -\alpha(1-\beta)(c_{b_2} - c_{v_2})\int_A^{z_{b_2}+q_2} F(\epsilon)d\epsilon, \qquad (11)\end{aligned}$$

respectively, subject to the conditions $w_{b_1}-(c_{b_1}-c_{v_1})-w_1 \geq 0$ and $w_{b_2}-(c_{b_2}-c_{v_2})-w_2 \geq 0$. When retailer is the Stackelberg leader, then $z_{b_1},\ z_{b_2}, P_b$ are the decision variables of the retailer and $w_{b_i},\ c_{b_i}$ are the decision variables of the competitive suppliers subject to the constraints $c_i < c_{b_i} < w_{b_i} < P_b$ for $i = 1,\ 2$. In this market scenario, the common retailer maximizes her expected profit taking into consideration the suppliers' reaction functions. On the other hand, both suppliers condition on their wholesale prices and the marginal costs of their own products. Hence suppliers' reaction functions will be obtained from the first order optimality conditions $\frac{\partial\Pi^{BB}_{dS_i}}{\partial w_{b_i}} = 0$ and $\frac{\partial\Pi^{BB}_{dS_i}}{\partial c_{b_i}} = 0$ for $i = 1, 2$. As Stackelberg leader retailer will always want to increase her profit as much as possible, she will maximizes her profit subject to the conditions $w_{b_1} - (c_{b_1} - c_{v_1}) - w_1 = 0$ and $w_{b_2} - (c_{b_2} - c_{v_2}) - w_2 = 0$.

4. SCB model: Supply chain model with a backup supplier

In the case of common cause disruption, nothing is delivered to the retailer and the retailer may have to face a massive loss of business. Hence, to alleviate the negative effect of the supply disruption we now incorporate a backup supplier into the previous supply chain model as an alternative source of supply. This backup supplier is assumed to be perfectly reliable in the sense that he can deliver product in time up to a specific quantity reserved earlier. Here we examine whether the presence of the backup supplier helps the retailer to lessen the impact of supply disruption, on the whole supply chain.

Insert **Figure 4** here

Figure 4 represents the sequence of events and subsequent decisions of SCB model. This sequence is different from the previous one [Figure 3] with respect to the following aspects. In this case, after placing the order quantities, Q_1 and Q_2 from the unreliable primary suppliers the retailer can reserve a certain quantity I from the reliable backup supplier to make up the shortfall due to the uncertainty in supplying the product. We also assume that the delivery time from the backup supplier is almost negligible and can be able to supply up to I quantity product in negligible time. Dell computer and Spanish clothing retailer Zara are examples of such successful supply chains in modern industry with negligible lead time to manage their demand and supply risk (Martha and Subbakrishna, 2002). In this case, the backup supplier charges a per unit reservation cost which is directly proportional to the reserved quantity. Then for the given wholesale prices and depending on the disruption condition, expected profit of the retailer is

$$\begin{aligned}\Pi_{dbR}^{WP}(Q_1, Q_2, I, P/w_1, w_2) \quad = \quad & \alpha\beta E\Big[P\min\Big\{Q_1 + Q_2 + I,\ D(P,\epsilon)\Big\} - w_1Q_1 - w_2Q_2 - w_3\min\Big\{I,\ \Big(D(P,\epsilon) - Q_1 - Q_2\Big)^+\Big\} \\ & -c_s\Big\{D(P,\epsilon) - Q_1 - Q_2 - I\Big\}^+ + c_v\Big\{Q_1 + Q_2 - D(P,\epsilon)\Big\}^+\Big] + \alpha(1-\beta)E\Big[P\min\Big\{Q_2 + I,\ D(P,\epsilon)\Big\} \\ & -w_2Q_2 - w_3\min\Big\{I,\ \Big(D(P,\epsilon) - Q_2\Big)^+\Big\} - (1-\eta)\gamma c_1Q_1 - c_s\Big\{D(P,\epsilon) - Q_2 - I\Big\}^+ + c_v\Big\{Q_2 - D(P,\epsilon)\Big\}^+\Big] \\ & +(1-\alpha)E\Big[(P - w_3)\min\Big\{I,\ D(P,\epsilon)\Big\} - c_s\Big\{D(P,\epsilon) - I\Big\}^+ - (1-\eta)\gamma c_1Q_1 - (1-\eta)\gamma c_2Q_2\Big] - c_rI, \qquad (12)\end{aligned}$$

which is a function of four decision variables. Here, the subscript *'dbR'* stands for retailer for decentralized supply chain with backup supplier. Again, for given order quantities and retail price, expected profit of the backup supplier is

$$\begin{aligned}\Pi_{bs}(w_3) \;=\;& rI + (w_3 - c_3)E\Big[\alpha\beta\min\Big\{I,\ \Big(D(P,\epsilon) - Q_1 - Q_2\Big)^+\Big\} \\ & + \alpha(1-\beta)\min\Big\{I,\ \Big(D(P,\epsilon) - Q_2\Big)^+\Big\} + (1-\alpha)\min\Big\{I,\ D(P,\epsilon)\Big\}\Big] \quad (13)\end{aligned}$$

whereas expected profits of the primary suppliers remain the same as those of SC model. Substituting $Q_i = d(P) + z_i$, for $i = 1, 2$ as before, we can convert the retailer's expected profit function as a function of safety stocks, reserve quantity, and retail price. Here we assume the complete sharing of information so that primary suppliers possess full information regarding the backup supplier's availability and terms of conditions of business with him. Since, in our current study, our main focus is on the revenues of Supplier 1, Supplier 2 and the common retailer, we don't care about the revenue of the backup supplier. That means the backup supplier is not a decision maker in the supply chain. The retailer's optimal decisions regarding reserve quantity I from the backup supplier are derived in the following propositions.

Proposition 4

For known z_1, z_2, P, expected profit Π_{dbR}^{WP} of the common retailer is concave with respect to its reserve quantity I from the backup supplier. Moreover, when both primary suppliers are perfectly reliable, then optimal reserve quantity $I^* = 0$ under the condition **A3** $(P + c_s - w_3 - c_r) < F\Big(d(P) + z_1 + z_2\Big)(P + c_s - w_3)$.

Proposition 5

Under uniform distribution, the optimal reserve quantities of the retailer from the backup supplier are always greater than zero under both market scenarios (a) when there is no common cause supply disruption and (b) when there is no supplier-specific disruption. Moreover, these optimal reserve quantities increase with the increase of both disruption probabilities under these market scenarios.

Proposition 4 and Proposition 5 provide the retailer with an important decision regarding optimal reserve quantity from the backup supplier. Proposition 4 implies that if both suppliers are perfectly reliable, then there is no need for the retailer to use backup supplier. Then SCB model converted to SC model *i.e.,* SC and SCB become equivalent. However, from Proposition 5 we see that if there exists supply disruption even with lower probabilities, optimal reserve quantities will always be positive and will increase with the increase of disruption probabilities. This implies that in the presence of the supply disruption, the retailer would always prefer to use the advantage of the backup supplier.

4.1. Equilibrium analysis: Retailer-Stackelberg game

In this model, to find the equilibrium solutions of a supply chain model consisting of two competitive primary suppliers, a secondary backup supplier, and a common retailer, we follow the same approaches as discussed in SC model. Proposition 2 is also valid for this Model.

4.1.1. Optimal strategies when suppliers are cooperative

Here also, to coordinate the suppliers and hence to enhance the supply chain efficiency, we consider the case when suppliers are cooperative in nature. Even though the backup supplier will not participate in this cooperative game, primary suppliers will always take into account the backup supplier's wholesale price in their wholesale price decisions. The objective function of the cooperative suppliers of the Nash bargaining game is

$$\begin{aligned}
&\max_{w_1,w_2}\ \Pi_{dbcS}^{WP}(w_1,w_2/z_1,z_2,P)\\
&\text{s.t.}\\
&w_3 \geq w_1 + \frac{(1-\alpha\beta)(1-\eta)\gamma c_1}{\alpha\beta},\\
&w_2 + \frac{(1-\alpha)(1-\eta)\gamma c_2}{\alpha} \geq w_1 + \frac{(1-\alpha\beta)(1-\eta)\gamma c_1}{\alpha\beta},\\
&w_3 \geq w_2 + \frac{(1-\alpha)(1-\eta)\gamma c_2}{\alpha}. \qquad (14)
\end{aligned}$$

The remaining approaches are the same as we discussed in Model 1.

4.2. Coordination with modified buyback contract

Now let us consider a coordinating mechanism through which we will try to coordinate the channel members of Model 2 for the competitive market scenario. Since the backup supplier is used as an emergency source, the common retailer offers the modified buyback contract to the primary suppliers only. If $\Pi_{dbR}^{BB}(Q_{b_1}, Q_{b_2}, I_b, P_b)$ denotes the expected profit of the retailer with modified buyback contract then for the given wholesale prices, it is given by

$$\begin{aligned}
\Pi_{dbR}^{BB}(Q_{b_1}, Q_{b_2}, I_b, P_b/w_1, w_2) \quad = \quad & \alpha\beta E\Big[P\min\Big\{Q_{b_1} + Q_{b_2} + I_b,\ D(P_b,\epsilon)\Big\} - w_{b_1}Q_{b_1} - w_{b_2}Q_{b_2} - w_3\min\Big\{I_b,\ \Big(D(P_b,\epsilon) - Q_{b_1} - Q_{b_2}\Big)^+\Big\} \\
& -c_s\Big\{D(P_b,\epsilon) - Q_{b_1} - Q_{b_2} - I_b\Big\}^+ + c_b\Big\{Q_{b_1} + Q_{b_2} - D(P_b,\epsilon)\Big\}^+\Big] + \alpha(1-\beta) \\
& \times E\Big[P\min\Big\{Q_{b_2} + I_b,\ D(P_b,\epsilon)\Big\} - w_{b_2}Q_{b_2} - w_3\min\Big\{I_b,\ \Big(D(P_b,\epsilon) - Q_{b_2}\Big)^+\Big\} - (1-\eta)\gamma c_1 Q_{b_1} \\
& -c_s\Big\{D(P_b,\epsilon) - Q_{b_2} - I_b\Big\}^+ + c_{b_2}\Big\{Q_{b_2} - D(P_b,\epsilon)\Big\}^+\Big] + (1-\alpha)E\Big[(P_b - w_3)\min\Big\{I_b,\ D(P_b,\epsilon)\Big\} \\
& -c_s\Big\{D(P_b,\epsilon) - I_b\Big\}^+ - (1-\eta)\gamma c_1 Q_{b_1} - (1-\eta)\gamma c_2 Q_{b_2}\Big] - c_r I_b. \qquad (15)
\end{aligned}$$

Since the leftover quantity of the retailer does not depend on the presence of the backup supplier the expressions of the primary suppliers remains the same as derived in SC model. As it is mentioned before, analytical solutions of this kind of problems is difficult. However, we can borrow computational study for gaining an insight into the management implications i.e. how well these models with different coordinating mechanisms work in practice.

5. Computational results and managerial explanations

In this section, we conduct a numerical analysis to investigate the key insights of our models. Due to the lack of closed forms of the analytical solutions, we find the equilibrium solutions numerically. To do so, we formulate each problem as non-linear programming problem and solve them using the built-in software Mathematica 7.0 (Wolfram 1996). Our main goals are (1) to find the equilibrium solutions of both models for different scenarios, (2) to investigate the implications of our proposed coordinating mechanisms, (3) to carry out a comparison study between our proposed models, (4) to do sensitivity analyzes with respect to some important parameters. In our numerical experiment, the stochastic demand follows truncated Normal distribution. Here, we have fixed the values of c_i $(i = 1, 2), c_s, c_v, c_r$ though those may have some impacts on the performance of supply chain coordinating mechanisms. Here, we investigate the effects of some other parameters like (i) disruption probabilities $(\alpha,\ \beta)$, (ii) price sensitivity parameter (b) and (iii) fractions of marginal cost $(\gamma,\ \beta)$, in three different possible scenarios. Parametric values of this numerical experiment are listed in Table 2. By varying the key parameters we create the following sets of problems: we take price sensitivity parameter $b = \{24, 27, 30\}$; common cause disruption probability $\alpha = \{0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0\}$; supplier specific disruption probability $\beta = \{0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0\}$; $\gamma = \{0.2, 0.3, 0.4\}$ and $\eta = \{0.2, 0.4, 0.6\}$. Again for positive demand realization, we assume that $A > -a$. We set $A = -(a-1)$ and $B = (a-1)$. Now the other parametric values are set as $c_1 = 1,\ c_2 = 1.1,\ c_3 = 6,\ w_3 = 12, c_r = 1,\ c_v = 0.5, c_{v_1} = 1.1,\ c_{v_2} = 1.2,\ c_s = 0.75,\ a = 600,\ \mu = 0,\ \sigma = 30,\ k = 100,\ l = 3$. Through experiment, we find that optimal solutions exist for $2.5 \leq l \leq 3$. Through experiment, we find that for those parametric selections, optimal solutions exist for $2.5 \leq l \leq 3$. These parametric values are chosen from the existing literature solely for the illustrative purpose. Using these parametric values we evaluate the optimal decisions of suppliers and the common retailer subject to the profit maximization for both proposed models in three different scenarios: (a) under WP contract when primary suppliers are competitive, (b) under WP contract when primary suppliers are cooperative and (c) under modified BB contract when primary suppliers are competitive.

Similar to our analytical study, in the numerical study also, we always consider Stackelberg game between each supplier and the common retailer. Further, for competitive suppliers, equilibrium solutions are obtained by playing non-cooperative Nash game between the suppliers whereas cooperative Nash bargaining game between two cooperative suppliers. Computational results are analyzed through tables as well as through different diagrams.

5.1. Analysis

5.1.1. The effect of supply disruption probabilities $(\alpha,\ \beta)$

Through experiment we find that feasible solutions exist for $\alpha \geq 0.3,\ \beta > 0.3$ (approximately) for SC model(Supply chain model without backup supplier) and $\alpha \geq 0.4,\ \beta > 0.3$ (approximately) for SCB model(Supply chain model with a backup supplier), under WP contract for both competitive and cooperative scenarios. This range of β validates our proclamation made in Proposition 3. Figures 5, 6 represent the variation of the expected profits with respect to common cause supply disruption probability $(1-\alpha)$ for competitive and cooperative scenarios for both models under WP contract. Figure 7 represents the effect of common cause failure probability $(1-\alpha)$ on the optimal ordered and reserved quantities for SCB model under WP contract for competitive suppliers. From Figure 5, it can be observed that for higher common cause disruption probability, SCB model gives the better result than that of SC model in consideration of total channel profit. From Figure 6, we see that in the presence of common cause failure probability, expected profit of the retailer is always higher in the later model (SCB) than that of SC model, whereas under the same circumstances the expected profits of the primary suppliers are always lower in SCB model than that of SC model. The underlying reason for these lower profits of the primary suppliers of SCB model can be interpreted as follows. From Figure 7, we can observe that with the increase of common cause disruption probability, optimal order quantities Q_1^* and Q_2^*, both decrease whereas optimal reserve quantity I^* increases for SCB model and this subsequently results in lower profits of primary suppliers in this model. This Figure 7 also endorses the conclusion made in Proposition 5. Thus, it can be concluded similar to Chopra (2007) that *with the increase of common cause disruption probability, the retailer would like to trust the backup supplier more than that of the risky primary suppliers in SCM model for both competitive and cooperative scenarios under WP contract.*

Again, the effects of supplier-specific disruption probability $(1-\beta)$ on the optimal solutions are depicted in Figures 8 and 9. Similar to common cause supply disruption, in this case too, expected profit of the common retailer is always higher in SCB model than that of SC model. Further, we can significantly observe from Figure 8 that with the increase of the supplier-specific disruption probability, expected profits of Supplier 1 decrease whereas expected profits of Supplier 2 increase for both models for both competitive and cooperative market scenarios. Before describing the underlying reason, we would like to remind the reader that Supplier 1 is affected by both types of failure and hence by supplier-specific failure whereas Supplier 2 is affected by common cause failure only. Hence, with the increase of supplier-specific disruption probability, Q_1^* (optimal order quantity of Supplier 1) decreases and Q_2^* (optimal order quantity of Supplier 2) increases for SC model, Q_1^* decreases and Q_2^* and the optimal reserve quantity I^* increase for SCB model (see Figure 9). This subsequently results in lower profit of Supplier 1 for both models.

Moreover, for higher disruption probabilities (common cause and supplier-specific), modified BB contract is not possible with the primary suppliers for both models. Hence, we can conclude that *When supply disruption probabilities are high, the retailer would prefer to follow SCB model under WP contract i.e., in that case, the retailer would prefer to rely on the reliable backup supplier rather than going into risky BB contract to enhance his profit.*

Insert **Figures 5 to 9** here

5.1.2. The effect of price sensitivity parameter

We investigate the effect of price sensitivity parameter (b) on the optimal solutions for the three different scenarios at $\alpha = 0.9,\ \beta = 0.9$. Tables 2, 4, 6 show this variation for SC model under WP contract when suppliers are competitive, under WP contract when suppliers are cooperative and under modified BB contract when suppliers are competitive, respectively. Similar results are found for SCB model in Tables 3, 5, 7, respectively. Here, we represent the supply chain performance in terms of supply chain efficiency (Lariviere and Porteus, 2001). In the present study, we consider a decentralized system under different scenarios. Hence, let us define the efficiency of the Stackelberg leader with respect to the expected profit of the decentralized system as $E_{dR} = \frac{\Pi_{dR}^*}{\Pi_d^*}$. It is to be noted that with the variation of l (wholesale-buyback price difference sensitive parameter), optimal solutions vary too, for modified BB contract under the competitive market scenario. Most feasible values of l are tabulated in Tables 6 and 7. From Tables 2 to 6, we can observe that, the higher the price sensitivity factor (b) of demand, the lower the channel efficiency would be for both models under all possible scenarios except for SCB model under modified BB contract. Here, with the increase of b, channel efficiency first decreases and then increases again (convex property). For both models the relation $E_{dR}^{WP} > E_{dR}^{BB} > E_{dcR}^{WP}$ holds in correspondence of supply chain efficiency whereas in consideration of the retailer's expected profit, $\Pi_{dR}^{BBRS^*} > \Pi_{dR}^{WPRS^*} = \Pi_{dcR}^{WPRS^*}$ holds for both models. Hence, *when disruption probabilities are very low, then as a Stackelberg leader the retailer would always prefer to follow modified BB contract rather than WP contract for both models.* Again, from Tables 2 to 6, it can be observed that the retailer's expected profit is always higher in SCB model than that of SC under WP contract. Hence, *for higher disruption probabilities, the common retailer would always prefer to follow SCB model under WP contract rather than SC model in order to increase his surplus.*

5.1.3. The effect of the fraction of marginal cost (γ, η)

Variation of the optimal solutions with respect to the fraction γ (the total proportion of the marginal cost in case of failure) are depicted in Tables 8 to 11 for both models under WP contract. With the increase of γ the channel profits and the optimal order quantities decrease for both models whereas the optimal reserve quantity increases in SCB model. Through experiment, we find that the impact of η (the proportion of marginal cost incurred by the disrupted supplier) on the optimal decisions is negligible.

Apart from the above analysis, a comparison study of the expected profits is depicted further in Table 12 under different possible scenarios. From this table, we can observe that for both models (i) retailer earns highest profit in modified BB contract under competitive market scenario, (ii) both primary suppliers earn highest profits in WP contract under cooperative market scenario and (iii) total channel profit is always highest in WP contract under cooperative market scenario.

6. Conclusions

In this study, we discuss the integrated effect of competition and coordination mechanisms on the optimal decisions in a supply chain with supply disruption, under the retailer-dominated scenario. We consider the market segment where two competing primary suppliers play horizontal Nash game among them whereas the monopoly retailer plays Stackelberg game with each of the primary suppliers. Through analytical as well as computational study we get the following conclusions from this study: (1) in presence of supply disruption even with lower probabilities, the retailer, as a Stackelberg leader will always prefer to follow SCB model rather than SC model in order to increase his profit for both WP and BB contracts, (2) for lower disruption probabilities, the retailer, as a Stackelberg leader, will always prefer to offer modified BB contract rather than WP contract for both models, (3) for SC model, Supplier 1 will not be encouraged to accept modified BB contract offered by the retailer whereas for SCB model, modified BB contract is profitable for all the channel members, (4) for higher disruption probabilities, the retailer will prefer to rely on the reliable backup supplier rather than going into risky modified BB contract to enhance his expected profit i.t., in this case, the retailer will prefer to follow SCB model under WP contract, (5) when the retailer offers WP contract, then suppliers will always prefer to play cooperative game among themselves, and finally, (6) for SC model, the mechanism of cooperative game can coordinate the channel members whereas for SCB model both cooperative game and modified BB contract can coordinate the channel members.

We now explore the limitations of our present study and possible extensions for future research in this research area. Throughout our study, we consider supply chain with monopoly retail channel. Although single retail market segment is common in literature, horizontal competition among multiple retailers will definitely affect the supply chain dynamics in addition to horizontal competition among suppliers. In our study, both suppliers sell the same product, brand differentiation is not considered. It would be interesting to incorporate the concept of brand substitution into our proposed models. Moreover, for random demand, additive demand shock is considered throughout this study. Our models can be extended for multiple demand shock, also and it would be interesting to see how this multiplicative demand shock will change our results. Further, in our study, supply disruption is considered only. Other disruptions may be investigated in addition of supply disruption. Other coordinating mechanisms may be addressed further in our proposed models.

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Table 2: Dependence of the optimal solutions on the price sensitivity parameter b under WP contract for SC model when suppliers are competitive

σ	w_1^*	w_2^*	P^*	Q_1^*	Q_2^*	$\Pi_{dR}^{WPRS^*}$	$\Pi_{dS_1}^{WPRS^*}$	$\Pi_{dS_2}^{WPRS^*}$	$\Pi_{d}^{WPRS^*}$	E_{dR}^{WP}
b=24										
10	4.606	5.183	16.987	92.278	100.962	1912.13	287.391	382.249	2581.77	0.7406
20	4.642	5.214	16.949	93.143	101.693	1856.70	292.799	387.804	2537.3	0.7318
30	4.676	5.241	16.904	93.959	102.352	1801.33	297.956	392.849	2492.14	0.7228
b=27										
10	4.164	4.693	15.138	91.890	100.355	1680.84	253.311	335.701	2269.85	0.7405
20	4.196	4.721	15.105	92.765	101.097	1631.16	258.160	340.687	2230.22	0.7314
30	4.227	4.746	15.066	93.594	101.768	1581.95	262.796	345.222	2189.97	0.7224
b=30										
10	3.811	4.301	13.659	91.504	99.750	1495.89	226.070	298.503	2020.46	0.7404
20	3.840	4.327	13.629	92.389	100.503	1451.20	230.467	303.025	1984.69	0.7312
30	3.868	4.349	13.595	93.230	101.183	1406.54	234.680	307.142	1948.36	0.7219

Table 3: Dependence of the optimal solutions on the price sensitivity parameter b under WP contract for SCB model when suppliers are competitive

σ	w_1^*	w_2^*	P^*	Q_1^*	Q_2^*	I^*	$\Pi_{dbR}^{WPRS^*}$	$\Pi_{dbS_1}^{WPRS^*}$	$\Pi_{dbS_2}^{WPRS^*}$	$\Pi_{db}^{WPRS^*}$	E_{dbR}^{WP}
b=24											
10	4.418	4.953	17.193	87.778	95.437	84.919	1942.81	260.043	341.555	2544.41	0.7636
20	4.372	4.915	17.122	86.676	94.528	79.137	1906.95	253.556	335.081	2495.59	0.7641
30	4.345	4.867	17.067	86.015	93.372	80.623	1870.22	249.700	326.937	2446.86	0.7643
b=27											
10	4.084	4.623	15.182	89.719	98.460	16.003	1689.19	241.487	323.149	2253.83	0.7495
20	4.043	4.587	15.201	88.611	97.481	31.847	1647.96	235.557	316.755	2200.27	0.7490
30	4.018	4.553	15.213	87.943	96.565	46.162	1606.36	232.021	310.824	2149.21	0.7436
b=30											
10	3.783	4.277	13.683	90.661	99.027	5.865	1496.91	221.924	294.192	2013.03	0.7436
20	3.786	4.280	13.681	90.764	99.118	11.739	1453.20	222.429	294.730	1970.36	0.7375
30	3.791	4.283	13.678	90.904	99.189	17.518	1409.47	223.114	295.156	1927.74	0.7315

Table 4: Dependence of the optimal solutions on the price sensitivity parameter b under WP contract for SC model when suppliers are cooperative

σ	w_1^*	w_2^*	P^*	Q_1^*	Q_2^*	$\Pi_{dcR}^{WPRS^*}$	$\Pi_{dcS_1}^{WPRS^*}$	$\Pi_{dcS_2}^{WPRS^*}$	$\Pi_{dc}^{WPRS^*}$	E_{dcR}^{WP}
b=24										
10	11.971	11.985	16.987	92.278	100.962	1912.13	871.726	966.584	3750.44	0.5098
20	11.971	11.985	16.949	93.143	101.693	1856.70	879.198	974.203	3710.10	0.5004
30	11.971	11.985	16.904	93.959	102.352	1801.33	886.229	981.122	3668.68	0.4910
b=27										
10	11.971	11.985	15.138	91.890	100.355	1680.84	873.185	955.575	3509.60	0.4789
20	11.971	11.985	15.105	92.765	101.097	1631.16	880.772	963.299	3475.49	0.4693
30	11.971	11.985	15.066	93.594	101.768	1581.95	887.907	970.333	3440.19	0.4598
b=30										
10	11.971	11.985	13.659	91.504	99.750	1495.89	873.419	945.852	3315.16	0.4512
20	11.971	11.985	13.629	92.389	100.503	1451.20	881.106	953.664	3285.97	0.4416
30	11.971	11.985	13.595	93.230	101.183	1406.54	888.344	960.806	3255.69	0.4320

Table 5: Dependence of the optimal solutions on the price sensitivity parameter b under WP contract for SCB model when suppliers are cooperative

σ	w_1^*	w_2^*	P^*	Q_1^*	Q_2^*	I^*	$\Pi_{dbcR}^{WPRS^*}$	$\Pi_{dbcS_1}^{WPRS^*}$	$\Pi_{dbcS_2}^{WPRS^*}$	$\Pi_{dbc}^{WPRS^*}$	E_{dbcR}^{WP}
b=24											
10	11.972	11.985	17.193	87.778	95.437	84.919	1942.81	830.594	912.106	3685.51	0.5271
20	11.972	11.985	17.122	86.676	94.528	79.137	1906.95	821.083	902.608	3630.64	0.5252
30	11.972	11.985	17.067	86.015	93.372	80.623	1870.22	814.497	891.733	3576.45	0.5229
b=27											
10	11.972	11.985	15.182	89.719	98.460	16.003	1689.19	854.314	935.976	3479.48	0.4855
20	11.972	11.985	15.201	88.611	97.481	31.847	1647.96	844.661	925.859	3418.48	0.4821
30	11.972	11.985	15.213	87.943	96.565	46.162	1606.36	838.283	917.087	3361.73	0.4778
b=30											
10	11.972	11.985	13.683	90.661	99.027	5.865	1496.91	866.091	938.359	3301.36	0.4534
20	11.972	11.985	13.681	90.764	99.118	11.739	1453.20	866.995	939.296	3259.49	0.4458
30	11.972	11.985	13.678	90.904	99.189	17.518	1409.47	868.114	940.156	3217.74	0.4380

Table 6: Dependence of the optimal solutions on the price sensitivity parameter b under modified BB contract for SC model when suppliers are competitive at $\sigma = 30$

l	$w_{b_1}^*$	$w_{b_2}^*$	P_b^*	$c_{b_1}^*$	$c_{b_2}^*$	$Q_{b_1}^*$	$Q_{b_2}^*$	$\Pi_{dR}^{BBRS^*}$	$\Pi_{dS_1}^{BBRS^*}$	$\Pi_{dS_2}^{BBRS^*}$	$\Pi_{d}^{BBRS^*}$	E_{dR}^{BB}
b=24												
2.7	4.676	5.241	15.081	1.1	1.2	37.485	228.328	1986.55	118.871	876.371	2981.79	0.6662
b=27												
2.8	4.227	4.746	13.458	1.1	1.2	39.681	226.275	1749.66	111.417	767.584	2628.66	0.6656
b=30												
2.7	3.868	4.349	12.160	1.1	1.2	41.928	224.328	1560.45	105.542	680.948	2346.94	0.6649

Table 7: Dependence of the optimal solutions on the price sensitivity parameter b under modified BB contract for SCB model when suppliers are competitive at $\sigma = 30$

l	$w_{b_1}^*$	$w_{b_2}^*$	P_b^*	$c_{b_1}^*$	$c_{b_2}^*$	$Q_{b_1}^*$	$Q_{b_2}^*$	I_b^*	$\Pi_{dbR}^{BBRS^*}$	$\Pi_{dbS_1}^{BBRS^*}$	$\Pi_{dbS_2}^{BBRS^*}$	$\Pi_{db}^{BBRS^*}$	E_{dbR}^{BB}
b=24													
3.0	8.85	9.42	15.26	5.60	5.81	314.38	346.39	6.91	1946.44	501.61	1008.44	3486.49	0.5582
b=27													
2.7	7.77	8.39	13.77	4.85	5.04	295.69	329.63	6.17	1670.83	471.24	931.93	3074.00	0.5435
b=30													
2.7	7.53	8.13	12.92	4.84	5.04	323.34	358.62	14.63	1586.26	407.97	860.37	2854.60	0.5557

Table 8: Dependence of the optimal solutions on γ when $\eta = 0.4$ under WP contract for SC model when suppliers are competitive

γ	w_1^*	w_2^*	P^*	Q_1^*	Q_2^*	$\Pi_{dR}^{WPRS^*}$	$\Pi_{dS_1}^{WPRS^*}$	$\Pi_{dS_2}^{WPRS^*}$	$\Pi_{d}^{WPRS^*}$	E_{dR}^{WP}
0.2	3.868	4.349	13.595	93.230	101.183	1406.54	234.680	307.142	1948.36	0.72191
0.3	3.872	4.353	13.601	93.049	101.161	1403.66	233.768	307.007	1944.44	0.72188
0.4	3.875	4.357	13.606	92.865	101.136	1400.78	232.843	306.854	1940.48	0.72187

Table 9: Dependence of the optimal solutions on γ when $\eta = 0.4$ under WP contract for SC model when suppliers are cooperative

γ	w_1^*	w_2^*	P^*	Q_1^*	Q_2^*	$\Pi_{dcR}^{WPRS^*}$	$\Pi_{dcS_1}^{WPRS^*}$	$\Pi_{dcS_2}^{WPRS^*}$	$\Pi_{dc}^{WPRS^*}$	E_{dcR}^{WP}
0.2	11.972	11.985	13.595	93.230	101.183	1406.54	888.344	960.806	3255.69	0.4320
0.3	11.958	11.978	13.601	93.049	101.161	1403.66	885.581	958.820	3248.06	0.4321
0.4	11.944	11.971	13.606	92.865	101.136	1400.78	882.794	956.805	3240.38	0.4323

Table 10: Dependence of the optimal solutions on γ when $\eta = 0.4$ under WP contract for SCB model when suppliers are competitive

γ	w_1^*	w_2^*	P^*	Q_1^*	Q_2^*	I^*	S_b^*	$\Pi_{dbR}^{WPRS^*}$	$\Pi_{dbS_1}^{WPRS^*}$	$\Pi_{dbS_2}^{WPRS^*}$	$\Pi_{db}^{WPRS^*}$	E_{dbR}^{WP}
0.2	3.791	4.283	13.678	90.904	99.189	17.518	120.869	1409.47	223.114	295.156	1927.74	0.7311
0.3	3.793	4.286	13.684	90.699	99.146	17.700	122.122	1406.65	222.110	294.899	1923.66	0.7312
0.4	3.796	4.290	13.690	90.494	99.103	17.881	123.373	1403.84	221.107	294.641	1919.59	0.7313

Table 11: Dependence of the optimal solutions on γ when $\eta = 0.4$ under WP contract for SCB model when suppliers are cooperative

γ	w_1^*	w_2^*	P^*	Q_1^*	Q_2^*	I^*	S_b^*	$\Pi_{dbcR}^{WPRS^*}$	$\Pi_{dbcS_1}^{WPRS^*}$	$\Pi_{dbcS_2}^{WPRS^*}$	$\Pi_{dbc}^{WPRS^*}$	E_{dbcR}^{WP}
0.2	11.972	11.985	13.678	90.904	99.189	17.518	120.869	1409.47	868.114	940.156	3217.74	0.4380
0.3	11.958	11.978	13.684	90.699	99.146	17.700	122.122	1406.65	865.186	937.975	3209.81	0.4382
0.4	11.944	11.971	13.690	90.494	99.103	17.881	123.373	1403.84	862.268	935.802	3201.91	0.4384

Table 12: A comparative study of optimal profits in different scenarios

Channel member	**SC model**	**SCB model**
Retailer	$\Pi_{dR}^{BBRS^*} > \Pi_{dR}^{WPRS^*} = \Pi_{dcR}^{WPRS^*}$	$\Pi_{dbR}^{BBRS^*} > \Pi_{dbR}^{WPRS^*} = \Pi_{dbcR}^{WPRS^*}$
Supplier 1	$\Pi_{dcS_1}^{WPRS^*} > \Pi_{dS_1}^{WPRS^*} > \Pi_{dS_1}^{BBRS^*}$	$\Pi_{dbcS_1}^{WPRS^*} > \Pi_{dbS_1}^{BBRS^*} > \Pi_{dbS_1}^{WPRS^*}$
Supplier 2	$\Pi_{dcS_2}^{WPRS^*} > \Pi_{dS_2}^{BBRS^*} > \Pi_{dS_2}^{WPRS^*}$	$\Pi_{dbcS_2}^{WPRS^*} > \Pi_{dbS_2}^{BBRS^*} > \Pi_{dbS_2}^{WPRS^*}$
Total Supply chain	$\Pi_{dc}^{WPRS^*} > \Pi_{d}^{BBRS^*} > \Pi_{d}^{WPRS^*}$	$\Pi_{dbc}^{WPRS^*} > \Pi_{db}^{BBRS^*} > \Pi_{db}^{WPRS^*}$

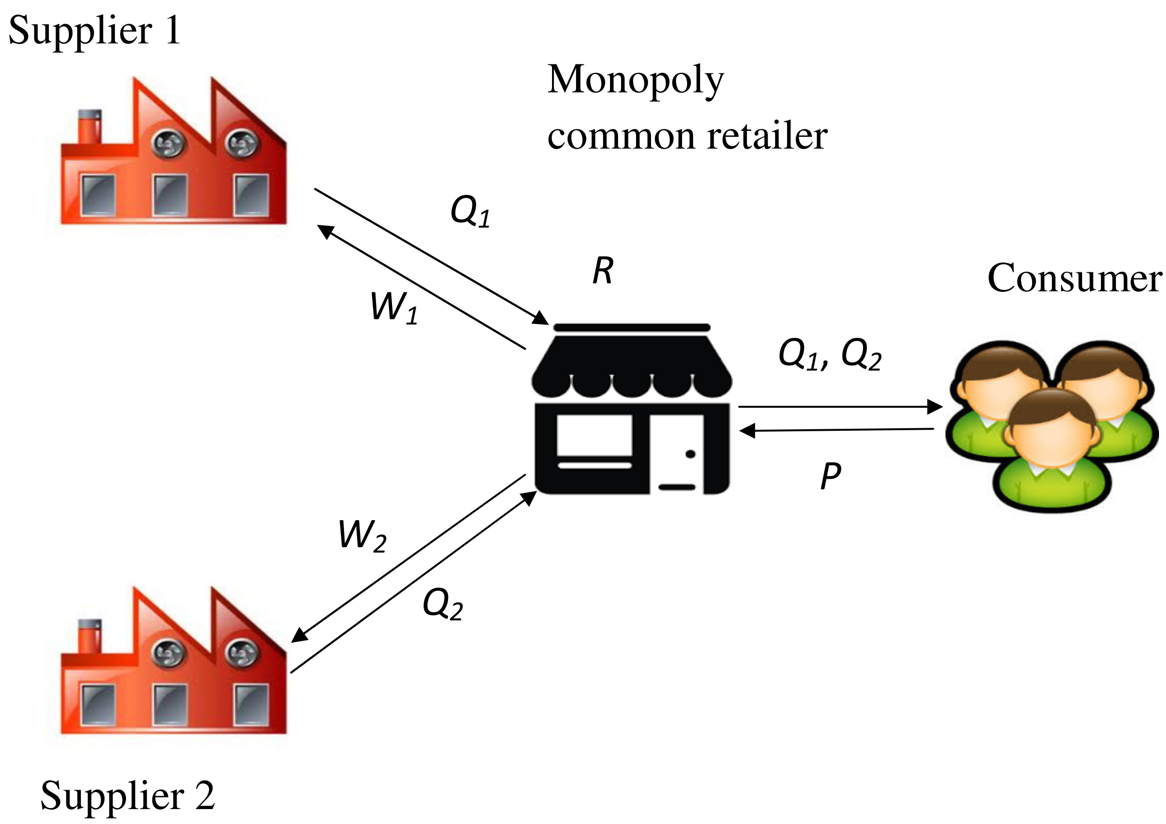

Figure 1: Schematic diagram of SC model

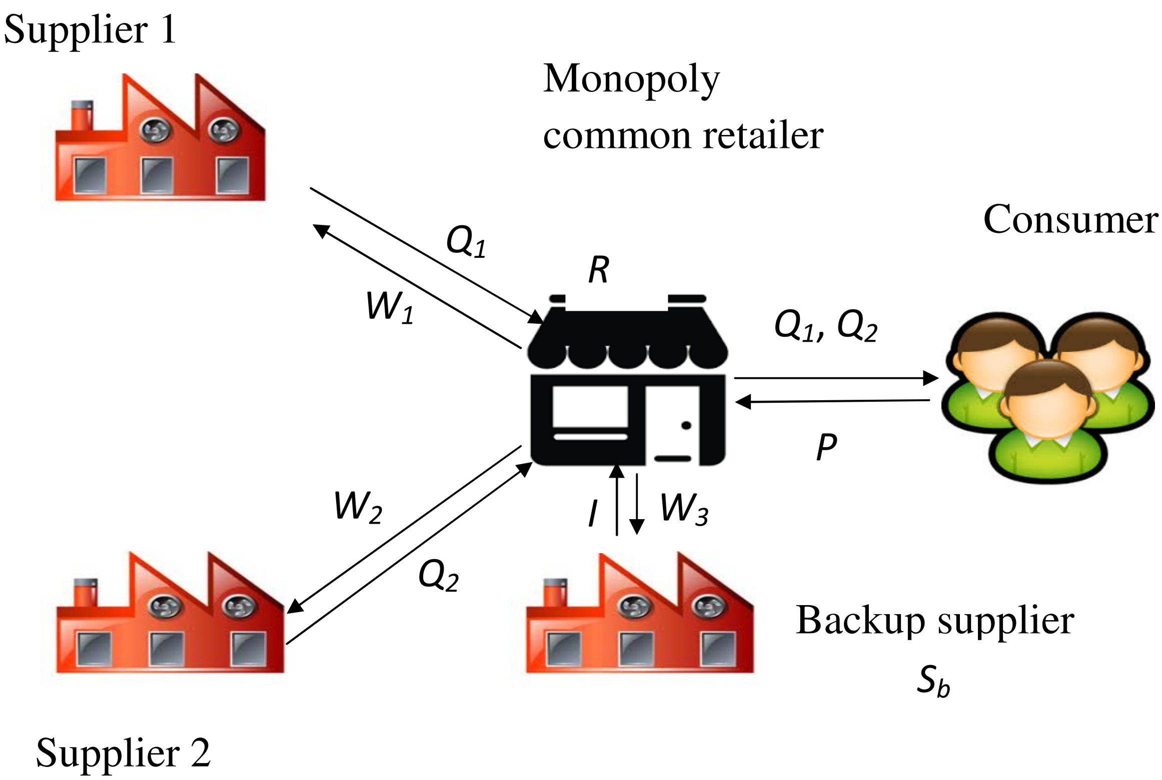

Figure 2: Schematic diagram of SCB model

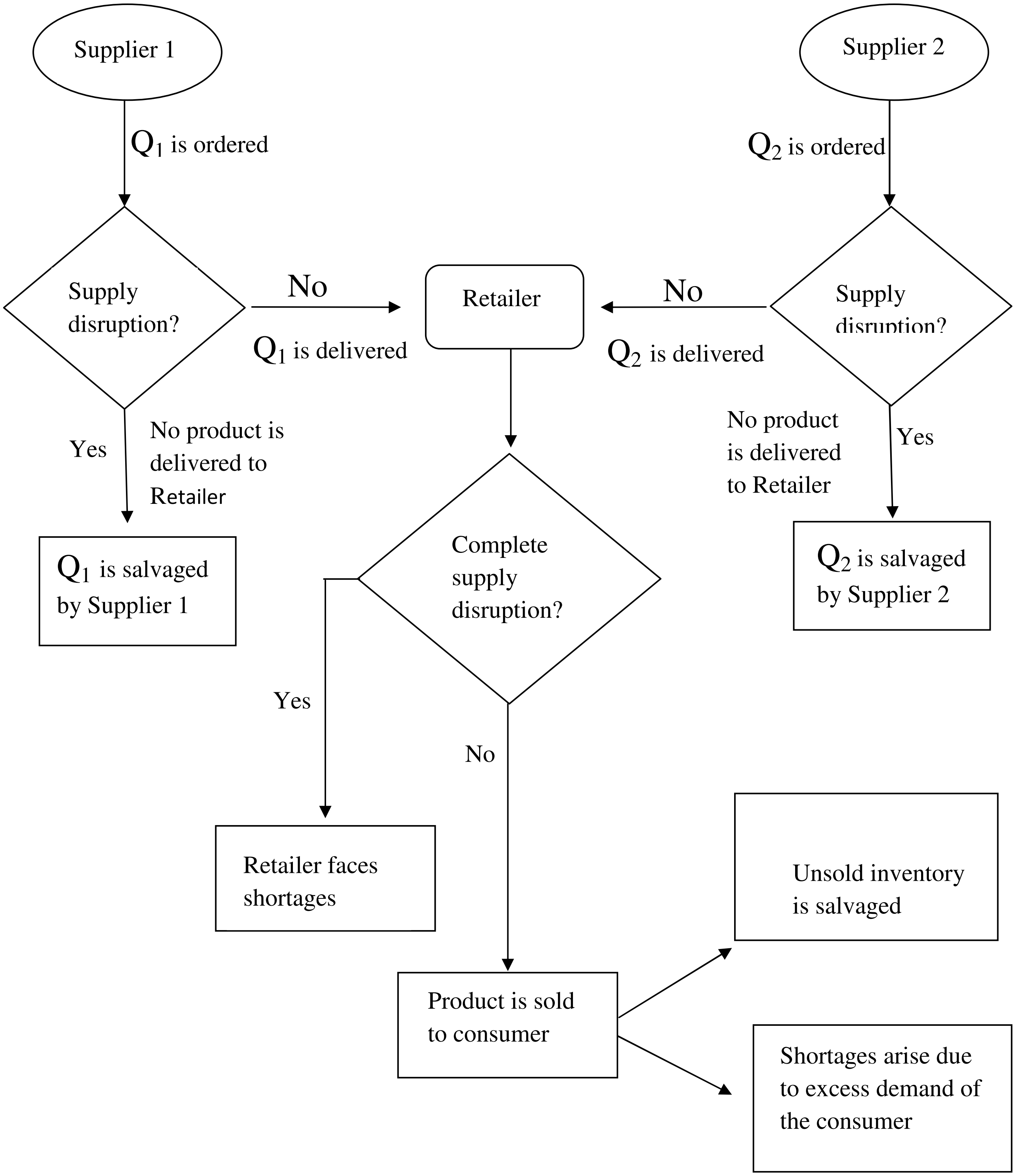

Figure 3: Sequence of events and subsequent decisions of SC model

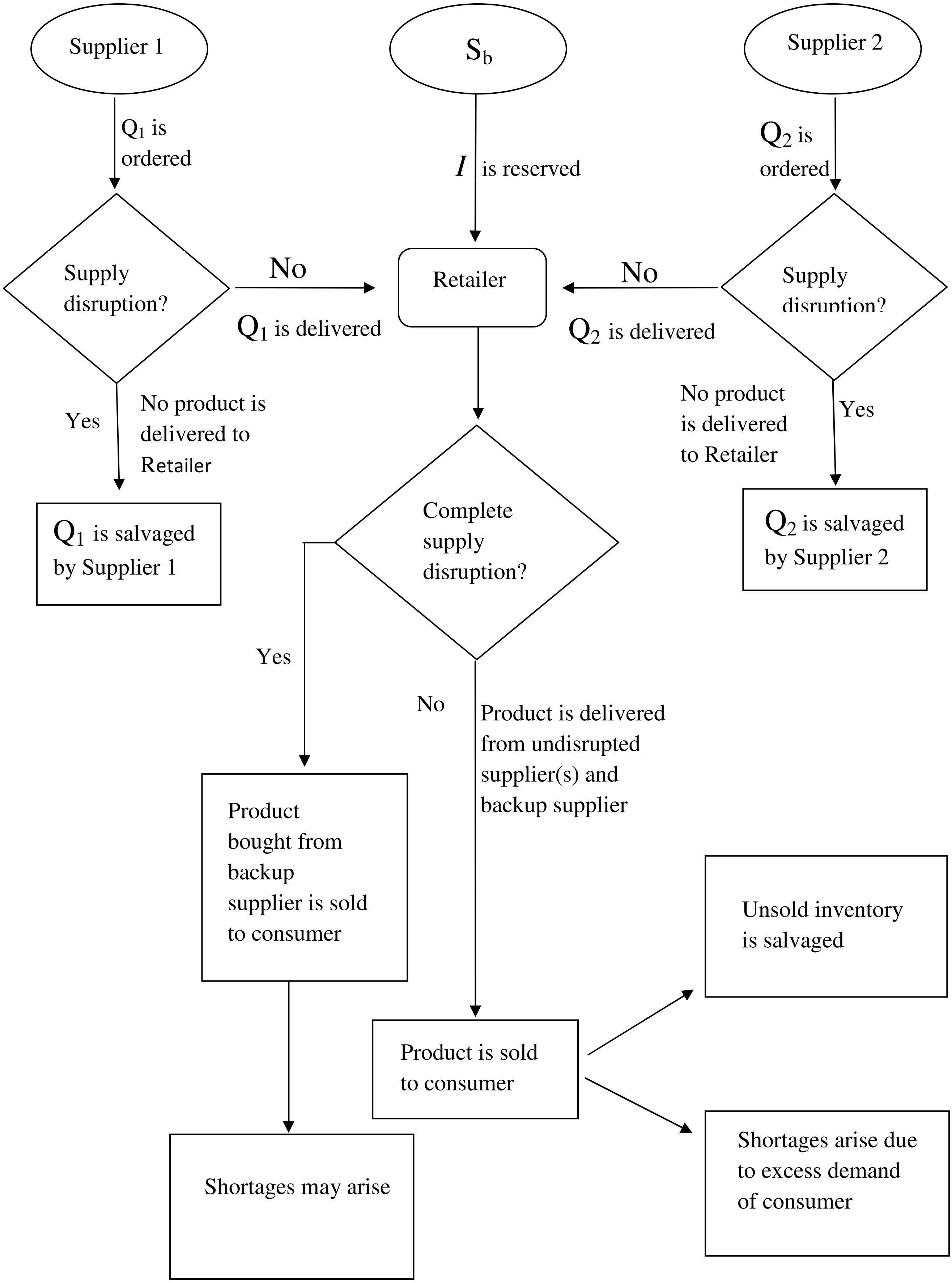

Figure 4: Sequence of events and subsequent decisions of SCB model

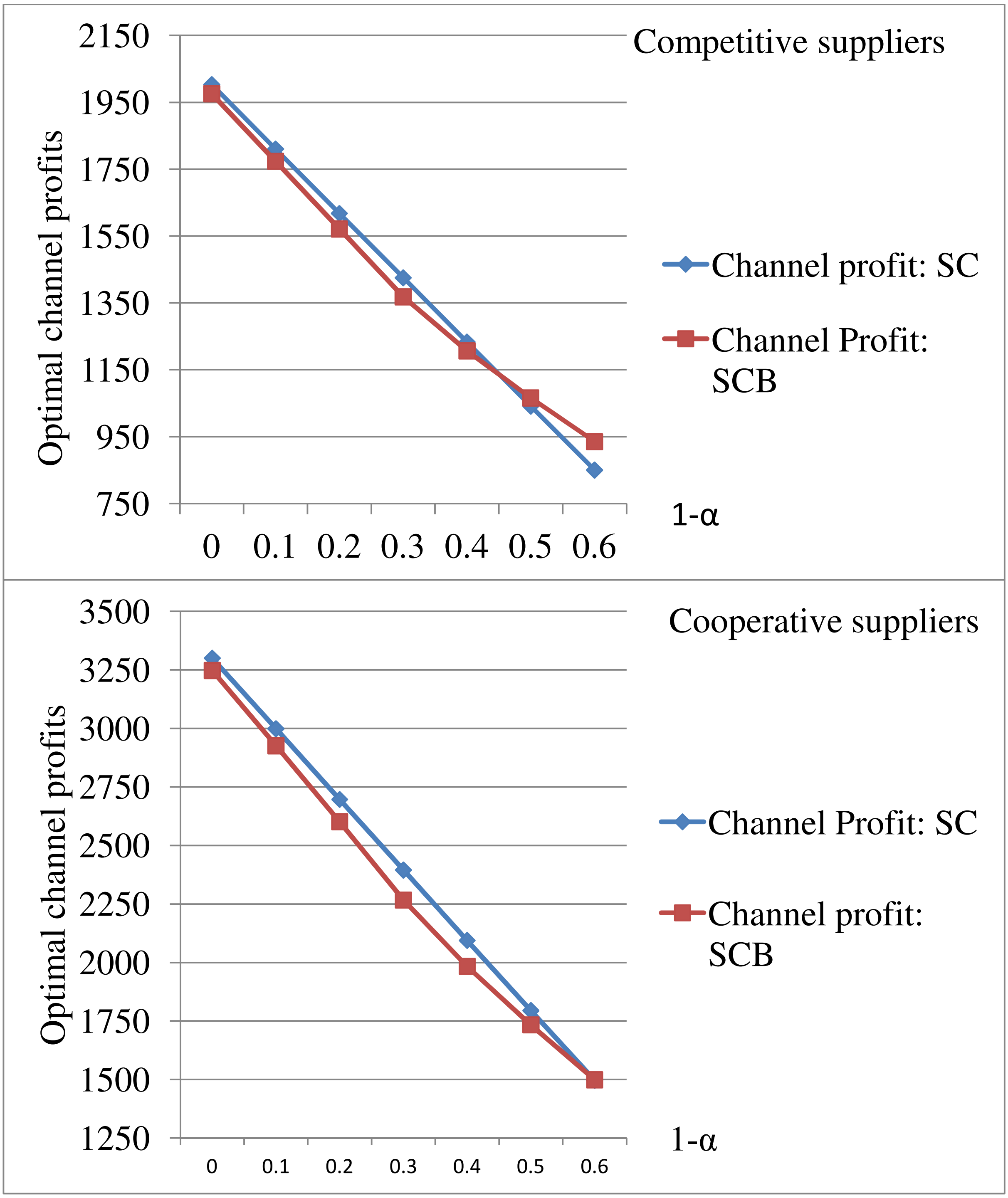

Figure 5: Variation of total channel profits of SC and SCB models with respect to common cause failure probability under WP contract

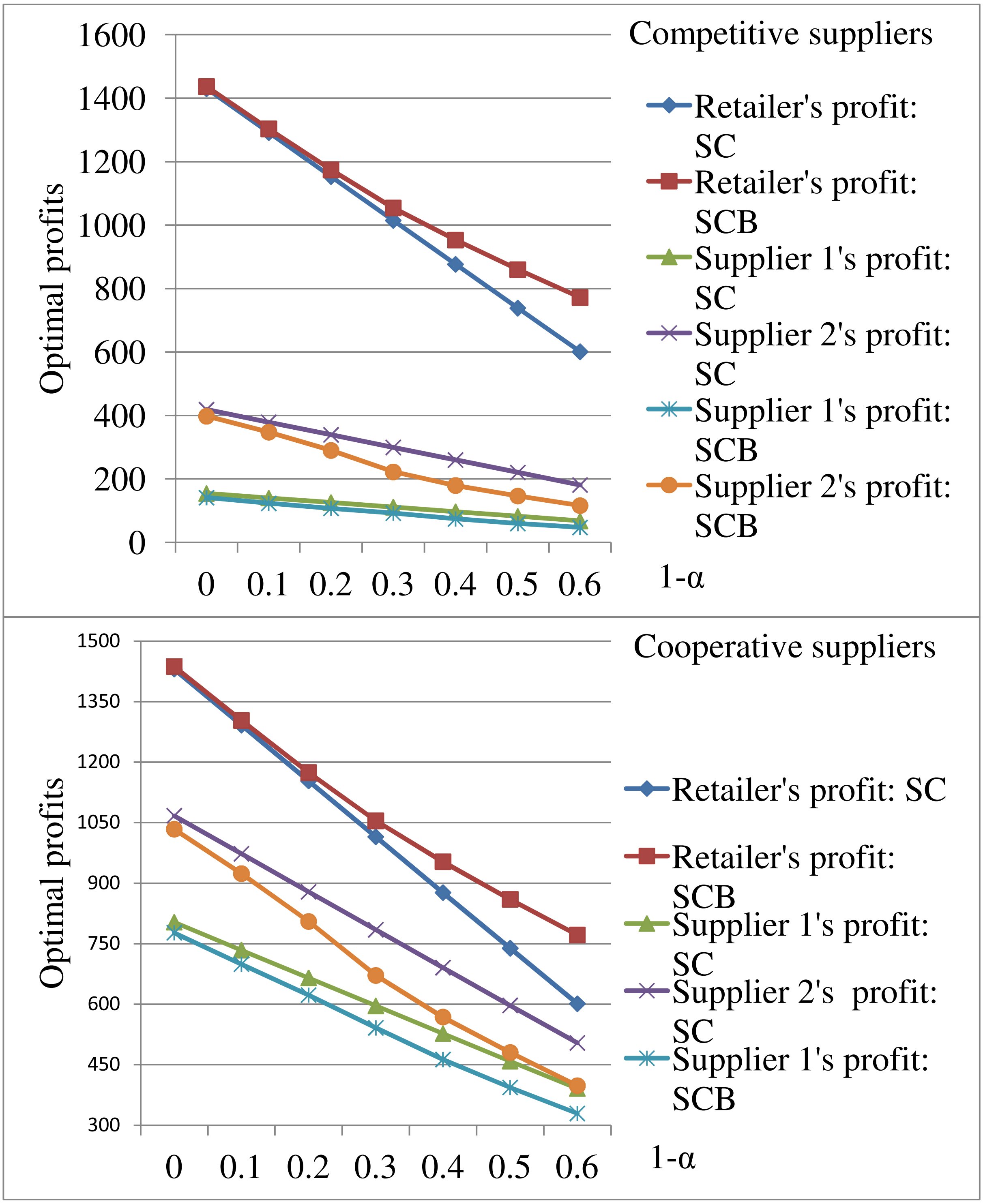

Figure 6: Variation of retailer's and suppliers' profits of SC model and SCB model with respect to common cause failure probability under WP contract

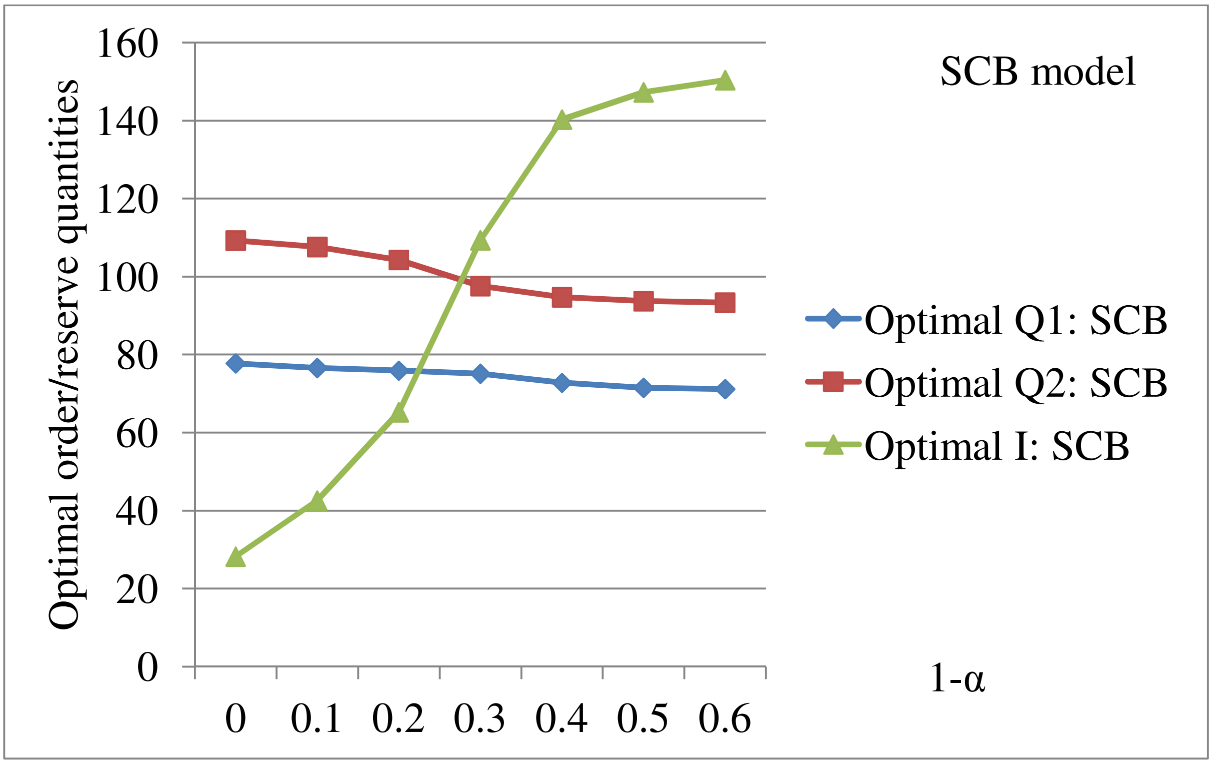

Figure 7: Variation of retailer's order quantities and reserve quantity of SCB model with respect to common cause failure probability under WP contract for competitive primary suppliers

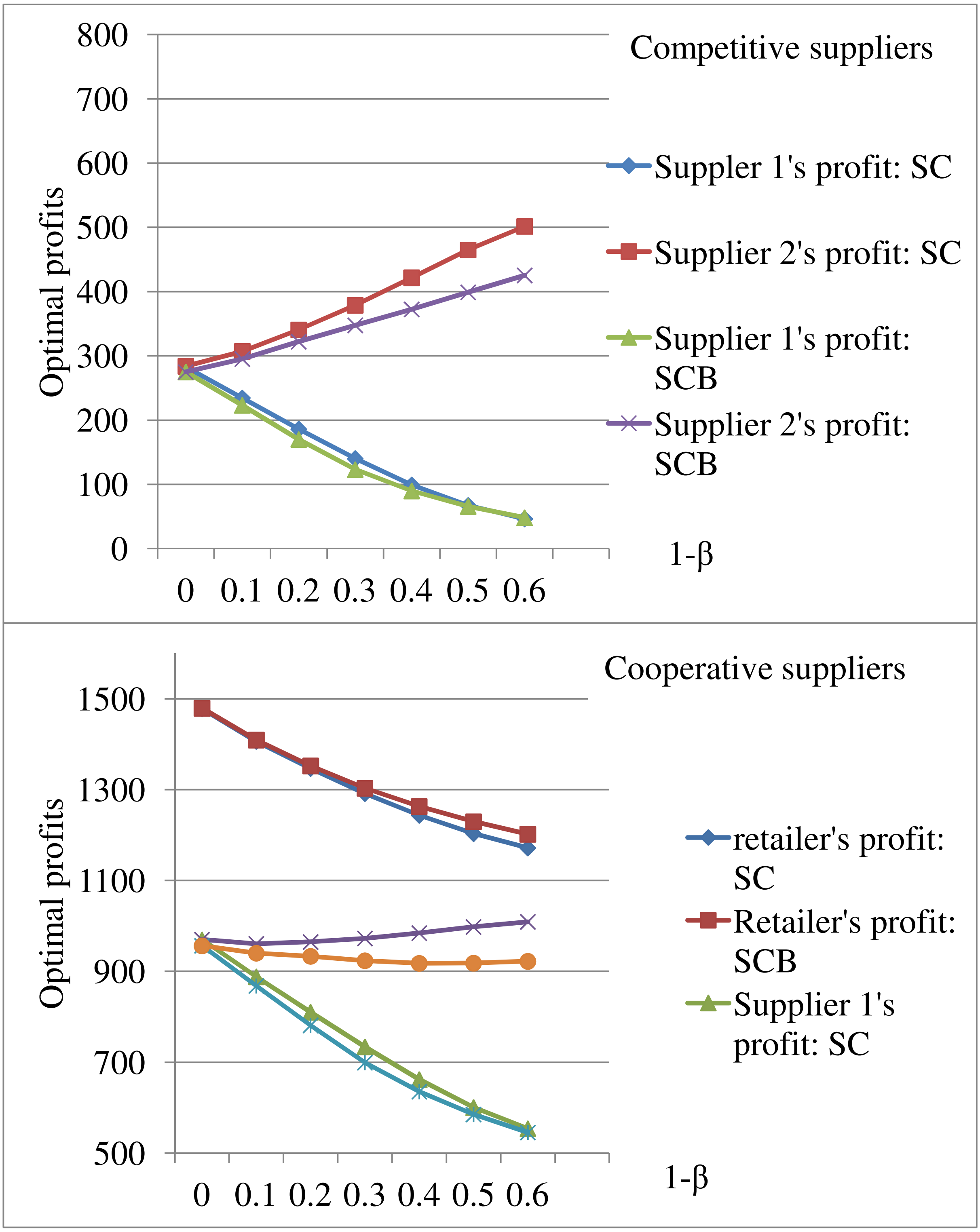

Figure 8: Variation of retailer's and suppliers' profits of SC model and SCB model with respect to supplier specific failure probability under WP contract

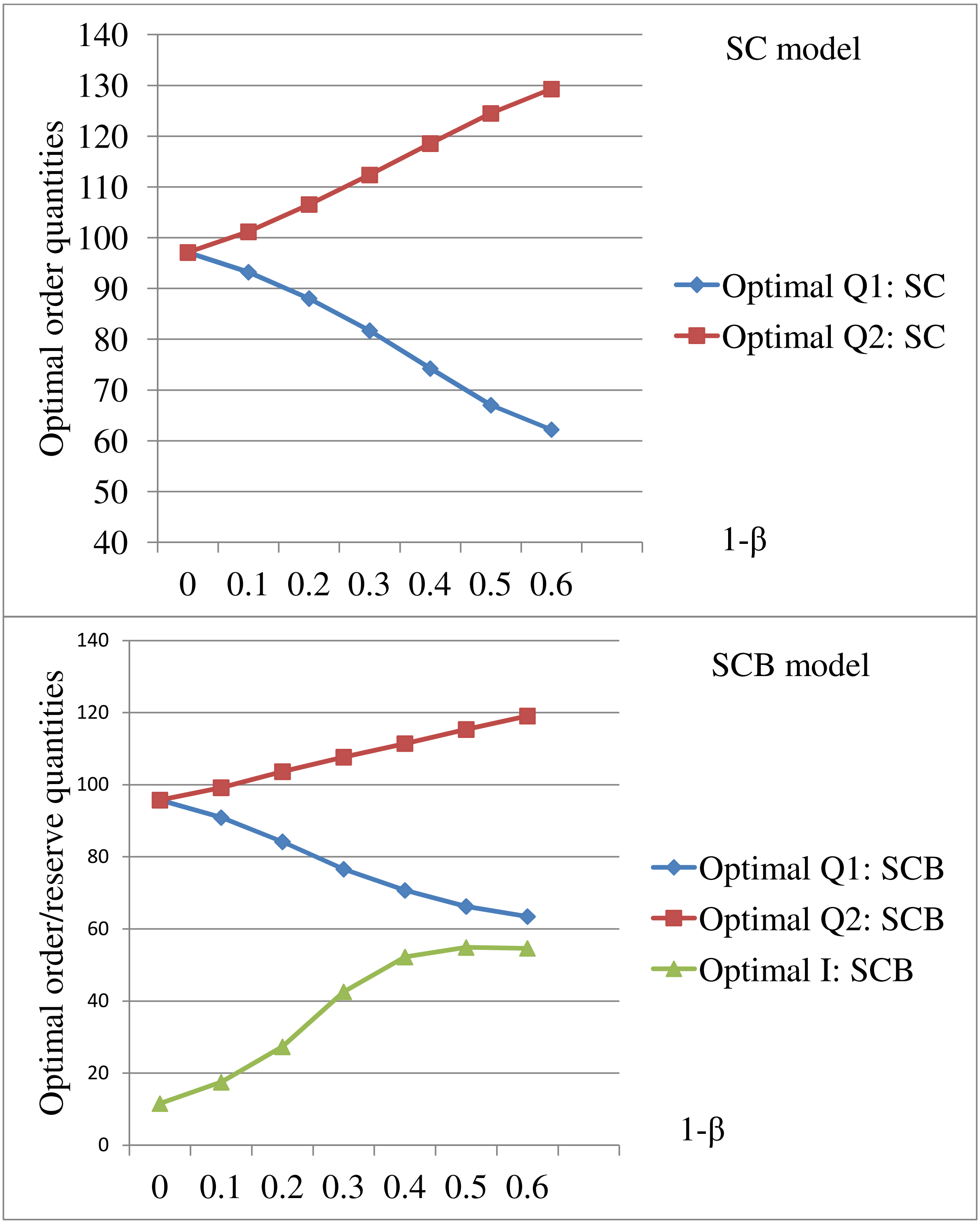

Figure 9: Variation of retailer's order quantities of SC model and SCB model with respect to supplier-specific failure probability under WP contract for competitive primary suppliers

Appendix A: Proof of Proposition 1

We will prove the result by establishing the concavity property of the retailer's expected profit function with respect to its decision variables. Again, to establish the concavity property, it is sufficient to show that the principal minors of the hessian matrix of that expected profit, are alternatively negative, positive and negative in order. If we denote these principal minors by $D_1(z_1, z_2, P)$, $D_2(z_1, z_2, P)$ and $D_3(z_1, z_2, P)$, then we just have to show that $D_1(z_1, z_2, P) < 0$, $D_2(z_1, z_2, P) > 0$ and $D_3(z_1, z_2, P) < 0$. Partially differentiating twice the objective function (2) of the retailer with respect to z_1, z_2 and P in turn we get

$$\begin{aligned}
\frac{\partial^2 \Pi_{dR}^{WP}}{\partial z_1^2} &= -\alpha\beta(P + c_s - c_v) f\Big(d(P) + z_1 + z_2\Big), \\
\frac{\partial^2 \Pi_{dR}^{WP}}{\partial z_2^2} &= -\alpha\beta(P + c_s - c_v) f\Big(d(P) + z_1 + z_2\Big) - \alpha(1-\beta)(P + c_s - c_v) f(z_2), \\
\frac{\partial^2 \Pi_{dR}^{WP}}{\partial P^2} &= -\alpha\beta\Big[2b\Big(1 - F\big(d(P) + z_1 + z_2\big)\Big) + 2b + b^2(P + c_s - c_v) f\Big(d(p) + z_1 + z_2\Big)\Big] - 2b\alpha(1-\beta).
\end{aligned}$$

Hence, $D_1(z_1, z_2, P) < 0$. Again, $\frac{\partial^2 \Pi_{dR}^{WP}}{\partial z_1 \partial z_2} = \frac{\partial^2 \Pi_{dR}^{WP}}{\partial z_2 \partial z_1} = -\alpha\beta(P + c_s - c_v) f\Big(d(P) + z_1 + z_2\Big)$. Thus, $D_2(z_1, z_2, P) = \Big(\frac{\partial^2 \Pi_{dR}^{WP}}{\partial z_1^2}\Big)\Big(\frac{\partial^2 \Pi_{dR}^{WP}}{\partial z_2^2}\Big) - \Big(\frac{\partial^2 \Pi_{dR}^{WP}}{\partial z_1 \partial z_2}\Big)^2 = \alpha^2\beta(P + c_s - c_v)^2 f(z_2) f\Big(d(P) + z_1 = z_2\Big)$ which implies that $D_2(z_1, z_2, P) > 0$. Now $D_3(z_1, z_2, P)$ can be written as

$$\begin{aligned}
D_3(z_1, z_2, P) &= \frac{\partial^2 \Pi_{dR}^{WP}}{\partial z_1^2}\Big[\frac{\partial^2 \Pi_{dR}^{WP}}{\partial z_2^2}\frac{\partial^2 \Pi_{dR}^{WP}}{\partial P^2} - \Big(\frac{\partial^2 \Pi_{dR}^{WP}}{\partial z_2 \partial P}\Big)^2\Big] + \frac{\partial^2 \Pi_{dR}^{WP}}{\partial z_1 \partial z_2}\Big[\frac{\partial^2 \Pi_{dR}^{WP}}{\partial z_2 \partial P}\frac{\partial^2 \Pi_{dR}^{WP}}{\partial P \partial z_1} \\
&\quad - \frac{\partial^2 \Pi_{dR}^{WP}}{\partial z_2 \partial z_1}\frac{\partial^2 \Pi_{dR}^{WP}}{\partial P^2}\Big] + \frac{\partial^2 \Pi_{dR}^{WP}}{\partial z_1 \partial P}\Big[\frac{\partial^2 \Pi_{dR}^{WP}}{\partial z_2 \partial z_1}\frac{\partial^2 \Pi_{dR}^{WP}}{\partial P \partial z_2} - \frac{\partial^2 \Pi_{dR}^{WP}}{\partial z_2^2}\frac{\partial^2 \Pi_{dR}^{WP}}{\partial P \partial z_1}\Big].
\end{aligned}$$

Now after differentiating partially we get the following results:
$\frac{\partial^2 \Pi_{dR}^{WP}}{\partial z_1 \partial P} = \frac{\partial^2 \Pi_{dR}^{WP}}{\partial P \partial z_1} = \alpha\beta\Big[1 - F\Big(d(P) + z_1 + z_2\Big) + b(P + c_s - c_v) f\Big(d(P) + z_1 + z_2\Big)\Big]$,
$\frac{\partial^2 \Pi_{dR}^{WP}}{\partial z_2 \partial P} = \frac{\partial^2 \Pi_{dR}^{WP}}{\partial P \partial z_2} = \alpha\beta\Big[1 - F\Big(d(P) + z_1 + z_2\Big) + b(P + c_s - c_v) f\Big(d(P) + z_1 + z_2\Big)\Big] + \alpha(1-\beta)\Big(1 - F(z_2)\Big)$.
Substituting the values of the partial derivatives and after some simplifications we get $D_3(z_1, z_2, P)$ as

$$\begin{aligned}
D_3(z_1, z_2, P) &= -\alpha^3\beta^2(1-\beta)(P + c_s - c_v) f(z_2)\Big[2b(P + c_s - c_v) f\Big(d(P) + z_1 + z_2\Big) - \overline{F}^2\Big(d(P) + z_1 + z_2\Big)\Big] \\
&\quad -\alpha^3\beta^2(1-\beta)(P + c_s - c_v) f(z_2)\Big[2b(P + c_s - c_v) f\Big(z_2\Big) - \overline{F}^2\Big(z_2\Big)\Big] \\
&< 0, \quad \text{under assumption } \mathbf{A1}.
\end{aligned}$$

Thus, we see that under assumption **A1** principal minors $D_1(z_1, z_2, P) < 0$, $D_2(z_1, z_2, P) > 0$ and $D_3(z_1, z_2, P) < 0$, which establishes the concavity property of the retailer's expected profit function with respect to her decision variables jointly. This completes the proof.

Appendix B: Proof of Proposition 2

If $\Pi_{dS_1}^{WPRS}$ and $\Pi_{dS_2}^{WPRS}$ denote Supplier 1's and Supplier 2's respective profits in Retailer-Stackelberg game under WP contract, then for given retail margin and safety stock, those profits can be rewritten as

$$\begin{aligned}
\Pi_{dS_1}^{WPRS}(w_1/z_1, m_1) &= \Big[\alpha\beta w_1 + (1-\alpha\beta)c_{v_1} - (\alpha\beta - \alpha\beta\eta\gamma + \eta\gamma)c_1\Big]\Big[a - b(m_1 + w_1) + z_1\Big], \\
\Pi_{dS_2}^{WPRS}(w_2/z_2, m_2) &= \Big[\alpha w_2 + (1-\alpha)c_{v_2} - (\alpha - \alpha\eta\gamma + \eta\gamma)c_2\Big]\Big[a - b(m_2 + w_2) + z_2\Big].
\end{aligned}$$

Since suppliers' profit functions are continuous and twice differentiable, to establish the existence of the Nash equilibrium, we proceed in the following way. The second order Jacobian matrix of the two suppliers are given by

$$J = \begin{bmatrix} \frac{\partial^2 \Pi_{dS_1}^{WPRS}}{\partial w_1^2} & \frac{\partial^2 \Pi_{dS_1}^{WPRS}}{\partial w_1 \partial w_2} \\ \frac{\partial^2 \Pi_{dS_2}^{WPRS}}{\partial w_2 \partial w_1} & \frac{\partial^2 \Pi_{dS_2}^{WPRS}}{\partial w_2^2} \end{bmatrix} = \begin{bmatrix} -2\alpha\beta b & 0 \\ 0 & -2\alpha b \end{bmatrix}$$

which is negative definite (Choi, 1991). This assures that the solutions given by (5) and (6) are Nash equilibrium between two suppliers. This completes the proof.

Appendix C: Proof of Proposition 3

In order to prove the existence of Stackelberg game where retailer is the Stackelberg leader, it is sufficient to show that the expected profit $\Pi_{dR}^{WPRS}\Big(z_1, z_2, P/w_1(z_1, P), w_2(z_2, P)\Big)$ of the retailer is quasiconcave with respect to z_1, z_2, P. Let $D_i(z_1, z_2, P),\ (i = 1, 2, 3)$ denote the principal minors of the hessian matrix of the above profit function. Now, the necessary and sufficient conditions for the quasi-concavity are $D_1(z_1, z_2, P) \leq,\ D_2(z_1, z_2, P) \geq 0,\ D_3(z_1, z_2, P) \leq 0$ and $D_1(z_1, z_2, P) < 0,\ D_2(z_1, z_2, P) > 0,\ D_3(z_1, z_2, P) < 0$ respectively. Proceeding in a similar way as described in Appendix A we get

$$D_1(z_1, z_2, P) = -\alpha\beta\Big[\frac{2}{b} + (P + c_s - c_v)f\big(d(P) + z_1 + z_2\big)\Big] < 0,$$

$$\begin{aligned} D_2(z_1, z_2, P) &= \frac{2\alpha\beta}{b}\Big[\alpha\beta\Big\{\frac{2}{b} + (P + c_s - c_v)f\big(d(P) + z_1 + z_2\big)\Big\} + \alpha(1-\beta)\Big\{\frac{2}{b} + (P + c_s - c_v)f\big(z_2\big)\Big\}\Big] \\ &\quad + \alpha\beta\Big[(P + c_s - c_v)f\Big(d(P) + z_1 + z_2\Big)\Big]\Big[\frac{2\alpha\beta}{b} + \alpha(1-\beta)\Big\{\frac{2}{b} + (P + c_s - c_v)f\big(z_2\big)\Big\}\Big] \\ &> 0 \end{aligned}$$

and after some simplification we get D_3 as

$$\begin{aligned} D_3(z_1, z_2, P) &= -2\alpha^3\beta^3\Big\{\frac{2}{b} + 2(P + c_s - c_v)f\big(d(P) + z_1 + z_2\big)\Big\}\overline{F}\big(d(P) + z_1 + z_2\big) - \alpha^3\beta^2(1-\beta)b(P + c_s - c_v)f\big(d(P) + z_1 + z_2\big) \\ &\quad \times\overline{F}\big(d(P) + z_1 + z_2\big)\Big\{\frac{2}{b} + (P + c_s - c_v)f\big(z_2\big)\Big\} - 4\alpha^3\beta(\beta^2 + 1)(P + c_s - c_v)f\Big(d(P) + z_1 + z_2\Big) \\ &\quad -4\alpha^3\beta(3\beta - 1)\Big\{\frac{2}{b} + (P + c_s - c_v)f\big(d(P) + z_1 + z_2\big)\Big\} - \alpha^3\beta(1-\beta)\Big\{\frac{2}{b} + (P + c_s - c_v)f\big(z_2\big)\Big\} \\ &\quad \times\Big[2b(P + c_s - c_v)f\big(d(P) + z_1 + z_2\big) - \beta\overline{F}^2\big(d(P) + z_1 + z_2\big)\Big] - 4\alpha^3\beta^3(P + c_s - c_v)f\big(d(P) + z_1 + z_2\big) \\ &\quad \times\Big[2 + \overline{F}\big(d(P) + z_1 + z_2\big) + b(P + c_s - c_v)f\big(d(P) + z_1 + z_2\big)\Big] - \frac{2\alpha^3\beta(1-\beta)}{b}\Big[2b(P + c_s - c_v)f(z_2) - (1-\beta)\overline{F}^2(z_2\Big] \\ &\quad -4\alpha^3\beta(1-\beta)(P + c_s - c_v)f(z_2) - \alpha^3\beta^3(1-\beta)b(P + c_s - c_v)f\big(d(P) + z_1 + z_2\big)\Big\{\frac{2}{b} + (P + c_s - c_v)f(z_2)\Big\} \\ &\quad -\frac{8\alpha^3\beta(1-\beta)}{b}\Big[\big(1 + F(z_2)\big) + \beta\big(1 - F(z_2)\big)\Big] - \alpha^3\beta(1-\beta)^2(P + c_s - c_v)f\big(d(P) + z_1 + z_2\big)\Big[2b(P + c_s - c_v)f(z_2) - \overline{F}^2(z_2)\Big] \\ &\quad < 0 \ \text{ under assumption } \mathbf{A1} \text{ and for } \frac{1}{3} \leq \beta \leq 1. \end{aligned}$$

Hence, $\Pi_{dR_S}^{WP}(z_1, z_2, P)$ is quasiconcave function with respect to z_1, z_2, P. Thus there exists a Stackelberg game where retailer is the Stackelberg leader. This completes the proof.

Appendix D: Proof of Proposition 4

Differentiating retailer's expected profit Π_{dbR}^{WP} with respect to I for known z_1, z_2, P we get

$$\frac{d\Pi_{dbR}^{WP}}{dI} = (P + c_s - w_3 - c_r) - (P + c_s - w_3)\Big[\alpha\beta F\Big(d(P) + z_1 + z_2 + I\Big) + \alpha(1-\beta)F(z_2 + I) + (1-\alpha)F\Big(I - d(P)\Big)\Big].$$

Differentiating it again with respect to I we get

$$\begin{aligned} \frac{d^2\Pi_{dbR}^{WP}}{dI^2} &= -(P + c_s - w_3)\Big[\alpha\beta f\Big(d(P) + z_1 + z_2 + I\Big) + \alpha(1-\beta)f(z_2 + I) + (1-\alpha)f\Big(I - d(P)\Big)\Big] \\ &< 0. \end{aligned}$$

Hence, for known z_1, z_2, P the profit function Π_{dbR}^{WP} is strictly concave with respect to I. Again, when both primary suppliers are perfectly reliable then $\alpha = 1,\ \beta = 1$. Then

$$\frac{d\Pi_{dbR}^{WP}}{dI} = (P + c_s - w_3 - c_r) - (P + c_s - w_3)\Big[\alpha\beta F\Big(d(P) + z_1 + z_2 + I\Big)\Big].$$

Now from the first order optimality condition we get $I^* = F^{-1}\Big(\frac{P+c_s-w_3-c_r}{P+c_s-w_3}\Big) - \Big(d(P) + z_1 + z_2\Big)$. Now if $F\Big(d(P) + z_1 + z_2\Big) > \frac{P+c_s-w_3-c_r}{P+c_s-w_3}$ *i.e.*, if $F^{-1}\Big(\frac{P+c_s-w_3-c_r}{P+c_s-w_3}\Big) - \Big(d(P) + z_1 + z_2\Big) < 0$ the profit function (24) reaches its maximum value at $I < 0$. So within $[0, d(P,\epsilon)$, the function Π_{dbR}^{WP} decreases with I. Therefore if $(P + c_s - w_3 - c_r) < F\Big(d(P) + z_1 + z_2\Big)(P + c_s - w_3)$ then $I^* = 0$. This completes the proof.

Appendix E: Proof of Proposition 5

Case (a): When there is no common cause supply disruption then $\alpha = 1$ and $0 < \beta < 1$. Then proceeding in similar way as mentioned in the Appendix D we get optimal reserve quantity under uniform distribution in $[A, B]$ as

$$I^* = \beta\Big(d(P) + z_1\Big) + (B - z_2) - \frac{(B - A)(P + c_s - w_3 - c_r)}{(P + c_s - w_3)}.$$

Now $I^* \leq 0$ implies $\beta < 0$, which contradicts our assumption that $0 < \beta < 1$. Hence, I^* must be positive always for this case. Moreover, this I^* increases with the increase of β, since $\frac{\partial I^*}{\partial \beta} = \Big(d(P) + z_1\Big) > 0$.

Case (b) When there is no supplier-specific disruption, then $\beta = 1$ and $0 < \alpha < 1$. For this case

$$I^* = \alpha\Big(2d(P) + z_1 + z_2\Big) + \Big(B + d(P)\Big)) - \frac{(B - A)(P + c_s - w_3 - c_r)}{(P + c_s - w_3)}.$$

Now if $I^* \leq 0$ then $\alpha < 0$, which contradicts our assumption that $0 < \alpha < 1$. Hence, I^* must always be positive for this market scenario. Again, $\frac{\partial I^*}{\partial \alpha} = \Big(2d(P) + z_1 + z_2\Big) > 0$ implies this I^* is increasing with α. This completes the proof of the proposition.

Appendix F:

The expected profit of the retailer under modified buyback contract for Model 1 is

$$\begin{aligned}
\Pi_{dR}^{BB}(z_{b_1}, z_{b_2}, P_b/w_{b_i}, c_{b_i}) &= \alpha\beta\Big[P\Big(d(P_b) + \mu\Big) - w_{b_1}\Big(d(P_b) + z_{b_1} + q_1\Big) - w_{b_2}\Big(d(P_b) + z_{b_2} + q_2\Big) \\
&\quad + c_b \int_A^{d(P_b)+z_{b_1}+z_{b_2}+q_1+q_2} F(\epsilon)d\epsilon - (P_b + c_s)\Big\{\Big(\mu - (d(P_b) + z_{b_1} + z_{b_2} + q_1 + q_2)\Big) \\
&\quad + \int_A^{d(P_b)+z_{b_1}+z_{b_2}+q_1+q_2} F(\epsilon)d\epsilon\Big\}\Big] + \alpha(1-\beta)\Big[(P_b - w_{b_2})\Big(d(P_b) + \mu\Big) \\
&\quad - (w_{b_2} - c_{b_2})\int_A^{z_{b_2}+q_2} F(\epsilon)d\epsilon - (P_b + c_s - w_{b_2})\Big\{\Big(\mu - (z_{b_2} + q_2)\Big) + \int_A^{z_{b_2}+q_2} F(\epsilon)d\epsilon\Big\} \\
&\quad - (1-\eta)\gamma c_1\Big(d(P_b) + z_{b_1} + q_1\Big)\Big] + (1-\alpha)\Big[-c_s\Big(d(P_b) + \mu\Big) \\
&\quad - (1-\eta)\gamma c_1\big(d(P_b) + z_{b_1} + q_1\big) - (1-\eta)\gamma c_2\big(d(P_b) + z_{b_2} + q_2\big)\Big], \\
\text{where } E(\epsilon) &= \mu \text{ and } c_b = c_{b_1} + c_{b_2}.
\end{aligned}$$